THE EVALUATION OF ACQUISITION PROBABILITY IN IMAGE MATCHING[1]

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ABSTRACT:

In modern navigation and guidance systems, image matching is often used as an efficient approach to increase the registration accuracy. Acquisition probability of image matching is one of the most important parameters in registration accuracy analysis of image matching. It represents the correctness of the position estimated by the navigation and guidance system with respect to the real position in flight. For example, in missile homing guidance, it is the probability of hitting a target. So, it is the main basis for designing a navigation and guidance system. In image matching, Mean Absolute Difference (MAD) is one of the most often used algorithms. It has a lot of advantages such as high registration accuracy, high noise robustness and can be easily realized by hardware etc. In this paper, first, the acquisition probability for the MAD algorithm is derived based on the image pixel-correlation model. Then, in order to evaluate the value of acquisition probability for the MAD algorithm, an approximation formula is given. Finally, the experiments with different optical aerial photographs and infrared remoted sensing photographs have been conducted on a IBM-PC microcomputer system and a S575 image processing system . By the experimental comparion to the evaluation of Johnson it is demonstrated that the evaluation of acquisition probability for the MAD algorithm proposed in this paper is more accurate and close to the real acquisition probability.

Key Words : accuracy, image matching, navigation and registration.

1. DEFINITION

1.1 Image Matching Point

Suppose S and R represent sensed image ($m \times n$) and reference image ($M \times N$) respectively . The purpose of image matching is to determine the position (i_0, j_0) where the reference subimage is most similar to the sensed image S by translating the reference subimage $R_{i,j}$ in the searching area G, as shown in Fig. 1. The position (i_0, j_0) is called matching point between image S and image R. For the MAD algorithm, the similarity between two images is measured with the mean absolute difference function $f(i, j)$:

$$f(i,j) = \frac{1}{mn}\sum_{k=1}^{m}\sum_{g=1}^{n} | R(i+k-1, j+g-1) - S(k,g) | \quad (1)$$

where $\quad 0 \leq i \leq M - n + 1; 0 \leq j \leq N + n - 1$

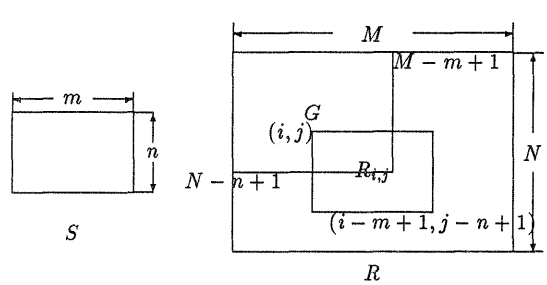

Fig. 1 The matching area of reference image

The smaller the MAD value, the more similar the two images. Therefore, for the MAD algorithm, the image matching point is the minimum of $f(i, j)$; it can be mathmatically expressed as:

$$f(i_0, j_0) = \min_{(i,j)\in G} f(i,j)$$

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1.2 Acquisition Probability

Suppose the correct matching point is (i^*, j^*). In practical systems, often it is the target point. So, it will be called 'target point' in this paper. A good image matching system should make the matching point as close as possible to the target point. But because of the influence of noise, they generally do not coincide even if there exist no geometrical distortions between the two images. Therefore, we define acquisition probability P_a as the probability where the image matching point coincides with the target point. For the MAD algorithm, we have

$$P_a = P\{f(i,j) > f(i^*,j^*), \forall(i,j) \in G, (i \neq i^*) \cap (j \neq j^*)\} \quad (2)$$

2. EVALUATION OF ACQUISITION PROBABILITY

2.1 Johnson's Evaluation

Different image correlation models result in different evaluation approaches and results. In [2], based on the image model of pixel-independency, the following evaluation of acquisition probability for the MAD algorithm is derived:

$$P_a = \frac{1}{\sqrt{2\pi}\sigma_0} \int_{-\infty}^{+\infty} exp\{-\frac{(x-\overline{x}_0)^2}{2\sigma_0^2}\} P_{ij}^K(x) dx \quad (3)$$

and

$$P_{ij}(x) = \frac{1}{\sqrt{2\pi}\sigma_1} \int_{x+\overline{x}_0}^{+\infty} exp\{-\frac{(x_1-\overline{x}_1)^2}{2\sigma_1^2}\} dx_1 \quad (4)$$

where $K = N_{G-1}$, N_G is the number of all points in the searching area G; $\overline{x}_0$, σ_0^2 represent the mean and variance of $f(i^*,j^*) = x'$ respectively (see [1]); similarly, $\overline{x}_1$, σ_1^2 represent the mean and variance in mismatching point(i,j) and $f(i,j) = x_1$, $x = x' - \overline{x}_0$.
(3) also can be expressed as

$$\begin{aligned} P_a &= \frac{1}{\sqrt{2\pi}\sigma_0} \int_{-\infty}^{+\infty} exp\{-\frac{(x-\overline{x}_0)^2}{2\sigma_0^2}\} \times \\ & P\{f(i,j) > x', \forall(i,j) \in G, (i \neq i^*) \cap (j \neq j^*)\} dx \end{aligned} \quad (5)$$

In contrast to [2], we use the expression $f(i,j) > x'$ instead of $f(i,j) \geq x'$ in (5). In case of equality, at least two minimum points in the searching area G appear and the matching point can not be determined.

2.2 The Acquisition Probability Based on the Pixel-Correlation Model

In real images, neighbour pixels generally correlated[4]. Therefore, the evaluation of acquisition probability by (3) is not accurate. In order to get a more accurate evaluation, an image correlation model must be used. In [1], based on the image correlation model proposed in Reference [4], the probability density distribution function in a single-valley area containing the minimum point has been given. Now, we will give the probability density distribution function in the whole searching area G. In order to simplify the analysis, we transfer the 2-D searching area into a 1-D sequence by scanning. So, we have

$$f(g) = f(i,j) \quad (6)$$

here $g = (i-1)m' + j$; (i,j) is a mismatching point in the searching area$(m' \times n')$.

For every $f(g)$, it is satisified with the following Gauss distribution:

$$f(g) \sim N(\overline{x}_g, \sigma_g^2) \quad (7)$$

where

$$\overline{x}_g = \sqrt{\frac{2}{\pi}[2\sigma_R^2(1-exp\{-\frac{|i_g-i^*|}{\lambda_x} - \frac{|j_g-j^*|}{\lambda_y}\}) + \sigma_n^2]} \quad (8)$$

$$\sigma_g^2 = (1-\frac{2}{\pi})[2\sigma_R^2(1-exp\{-\frac{|i_g-i^*|}{\lambda_x} - \frac{|j_g-j^*|}{\lambda_y}\}) + \sigma_n^2] \quad (9)$$

For random sequence $\{f(g)\}(g = 1, 2, \ldots, N_{G-1})$, suppose their joint probability distribution is the N_{G-1}-dimensional joint Gauss distribution

$$P(y_1, y_2, \ldots, y_K) = \frac{1}{2\pi^{K/2}(det\sum)^{(1/2)}} exp\{-\frac{1}{2}(Y-\overline{Y})^T \sum^{-1}(Y-\overline{Y})\}$$

with

$$\begin{cases} K = & N_{G-1} \\ Y = & (y_1, y_2, \ldots, y_K)^T \\ \overline{Y} = & (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_K)^T \\ \sum = & \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \ldots & \sigma_{1K}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \ldots & \sigma_{2K}^2 \\ \ldots & & & \\ \sigma_{K1}^2 & \sigma_{K2}^2 & \ldots & \sigma_{KK}^2 \end{bmatrix} \end{cases} \quad (10)$$

σ_{kg}^2 are the covariances of $f(k)$ and $f(g)$:

$$\sigma_{kg}^2 = \sigma_k \sigma_g \quad (11)$$

$$1 \leq k, g \leq N_{G-1}$$

So we have

$$\begin{aligned} & P\{f(i,j) > x', \forall(i,j) \in G, (i \neq i^*) \cap (j \neq j^*)\} = \\ & \int_{x+\overline{x}_0}^{+\infty} \int_{x+\overline{x}_0}^{+\infty} \cdots \int_{x+\overline{x}_0}^{+\infty} p\{y_1, y_2, \ldots, y_K\} dy_1 dy_2 \ldots dy_K = \\ & \int_W^{+\infty} \frac{1}{(2\pi)^{K/2}(det\sum)^{1/2}} exp\{-\frac{1}{2}(Y-\overline{Y})^T \sum^{-1}(Y-\overline{Y})\} dY \end{aligned} \quad (12)$$

with $W = (w, w, \ldots, w)$, $w = x + \overline{x}_0 = x'$
According to (5), the acquisition probability for the MAD algorithm is:

$$\begin{aligned} P_a = & \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_0} exp\{-\frac{(x-\overline{x}_0)^2}{2\sigma_0^2}\} \times \\ & \int_W^{+\infty} \frac{1}{(2\pi)^{K/2}(det\sum)^{1/2}} \times \\ & exp\{-\frac{1}{2}(Y-\overline{Y})^T \sum^{-1}(Y-\overline{Y})\} dY dx \end{aligned} \quad (13)$$

$W, Y, \overline{Y}$ and $\sum$ can be determined by (8) $\sim$ (11) when the reference image's variance σ_R^2, the signal to noise ratio SNR, the correlation length λ_x, λ_y and m, n, M, N are known. So, the acquisition probability for the MAD algorithm can be calculated from (13).

2.3 The Evaluation of Acquisition Probability for the MAD Algorithm

In image matching, usually N_G is very big. For example, when the reference image is of size 64×64 and the sensed image is of size 32x32, $N_G - 1 = (64-32+1)(64-32+1) = 1088$.

So, according to (13), a 1089-dimensional numerical integration must be calculated in order to evaluate acquisition probability P_a. Obviously this is impossible. Therefore, we must simplify (13) and get its approximation. More than 10,000 experiments with different images and with different SNR illustrated that the city-block distance between the matching point(i_0, j_0) and the target point(i^*, j^*) does not exceed 1 when there are no geometrical distortions between the reference and the sensed images and the sensed image is not very small (for example more than 1000 pixels). That is, the following conclusion is tenable when (i, j) is satisfied with condition $\mid i - i^* \mid + \mid j - j^* \mid > 1$:

$$P\{f(i,j) > f(i^*, j^*)\} = 1 \tag{14}$$

So

$$\begin{aligned} & P\{f(i,j) > x', \forall(i,j) \in G, (i \neq i^*) \cap (j \neq j^*)\} \\ = \quad & P\{f(i,j) > x', \mid i - i^* \mid + \mid j - j^* \mid = 1\} \end{aligned} \tag{15}$$

In the searching area G, there are only four points which satisfy the condition $\mid i - i^* \mid + \mid j - j^* \mid = 1$; they are $(i^* - 1, j^*), (i^*, j^* - 1), (i^* + 1, j^*)$ and $(i^*, j^* + 1)$.
Let $y_1 = f(i^* - 1, j^*), y_2 = f(i^*, j^* - 1), y_3 = f(i^* + 1, j^*)$, $y_4 = f(i^*, j^* + 1)$ and suppose $\lambda_x = \lambda_y = \lambda$, then

$$\begin{aligned} & E\{y_1\} = E\{y_2\} = E\{y_3\} = E\{y_4\} \\ = & \sqrt{\frac{2}{\pi}[2\sigma_R^2(1 - e^{-1/\lambda}) + \sigma_n^2]} = \overline{y}_f \end{aligned} \tag{16}$$

$$\overline{Y} = (\overline{y}_f, \overline{y}_f, \overline{y}_f, \overline{y}_f)$$
$$Y = (y_1, y_2, y_3, y_4)$$

Because

$$\begin{aligned} & \sigma_{11}^2 = \sigma_{22}^2 = \sigma_{33}^2 = \sigma_{44}^2 \\ = \quad & (1 - \tfrac{2}{\pi})[2\sigma_R^2(1 - e^{-1/\lambda}) + \sigma_n^2] = \sigma_f^2 \end{aligned} \tag{17}$$

$$\begin{aligned} & \sigma_{13}^2 = \sigma_{24}^2 = \sigma_{12}^2 = \sigma_{23}^2 = \sigma_{34}^2 = \sigma_{14}^2 = \sigma_h^2 \\ = \quad & (1 - \tfrac{2}{\pi})[2\sigma_R^2(1 - e^{-2/\lambda}) + \sigma_n^2] \end{aligned} \tag{18}$$

So

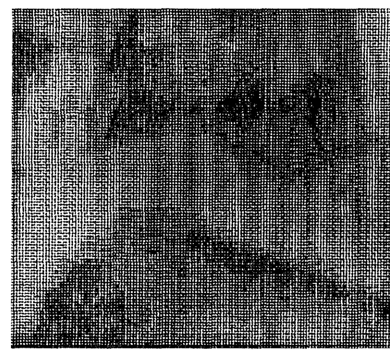
$$\sum = \begin{bmatrix} \sigma_f^2 & \sigma_h^2 & \sigma_h^2 & \sigma_h^2 \\ \sigma_h^2 & \sigma_f^2 & \sigma_h^2 & \sigma_h^2 \\ \sigma_h^2 & \sigma_h^2 & \sigma_f^2 & \sigma_h^2 \\ \sigma_h^2 & \sigma_h^2 & \sigma_h^2 & \sigma_f^2 \end{bmatrix} \tag{19}$$

Now, with (16) $\sim$ (19), the numerical integration can be reduced from 1089-D to 5-D. Using a number theory net approach, the acquisition probability can be evaluated.

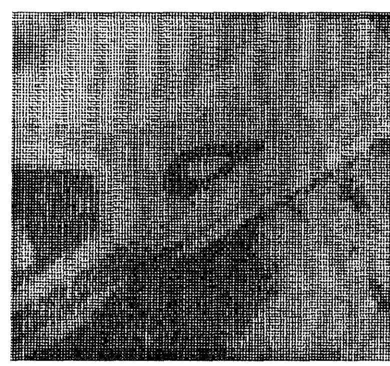
3. EXPERIMENTS

In order to compare the evaluation of the proposed approach with the approach of Johnson, the experiments with different optical aerial photographs and infrared remoted sensing photographs have been conducted. Part of the images which have been used in the experiments are shown in Fig. 2. In Fig. 2, (a)~(c) there are three images (64×64) which are produced from three different optical aerial photographs and quantized with 16 grey levels (0~ 15); (d) is a 128×128 image which comes from an infrared remoted sensing photograph quantized with 256 grey levels (0~ 255). The image matching experiments using optical aerial image as reference images were completed on a IBM-PC microcomputer system (with PC-EYE frame grabber). In each searching area of the reference images, we choose 100 points as target points. The correspondent sensed images with differnet SNR are simulated by means of the approach proposed in [3].

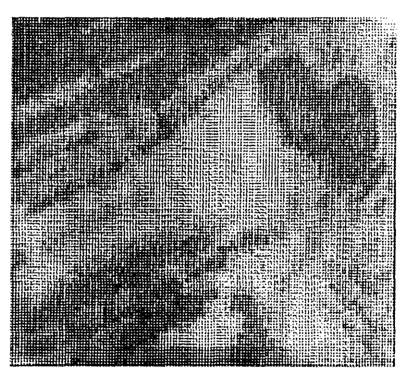
Similar simulation was done with infrared remoted sensing images on a S575 image processing system (host computer is VAX-11/730, equipped with Model75 array processor). Part of the experimental results are shown in Table 1. From Table 1, it can be seen that the evaluation value of the acquisition probability proposed by Johnson is much smaller than the real one. So, it is further confirmed that the evaluation based on the pixel-independent assumpation is not accurate enough; using the image correlation model, a more accurate evaluation of the acquisition probability can be obtained. It will provide a more reliable theoretical basis for the parameter design of a practical navigation and guidance system.



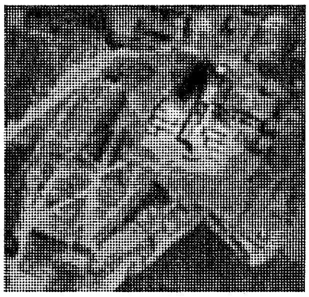
(a)



(b)



(c)



(d)
Fig. 2 The images used in the experiments

TABLE 1 CALCULATION OF ACQUISITION PROBABILITY FOR THE MAD ALGORITHM

Fig. 2		(a)	(b)	(c)	(d)
SNR=2	test value	1.000	1.000	1.000	1.000
	ours	0.938	0.955	0.968	0.938
	Johnson's	0.700	0.770	0.800	0.700
SNR=3	tset value	1.000	1.000	1.000	1.000
	ours	0.941	0.962	0.974	0.941
	Johnson's	0.750	0.800	0.860	0.750

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