IMPORTANCE OF MODERN MATHEMATICAL METHODS FOR THE DIGITAL PHOTOGRAMMETRY

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ABSTRACT

From a mathematical point of view modern photogrammetric tasks are to define characteristic transformations in suitable function spaces. In such an uniform model both image space and feature space are realisations for the digital photogrammetry and the GIS–technology integrated also the cartography. Two examples – the scene correspondence and an abstract description of the map updating – are presented with the help of functional analytical means like measures, norms, and scalar products as tools of an abstract characterization.

Keys: theory, system integration, algorithm

1. INTRODUCTION

The state of the art in modern geoinformatics is characterized by the integration of classic disciplines like photogrammetry and cartography with remote sensing and GIS–techniques. The separate development in modelling must be integrated in one model (see FOERSTNER 1991).

A mathematical model demonstrating a functional analytical description in geoscience especially remote sensing, photogrammetry, and cartography in one calculus was developed (see PROSS 1990 a,b, 1991 a).

The disciplines induce several views on the same data and information processing with emphasis on
remote sensing – data collection
photogrammetry – information derivation and structuring
cartography – information storage and presentation.

In section 2 we will define the image and the feature space and also the characteristic transformations in this frame and in the section 3 we formulate two tasks – the scene correspondence and the updating of maps – in such a model. An outview completes the paper.

2. SPACE DEFINITION

From the mathematical point of view it is necessary to define characteristic operations in suitable function spaces. Two kinds of spaces are modeled – the **image space** and the **feature space**.

In correspondence with LEBERLs Image Cubes (see LEBERL 1991) the *image space* is more–dimensional and the first two coordinates are the $x - y$ – geometry (in the small–scale information systems the altitute is a separate level and not a complete coordinate axis) added by time or spectral coordinates.

If the time is discrete or other thematic levels are contained the image space can be interpreted as slides.

The *feature space* is a generalized description of thematic information. In cartography respectivly in GIS–technologies these spaces are vector–oriented containing the objects, their geometry, and relations between them organized as a data base (see Figure 1).

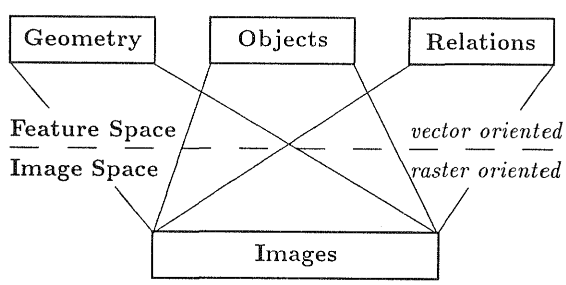


Figure 1: Feature and Image Space

On the base of such defined spaces specific thematic processes are represented as transformations. In addition to Figure 1 in Figure 2 the operator activities are shown.

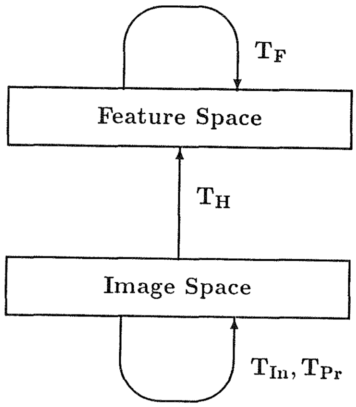


Figure 2: Operator in the Spaces

The two kinds of spaces permit a classification of the process-dependent operations. Such a generalized point of view leads to ideas of algorithmization as

- finding of suitable projections T_{Pr} and integrations T_{In} with suitable domains and weights within the image space ,
- introduction of a continuous time coordinate and the time operator

$$T_t(f(x,y)) = f_t(x,y) \quad , \tag{1}$$

- upgrading the problem from the image space to the feature space by hierarchic transformations T_H .

By using the functional analytical especially the HILBERT–Space–Methods the transformations can be designed and analized by a scalar product $< \, . \, , \, . \, >$ realized by integrals or series in the sense of L^2– and l^2–HILBERT–Spaces. Than the image space transformations T_I are being modeled

$$T_{In} \;=\; < \quad , \rho >_a \tag{2}$$

$$T_{Pr} \;=\; < \quad , \delta > \quad . \tag{3}$$

ρ is a weight-function corresponding to an area and δ is the DIRAC–Distribution depending on the projection direction (see also PROSS 1991 b).

Because the feature space is organized as a data system the transformations T_F within the feature space are designed as a generalized matrix.

The feature space exists not only in one qualitative level. Table 1 illustrates this fact.

Therefor it is necessary to redefine photogrammetric processes as specific transformations in the image and feature space.

	Qualitative Level		
	low	middle	high
geometry	*	*	*
objects	—	(*)	*
relations	(*)	*	*
characteristics	geometry (topology)	geometry topology	geometry topology semantics

Table 1: Feature Space Characterization

It is typical that the search for a solution of problems in science and technology corresponds to finding the extrema of functionals. Often such functionals are measures of the difference of structures (e.g. least square means and variation methods). POGGIO et.al. (1985) demonstrate such a calculus in computer vision theory.

In Table 2 the time characterization is compared with mathematical methods.

Time	Mathematical Methods
point	least square means
sequence	variation methods
continuum	functional analysis

Table 2: Characterization of Time

Depending on the kind of problems and on the data using we act in the scheme of Figure 3.

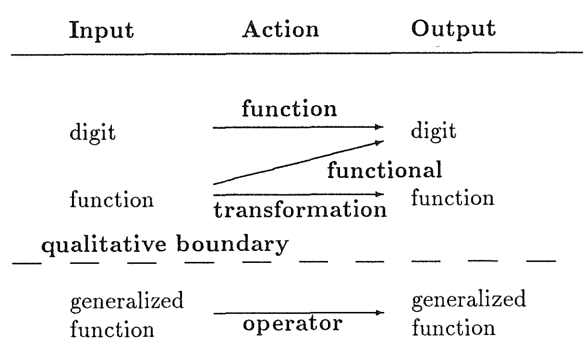


Figure 3: Mathematical Levels

In the next section we will formulate the scene correspondence and the updating of maps as examples in this calculus.

3. PROCESSES

In this section we will give two examples demonstrating the philosophy of an abstract description of photogrammetric tasks.

3.1 Scene Correspondence

The task of scene correspondence is to define the geometric relations between two or more images to the same domain. Thereby the time is running continuously and typically the time points are discrete on the time axis in the image space. Consequently these images also correspond to discrete levels in the image space.

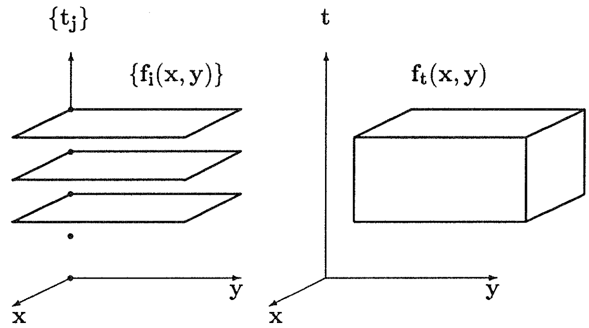
The correspondence problem is solved with the aid of image information (grey values, image frequencies, textural features etc.) by using the variation calculus and defining functionals to be minimized. The minimizing problem leads to the solution of EULER–Equations (systems of partial differential equations).

Because a sequence of images is a set of discrete planes in the image space one direction of generalization is the change over to the time continuum and to get a complete image space. In this model it is possible to define termini as image flow or trajectories of objects.

The **digital photogrammetry** transfers both the spaces and their transformations in a digital world and leads also to new notions as **softcopy photogrammetry** (see also LEBERL 1991). The transformations are from Type T_I acting nearly exclusive in the image space. As results of the discretization of the image space we get sequences of points. The correspondence is defined by the solution of attached difference equations being also an analogon in the digital sense to the differential equations. By using an iterative scheme it can be formulated as a complete digital photogrammetric task (see HAHN, PROSS 1992). Farther image processes can be designed by finding attached transformations T_I within the image space especially by separation and qualified algorithmization in geometry and time.

New kinds of mathematical means and methods for the representation like *stochastical* (see BUSCH, KOCH 1990) and *functional analytical methods* – see PROSS 1990 b, 1991 a for the use of HILBERT–Space–Methods – are the theoretical background.

Figure 4 shows the differences between the discrete and the continuous image space.



$f_{i+1}(x,y) = T_{i+1}(f_i(x,y))$ $\qquad$ $f_t(x,y) = T_t(f_0(x,y))$

Figure 4: Distrete and Continuous Image Space

3.2 Updating of Maps

By using digital image processing methods in photogrammetry and cartography the problem of map updating is rising from the images and symbolized description to an abstract desymbolized structure. The definition of characterized structures leads to a correspondence problem in such structures added by difference measures in structure and geometry.

Figure 5 shows the action of the hierarchical transformation T_H .

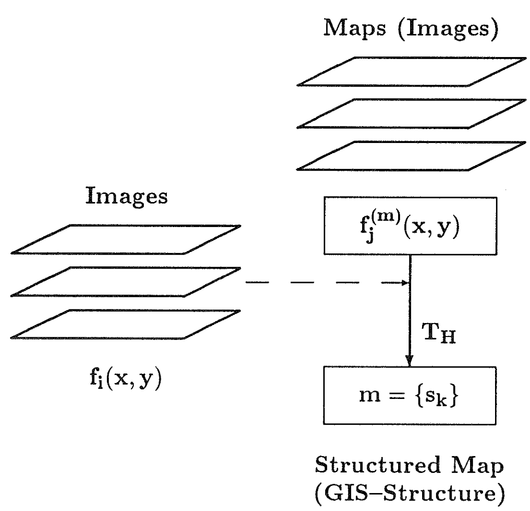


Figure 5: Hierarchical Transformation

The transformation T_H creates

$$\begin{aligned} m &= T_H(f_j^{(m)}(x,y)) \\ &= \{s_k\} \end{aligned} \qquad (4)$$

with

$$\begin{aligned} s_k &= (g_k, o_k, r_k|d) \\ &= (\hat{s}_k|d) \quad . \end{aligned} \qquad (5)$$

The *nonsymbolic description* $\hat{s}_k$ contains the geometry g_k of the objects o_k and the relations r_k between the objects o_k added by the set of symbols of the map d .

The *updating operator* U can be designed as

$$m^{(n+1)} = U(m^{(n)}) \tag{6}$$

with

$$s_j^{(n+1)} = U(\{s_j^{(n)}\}, \{f_i(x,y)\}) \quad . \tag{7}$$

Updating is a result of comparing the (cartographic) objects with topical structured image data. Comparing uses a difference operator D

$$D = (S, G) \tag{8}$$

separeted in two parts: S acting in the structure, G in the geometry. The difference operator D is a functional and weighted the structure differences by S and compares the geometry differences by G .

Based on the three kinds of cartographic objects points, lines, and areas the *structure operator* S forms a symbolic 4 by 3 matrix describing the several possibilities of transmission between the cartographic object groups (see Figure 6). The principle diagonal of S contains the *structure invariant part.* The upper right part of S is the *generalization part* corresponding to the dimension reduction in the structure and the upgrading in the object hierarchy. The last row in S is the zero–space corresponding with such *objects deleted during the generalization.*

After a geometric transformation to a reference geometry the structure is in coincidence. A difference measure $d(s_i, s_j)$ between the structures s_i and s_j is defined by

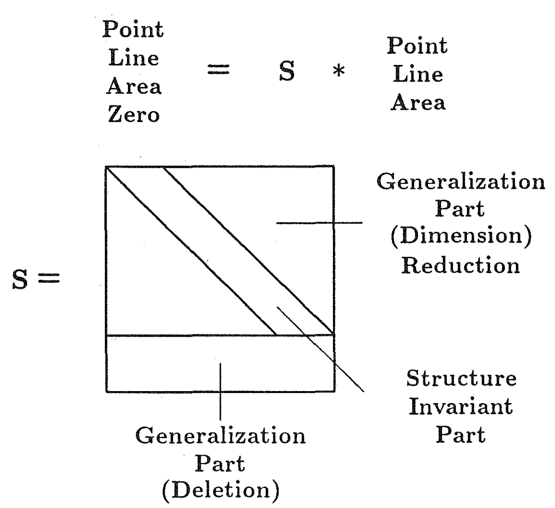


Figure 6: Structural Matrix

$$d_G(s_i, s_j) = \sum_{k=1}^{3} \alpha_k d_G^k(s_i^k, s_j^k) \quad . \tag{9}$$

d_G^1 is the point coincidence measure; d_G^2 that for lines, and d_G^3 that for areas; α_k are weights. A lot of difference functionals d is possible. The simplest ones are the point distances, the differences of line lengths and of square measures.

In classic maps line objects dominate. Therefor it is also adequate to integrate about the square differences of normalized length.

$$\begin{aligned} d^2(l_1, l_2) &= \|l_1 - l_2\|_2^2 \\ &= \int [(x(\hat{t}_1) - x(\hat{t}_2))^2 + \\ &\quad +(y(\hat{t}_1) - y(\hat{t}_2))^2] \, d\hat{t} \end{aligned} \tag{10}$$

with

$$\hat{t}_i = \frac{t_i - t_i(P_1)}{t_i(P_2) - t_i(P_1)} \tag{11}$$

$\hat{t}_i$ is the normalized length of the lines.

In summary a *correspondence measure* of two structures s_i and s_j is defined by

$$D(s_i, s_j) = \alpha d_G(s_i, s_j) + \beta d_S(s_i, s_j) \quad . \tag{12}$$

The structure measure d_S is derived from the structure matrix S. The weights α, β, and α_k estimate the significance of the several parts.

4. OUTVIEW

From a generalized point of view a structuring of processes and definition of suitable spaces and operators is very important.

By using a *connection–symbol* of operators $\circ$

$$(A \circ B)(x) = B(A(x)) \tag{13}$$

we can formulate a thematic processing with an operator sequence acting in the image and the feature space (see also Figure 2)

$$\{T_i^{j_1}\}_{j_1} \circ \{T_h^1\} \circ \{T_f^{k_1}\}_{k_1} \circ \{T_{h-1}^2\}. \tag{14}$$

For a concrete task the question is to formulate a *target functional* F that must be extremaly and depends on the sequences of operators

$$F = F(i_{j_\nu}, f_{k_\mu}, h_\nu, h_\mu^{-1}). \tag{15}$$

In the case of map updating (section 3.2) the operator sequence contains

T_i – image enhancement, map scanning

T_h – generation of a structured map representation from images and maps

T_f – comparison of structure, deletion and creation of objects

$T_{h^{-1}}$ – symbolization of a map structure.

By definition of suitable target functionals an automatic structure generation can be added by the computation of difference measures controlled by an operator sequence.

After the first practical steps it is necessary to separate the techniques into an interactive and an automatic part. The analyse of data structures leads to suitable interfaces in the data and information flows.

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