HOUGH TRANFORM IN DIGITAL PHOTOGRAMMETRY

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ABSTRACT

Digital Photogrammetry requires nowadays a combination of a totally automated environment and high accuracy. Image processing, pattern recognition, and computer vision facilitate these requirements and they are used more and more frequently. Hough transform, a method for detecting predefined shapes, has been used for more than two decades in those areas.
The present paper proposes the use of Hough transform in Digital Photogrammetry and in particular it concentrates on the development of a target detection technique based on the principles of this powerful transform.

Key Words: Target Detection, Hough Transform, Digital Photogrammetry.

1. INTRODUCTION

Hough Transform (HT), a method for detecting predefined shapes, has been always recognized as a unique promising method for shape and motion analysis in images that contain noise, missing and extraneous data. But until recently HT was not applied extensively due to the fact that it was computationally expensive. During the last few years, great progress has been achieved in overcoming this disadvantage thus making the Hough Transform a powerful tool.

Hough Transform has been used quite frequently in the areas of image processing, pattern recognition, and computer vision. Some applications of the Hough Transform in these fields are reported to be:
A system for detecting and locating mechanical parts using generalized HT [Arbuschi, Cantoni and Musso, 1983]. Detection of straight edges of roads and tracks [Inigo et al., 1984]. Automatic recognition of Hebrew characters [Kushnir, Abe and Matsumoto, 1983]. Detection of faint lines in synthetic aperture Radar(SAR) images [Skingley and Rye, 1987]. Analysis of cleavage cracks in minerals [Thomson and Sokolowska, 1988]. Determination of motion parameters from a sequence of images [Ballard and Kimball, 1983] [Fennema and Thompson, 1979]. Detection and removal of linear variations for illumination across images [Nixon, 1985]. Detection of tumors in chest radiographs [Ballard and Sklansky, 1976]. In recent years, HT was generalized to detect arbitrary shapes [Ballard, 1981] and expanded to detect shapes in 3 dimensions [Hederson and Fai, 1984].

In this paper the Hough Transform is investigated from the point of view of the digital photogrammetry, and in particular a target detection method based on HT is fully described.

2. GRADIENT (EDGE) IMAGE

In order to facilitate the target recognition and at the same time reduce the amount of information presented, images are transformed from gray level to magnitude and direction of the gray level changes using edge operators. The most common edge operator is the gradient operator, and the resultant image is then referred to as gradient image.

If the image is described as a function $f(x,y)$, where x,y are space indicators and f gray value for the particular pixel of the image, then the gradient of f at (x,y) is the vector:

$$\overline{G}[f(x,y)] = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} Gx \\ Gy \end{bmatrix}$$

which points at the direction of the maximum rate of increase of the function $f(x,y)$ (intensity).

The magnitude of the gradient $\overline{G}$ is:

$$G[f(x,y)] = mag\overline{G} = [Gx^2 + Gy^2]^{1/2}$$

and the direction of the gradient vector:

$$\Phi(x,y) = \tan^{-1}(Gx/Gy)$$

Using a 3×3 mask which is referred as Sobel operator:

x_1 x_2 x_3	-1 -2 -1	-1 0 1
x_4 x_5 x_6	0 0 0	-2 0 2
x_7 x_8 x_9	1 2 1	-1 0 1

Gx and Gy for the center pixel of the mask in the above equations are given by:

$$Gx = (x_7 + 2*x_8 + x_9) - (x_1 + 2*x_2 + x_3)$$

$$Gy = (x_3 + 2*x_6 + x_9) - (x_1 + 2*x_4 + x_7)$$

Setting a threshold in the magnitude of the gradient, every pixel that exceeds this threshold is mapped as edge point, otherwise it is mapped as background (zero gray value). In this way the edge image is generated and each pixel is assigned with the magnitude and direction of the gradient vector which will be used in forthcoming tasks.

3. HOUGH TRANSFORM FOR DETECTING CURVES

The classical Hough Transform is a curve detection technique that can be applied if little is known about the location of a boundary but its shape can be described as a parametric curve [Ballard and Brown, 1982]. Furthermore, Hough Transform has been expanded to detect arbitrary shapes [Ballard, 1981].

The method is attractive because it is quite unaffected by gaps, partial occlusion and noise. Hough Transform is regarded as an intermediate level vision task since it converts an iconic

representation of an image to a symbolic one [Lutofo et al., 1989].

[Duba and Hart, 1972] introduced and modified the Hough Transform [Hough, 1962] for detection of straight lines. In this approach a line is described in the image space as:

$$x*\cos\theta + y*\sin\theta = \rho \qquad (1)$$

where parameter θ is the angle relative to the positive horizontal direction and ρ the normal distance from the origin (see figure 1). The range of ρ is $-\sqrt{2}*D$ to $\sqrt{2}*D$, where D is the size (diagonal) of the image space. The range of θ is between $0°$ and $180°$ degrees.
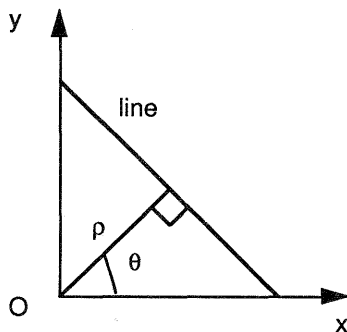

Fig. 1: Normal representation of a line.

Then a transformation from the image space (xy) to the parameter $(\rho\theta)$ space is carried out. Image points are mapped into sinusoidal curves in the parameter space. If for a number of image points, the corresponding sinusoidal curves in the parameter space intersect in a common point (ρ_K , θ_K), then these points lie on the straight line :

$$x*\cos\theta_K + y*\sin\theta_K = \rho_k$$

in the image.

The computational attractiveness of the Hough Transform arises from subdividing the parameter space into accumulator cells $A(\rho,\theta)$ (see figure 2).
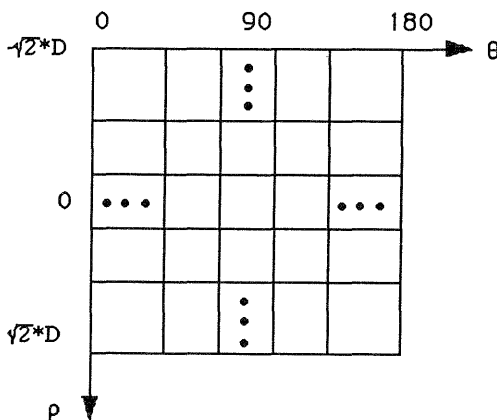

Fig. 2: Quantization of the parameter space into cells.

Initially accumulator cells are set to zero: $A(\rho,\theta) = 0$. Then for every point (x,y) in the gradient image the quantized M_θ values of θ are incremented by $\delta\theta$ and for each value of θ the corresponding ρ is calculated using equation (1). The calculated values of ρ are then quantized in M_ρ intervals of width $\delta\rho$ and the correspondent accumulator cells are incremented by one : $A(\rho,\theta) = A(\rho,\theta)+1$. The accumulator cells with the greater number of votes correspond to lines in image space.

As can be seen, the computational expense of the Hough Transform is $M_\theta*n$ computations of ρ, where n is the number of points (pixels) in the image. Using the gradient instead of the of the original image, the number of points in the image is reduced dramatically and the Hough Transform becomes computationally much cheaper.

In a noise-free image all points that belong to a line correspond to sinusoidal curves that intersect at a unique point (ρ_k,θ_k) in the parameter space. But images are not noise-free, so the point of intersection is not unique anymore and a number of intersections are distributed around the actual one (ρ_k,θ_k). It is clear now that local maxima in the parameter space describe the lines of the image. Working in this direction, [Harjoko, 1990] proposed the following procedure:
After every pixel of the image has been considered and the accumulator cells of the parameter space have been prepared, a cluster algorithm is performed in order to detect clusters in the parameter space. Each cluster is bounded using a boundary value threshold. The boundary value is chosen to be a fixed fraction of the peak value of the cluster. The standard deviation of the cluster is selected as the peakyness (local maximum) measure. For every bounded cluster the standard deviations σ_ρ, σ_θ and the means m_ρ, m_θ are calculated. The value in each cell is treated as the weight of the cell:

$$m_\rho = \frac{1}{s}*\Sigma\Sigma\ i*f(i,j)$$

$$m_\theta = \frac{1}{s}*\Sigma\Sigma\ j*f(i,j)$$

$$\sigma_\rho^2 = \frac{1}{s}*\Sigma\Sigma\ (i-m_\rho)^2*f(i,j)$$

$$\sigma_\theta^2 = \frac{1}{s}*\Sigma\Sigma\ (j-m_\theta)^2*f(i,j)$$

where $s = \Sigma\Sigma\ f(i,j)$, f(i,j) the value of the (i,j) cell,
and $i,j = -a$ to a , a is an integer part of the cluster size.

The mean values for each cluster are the wanted local maxima in the accumulator array and correspond to straight lines in the image. The last step is the transformation of the local maxima back to the image in order to get the analytical equation of the detected lines.

Hough transform cannot only detect straight lines, but it can also detect any predefined shape. But the order of computational expense increases with the complexity of the curve, and the Hough Transform tends to be computational expensive. A more efficient and computationally attractive Hough Transform for detecting analytical curves in images [Ballard, 1982] is making use of the edge pixel gradient directions as follows:
If f(x,a)=0 is an analytical curve, where x represents image points, and a the parameter vector, then

step1: Form an accumulator array A(a). Initialize it to zero.

step2: For each edge pixel x:

-Compute all **a** such that $f(x,a) = 0$ and $\frac{\partial f}{\partial x}(x,a) = 0$

-Increment the corresponding accumulator cell: $A(a) = A(a) + 1$

step3: After each edge pixel **x** has been considered, local maxima in the array A correspond to curves of f in the image.

The advantage of this method is clear if we take into account that the gradient direction for each edge pixel is already known from the proceeding edge detection procedure.

For example, the analytical form of the circle is given by:

$$f(x,y) = (x-a)^2+(y-b)^2 = r^2 \qquad (2)$$

where (a,b) represents the center of the circle and r the radius.

When we follow the edge pixels that lie on the circle the difference in gray level among the edge pixels is zero. That means that the magnitude of gradient is zero:

$$\frac{\partial f}{\partial x} = 0 \qquad \Rightarrow$$

$$\frac{\partial f}{\partial x} = 2*(x-a) + 2*(y-b)\frac{dy}{dx} = 0 \quad \Rightarrow \quad (x-a) + (y-b)\frac{dy}{dx} = 0$$

where $\frac{dy}{dx} = \tan(\Phi(x) - \frac{\pi}{2})$

and $\Phi(x)$ is the gradient direction as indicated from the edge operator.

Then:

$$(x-a) + (y-b)* \tan(\Phi(x) - \frac{\pi}{2}) = 0 \quad \Rightarrow$$

$$(x-a)*\cos(\Phi(x) - \frac{\pi}{2}) + (y-b)* \sin(\Phi(x) - \frac{\pi}{2}) = 0 \qquad (3)$$

Using the equations (2) and (3), it is obvious that the parameter locus is reduced from a cone to a line. It can be concluded that when using the gradient direction information, the number of free parameters is reduced by one.

The same algorithm can be used for all the analytical curves. For example an ellipse has 5 parameters. Using the gradient direction two of the parameters can be solved as a function of the other three. The computational effort in this case is $O(e*d^3)$, where e is the number of edge points and d the distinct values for each parameter.

Concluding the reference to the Hough Transform it has to be highlighted that this powerful tool not only detects a predefined shape, but it also gives its size, orientation, and center.

4. TARGET DETECTION USING HOUGH TRANSFORM

An important task in Digital Photogrammetry is the performance of highly accurate measurements of target positions on the digital image in order to perform a transformation between image coordinate and object coordinate systems. These measurements can be done either by operator or automatically. In the latter case there is no need for an operator but the target must be first detected and its center must be precisely located

using only the computer. Working in this direction [Trinder and Mikhail, 1982] introduced an algorithm for edge modeling based on least squares (edges,lines and cross targets), [Zhou, 1986] utilized an algorithm for locating ellipse centers based on the moment preserving method, [Trinder, 1988] proposed circles as targets utilizing again least squares, and [Mikhail, Akey and Mitchell, 1984] used Fourier descriptors and one dimensional moments for target location and recognition.

In the present paper the Hough Transform is utilized to detect and accurately measure the following target:
A white circle in a black background with four equally spaced diameters intersecting on its center (see figure 3).
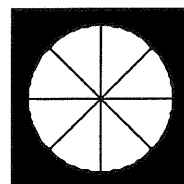

Fig. 3: Test target

A circle is the most appropriate shape for a target because the image of a circle can only be a circle or an ellipse depending on the observation direction (see figures 4 and 5).
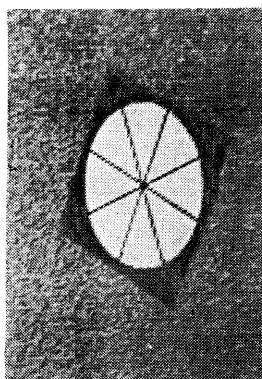

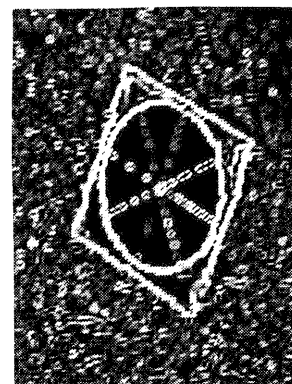Fig. 4: Digital image of the test target taken at an angle.

Fig. 5: Gradient (Edge) digital image of the test target of Fig.4

Hough Transform, as mentioned, cannot only detect circle and ellipse in an image but it can also directly give their centers which in the case of target detection is exactly what we are interested in.

An alternative solution for the target center determination is first to detect the circle-ellipse image of the target using the Hough Transform and then to mask the rest of the image outside of the target, making in this way the detection of the four lines inside the circle easier using again the Hough Transform. The point of intersection of these lines is the center of the target. [Harjoko, 1990] proposed the following algorithm for computing the intersection of lines that are detected using the Hough Transform:
N lines in the image space form a cluster of lines with a common point of intersection (x_t,y_t). These lines correspond to N associated clusters in parameter space. It has already been mentioned that the measurements of the peakness of these clusters result in a set of N pairs of standard deviations and means. These quantities are further used to compute the center of the target (x_t,y_t). The location of the target center is computed using least squares adjustment. For observations with different standard deviations each observation must be weighted

properly by introducing the weight :

$$w = \frac{k}{\sigma^2}$$

where σ is the standard deviation and k a proportionality constant.

Then the least squares criterion becomes:

$$\Phi = w_1*u_1+w_2*u_2+.......+w_N*u_N = min$$

where u_i are observation residuals ,

and the solution vector is given by :

$$X = (A^t*W*A)^{-1}*(A^t*W*F) \qquad (4)$$

The standard deviations of the (x_t, y_t) are the elements of :

$$(A^t*W*A)^{-1}$$

where A is a N by 1 matrix of the coefficients of the weights of the observations, W a diagonal weight matrix, and F an N by 1 matrix of constants indicating linearity of the condition equations.

Equation (4) is iterated until the desired accuracy is achieved. In any iteration the initial value of X was taken from the previous iteration.

Experiments have been carried out for the above described approach to evaluate the performance of the HT under various conditions such as: variation of angle of lines, length of line segments, number of lines in the target, noise contamination, and quantization of the parameter space. It is found that for a noisy image the best accuracy is given by four equally spaced lines with normal distance quantization equal to one, and angle quantization equal to one degree.

The main advantage of the proposed method for target detection and accurate measurement is that the whole procedure is totally automated. The target detection can be performed quickly and safely and then the center of the circular target can be found in the image either as the center of the corresponding ellipse-circle or as the intersection of four equally spaced diameters of the circle target.

The method works well even under the most noisy environment. No limitations have been observed. Practical experiments showed that the standard deviation of the target center position in repetitive image acquisitions with different conditions of illumination was in the subpixel level.

5. CONCLUSIONS

A target detection method based on the Hough Transform and utilizing a white circle with four diagonal black lines in a black background is proposed. The method provides automated and safe target detection even in the most inconvenient images (noisy,interrupted). It works in an gradient image and the accuracy of the target detection is of subpixel magnitude.

The purpose of the recent paper is to underline to the photogrammetrists the existence of a powerful tool named Hough Transform which in a digital image environment can offer a lot in the pattern recognition process. Except for the described target detection application of the Hough Transform, work is currently underway by the authors in applying HT in image matching techniques.

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