A Note on the Computational Testing of AC Optimal Power Flow Using the Current Voltage Formulations

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Background. We solve a set of IEEE test problems (14, 30, 57, 118 and 300 bus) on a different hardware platform with the same software and compare the results to those in O'Neill, Castillo and Cain (2012). The GAMS code to test the solvers is unchanged. In O'Neill, Castillo and Cain (2012), the problems were originally solved on a DELL Latitude laptop with an Intel® CoreTM i5-540M (2.53GHz, 3M cache), 4GB memory, 32-bit and a Vista operating system (32 bit). The new hardware is a DELL Latitude laptop with CoreTM i7-3520M (2.90GHz,) and 8GB memory, 64-bit with a Windows 7 operating system (64 bit). Below the old and new hardware platforms will be referred to as 32-bit and 64-bit, respectively.

We used the same GAMS code with two nonlinear solvers: IPOPT 3.8 and CONOPT 3.14 and two linear programs solvers: CPLEX 12.2.0.1 and GUROBI 4.0.0. Differences in solution times were recorded when the problems were run at different times.

Approach. We solve the nonlinear IV formulation of the test problems with quadratic costs (OPF2) to benchmark the solvers against the generally accepted optimal solution. Next, we solve the nonlinear IV formulation with a linear ten-step objection function (OPF) approximating the quadratic costs to benchmark the linear objective function and examine how the software performs with a linear objective function. We then solve a series of linearized IV formulations (OPFlin). Finally, we make an additional run through the nonlinear IV formulation with the linear objective function starting with the optimal solution to the last iteration of the linearized IV formulation OPFlin (OPFp). We also solve and present the BTHETA formulation for contrast. We apply IPOPT and CONOPT to OPF2, OPF and OPFp; we apply CPLEX 12.2.0.1 and GUROBI 4.0.0 to OPFlin and BTHETA. Therefore we consider for each of the five problems, the above nonlinear or linear solver; we refer to this as the five solver-problem combinations.

Starting Points. We solve each test problem with ten different starting points. The first starting point is a 'flat' start (in polar coordinates, voltage magnitude = 1 and voltage angle = 0) for all buses. The remaining nine points are uniformly random points that satisfy the voltage magnitude constraints. The starting value for current is $\underline{I} = Y\underline{V}$ where $\underline{V}$ is the randomly generated point. This $\underline{I}$, $\underline{V}$ point satisfies the

current flow equations but not necessarily the generator and load constraints at each bus.

Test Problems. Summary parameters, the widely accepted quadratic optimal value, and the step function approximation of the objective function for each test problem are in Table 1.

Table 1. Test problem parameters and optimal solutions

Nodes	Branches	Generators		Demand	V^{max}	V^{min}	Best Known Optimal Value	
		Number	Capacity				Step Function	Quadratic
14	20	5	7.724	2.590	1.06	0.94	82.76	80.81
30	41	6	326.80	42.42	1.10	0.95	5.918	5.745
57	80	7	326.78	235.26	1.06	0.94	423.7	417.4
118	186	54	99.66	42.42	1.06	0.94	1311	1297
300	411	69	19.76	12.51	1.06	0.94	7488	7197

Results

Speedup. Table 2 presents the average speedups. We define speedup as 1- (average time for 64-bit)/(average time for 32-bit). With the exception of four solver-problem combinations the speedups ranged from 0.18 to 0.95. Based on processor speed alone, a speedup of about 0.15 is expected. The four solver-problem combinations without speedups were in solving the smaller problems(14 and 30 bus). For the nonlinear solvers on the larger (57, 118 and 300 bus) problems, the speedups were between 0.42 and 0.95. For the linear solvers on the larger (57, 118 and 300 bus) problems, the speedups were between 0.27 and 0.73. There are no obvious reasons why the smaller problems did not result in similar speedups to the larger problems.

Objective function. For the 14 and 30 bus problems, the same objective function values were found in almost all comparative instances. For the 57 bus problem, CONOPT-OPF2 performed better on the 64-bit machine, but for the remaining problems, the same objective function values were found in almost all comparative instances. For the 118 bus problem, CONOPT and IPOPT had problems solving OPF2 and CONOPT also had problems solving OPF. For the 300 bus problem, CONOPT-OPF2 performed better on the 64-bit machine, but for the remaining problems, the same objective function values were found in almost all comparative instances. Appendix A contains more detailed results.

Additional Results for 64-bit machine. For the 30 bus problem, 2 of 10 instances of IPOPT-OPF declared local optimality with active penalty variables. For the 57 bus problem, 3 of 10 instances of CONOPT-OPF2 declared local optimality with active penalty variables. For the 118 bus problem, 9 of 10 instances of CONOPT-OPF2, declared local optimality with active penalty variables; 3 of 10 instances of CONOPT-OPF declared local optimality with active penalty variables and 1 of 10 instances of CONOPT-OPFp declared local optimality with active penalty.

Table 2. Fractional Speedups with New Hardware

		14bus	30bus	57bus	118bus	300bus
OPF2	CONOPT	*0.38*	*0.63*	*0.53*	0.61	0.67
OPF2	IPOPT	*0.42*	*0.61*	*0.64*	0.72	0.95
OPF	CONOPT	*-0.24*	*0.29*	*0.53*	0.76	0.59
OPF	IPOPT	*0.27*	*0.20*	*0.64*	0.79	0.79
OPFp	CONOPT	*-0.70*	*-0.10*	*0.53*	0.59	0.44
OPFp	IPOPT	*0.42*	*0.53*	*0.65*	0.80	0.42
BTHETA	GUROBI	0.00	*-2.06*	*0.65*	0.71	0.88
BTHETA	CPLEX	*-0.32*	*0.67*	*0.45*	0.69	0.90
OPFlin	GUROBI	*0.18*	0.72	*0.68*	0.66	0.42
OPFlin	CPLEX	*0.49*	*0.66*	*0.42*	0.73	0.27

Reference

Richard P O'Neill, Anya Castillo and Mary Cain, "The Computational Testing of AC Optimal Power Flow Using the Current Voltage Formulations," FERC Staff Technical Paper, December 2012, http://www.ferc.gov/industries/electric/indus-act/market-planning/opf-papers/acopf-3-iv-linearization-testing.pdf

Appendix

Solution Statistics. The summary statistics are over the set of starting points. They include

OPTVAVG is the average optimal solution with penalty variables set to zero if necessary.
OPTVCV is the coefficient of variation (standard deviation/mean) of optimal values
. CPUAVG is the average cpu time.
CPUCV is the coefficient of variation (standard deviation/mean) of CPU times
CPUMAX is the maximum cpu time.
CONVRG for non-linear solvers is the number that reported convergence over the ten starting points, that is, did not exceed the maximum time limit of 1000 seconds.
NUMLPS is the total number of linear programs.
MAXLPS is the number of times the maximum number of linear programs was reached.
MINOPF is the minimum objective function value found.
TEPSIN is the number of starting points that declared a local optimum with nonzero penalty variables.

The 32-bit hardware was run in March 2012. The 64-bit hardware was run in August/September 2013. The results for the 64-bit hardware are in bold and italics.

Table for 14 bus problem.

type	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
CONOPT	80.82	0.000	0.083	0.065	0.109	10	0	80.82
	80.82	***0.000***	***0.056***	***0.587***	***0.125***	***10***	***0***	***80.82***
	80.82	***0.000***	***0.047***	***0.635***	***0.094***	***10***	***0***	***80.82***
IPOPT	80.82	0.000	0.432	0.062	0.562	10	0	80.82
	80.82	***0.000***	***0.282***	***0.074***	***0.421***	***10***	0	***80.82***
	80.82	***0.000***	***0.219***	***0.380***	***0.406***	***10***	***0***	***80.82***
OPF							0	
CONOPT	82.76	0.000	0.056	0.152	0.109	10	0	82.76
	82.76	***0.000***	***0.076***	***0.460***	***0.109***	***9***	0	***82.76***
	82.76	***0.000***	***0.063***	***0.742***	***0.140***	***10***	***0***	***82.76***
IPOPT	82.76	0.000	0.446	0.058	0.546	10	0	82.76
	82.76	***0.000***	***0.356***	***0.064***	***0.452***	***10***	***0***	***82.76***
	82.76	***0.000***	***0.295***	***0.321***	***0.531***	***10***	***0***	***82.76***
OPFp							0	
CONOPT	82.76	0.000	0.020	0.307	0.062	10	0	82.76
	82.76	***0.000***	***0.039***	***0.606***	***0.078***	***8***	***0***	***82.76***
	82.76	***0.000***	***0.029***	***0.414***	***0.047***	***10***	***0***	***82.76***
IPOPT	82.76	0.000	0.237	0.039	0.281	10	0	82.76
	82.76	***0.000***	***0.120***	***0.049***	***0.265***	***10***	***0***	***82.76***
	82.76	***0.000***	***0.154***	***0.382***	***0.265***	***10***	***0***	***82.76***
BTHETA						NUMLP	MAXLPS	
GUROBI	76.59	0.000	0.002	0.050	0.002	10	0	
	76.59	0.000	0.002	0.050	0.002	10	0	
	76.59	0.000	0.002	0.153	0.004	10	0	
	76.59	***0.000***	***0.002***	***0.323***	***0.002***	***10***	***0***	
	76.59	***0.000***	***0.002***	***0.471***	***0.003***	***10***	***0***	
CPLEX	76.59	0.000	0.009	0.049	0.011	10	0	
	76.59	0.000	0.010	0.111	0.02	10	0	
	76.59	***0.000***	***0.007***	***0.152***	***0.008***	***10***	***0***	
	76.59	***0.000***	***0.018***	***2.190***	***0.128***	***10***	***0***	
OPFlin								
GUROBI	82.40	0.003	0.022	0.306	0.003	56	0	
	82.40	0.003	0.022	0.281	0.003	56	0	
	82.40	0.003	0.024	0.332	0.004	56	0	
	82.35	***0.003***	***0.018***	***0.321***	***0.028***	***55***	***0***	
	82.40	***0.003***	***0.019***	***0.323***	***0.033***	***56***	***0***	
CPLEX	82.37	0.003	0.063	0.125	0.008	62	0	
	82.37	0.003	0.065	0.214	0.008	62	0	
	82.37	***0.003***	***0.018***	***0.321***	***0.028***	***62***	***0***	
	82.37	***0.003***	***0.047***	***0.297***	***0.069***	***62***	***0***	

Table for 30 bus problems

	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
OPF2								
CONOPT	5.745	0.000	0.175	0.098	0.296	10	0	5.745
	5.745	***0.000***	***0.073***	***0.471***	***0.156***	10	0	***5.745***
	5.745	***0.000***	***0.056***	***0.350***	***0.093***	10	0	***5.745***
IPOPT	5.745	0.000	1.019	0.084	1.700	10	0	5.745
	5.745	***0.000***	***0.379***	***0.266***	***0.530***	***10***	***0***	***5.745***
	5.745	***0.000***	***0.415***	***0.234***	***0.671***	***10***	***2***	***5.745***
OPF								
CONOPT	5.918	0.000	0.097	0.138	0.188	10	0	5.918
	5.918	***0.000***	***0.076***	***0.46***	***0.140***	10	0	***5.918***
	5.918	***0.000***	***0.061***	***0.425***	***0.109***	10	0	***5.918***
IPOPT	5.919	0.000	0.760	0.087	1.235	8	0	5.918
	5.919	***0.000***	***0.591***	***0.207***	***0.811***	***8***	***0***	***5.918***
	5.919	***0.000***	***0.634***	***0.191***	***0.780***	***8***	***0***	***5.918***
OPFp								
CONOPT	5.918	0.000	0.04	0.189	0.078	10	0	5.918
	5.918	***0.000***	***0.048***	***0.695***	***0.094***	***10***	***0***	***5.918***
	5.918	***0.000***	***0.040***	***0.716***	***0.093***	***10***	***0***	***5.918***
IPOPT	5.919	0.000	0.772	0.07	1.154	10	0	5.918
	5.919	***0.000***	***0.395***	***0.232***	***0.499***	***10***	***0***	***5.918***
	5.919	***0.000***	***0.328***	***0.283***	***0.499***	***10***	***0***	***5.918***
OPFlin						NUMLP	MAXLPS	
GUROBI	5.909	0.004	0.167	0.185	0.020	80	0	
	5.909	0.004	0.185	0.191	0.028	80	0	
	5.909	0.004	0.189	0.216	0.019	80	0	
	5.910	***0.004***	***0.050***	***0.199***	***0.061***	***80***	***0***	
	5.801	***0.002***	***0.050***	***0.193***	***0.065***	***77***	***0***	
CPLEX	5.910	0.004	0.300	0.202	0.030	88	0	
	5.910	0.004	0.308	0.181	0.045	88	0	
	5.910	***0.004***	***0.104***	***0.203***	***0.129***	***77***	***5***	
	5.910	***0.004***	***0.102***	***0.393***	***0.204***	***80***	***5***	
BTHETA								
GUROBI	5.659	0.000	0.008	0.026	0.009	10	0	
	5.659	0.000	0.009	0.075	0.013	10	0	
	5.659	0.000	0.009	0.048	0.011	10	0	
	5.659	***0.000***	***0.003***	***0.269***	***0.004***	***10***	***0***	
	5.801	***0.002***	***0.050***	***0.193***	***0.065***	***77***	***0***	
CPLEX	5.659	0.000	0.024	0.017	0.028	10	0	
	5.659	0.000	0.028	0.11	0.056	10	0	
	5.659	***0.000***	***0.009***	***0.16***	***0.011***	***10***	***0***	
	5.659	***0.000***	***0.008***	***0.189***	***0.012***	***10***	***0***	

Note: For the IPOPT-OPF, 2 of 10 instances declared local optimality with penalty variables in the objective function.

Table for 57bus problem

OPF2	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
CONOPT	456.72	0.054	0.817	0.130	1.279	9	2	417.38
	417.38	***0.000***	***0.445***	***0.408***	***0.686***	***10***	***2***	***417.38***
	417.38	***0.000***	***0.325***	***0.403***	***0.499***	***10***	***3***	***417.38***
IPOPT	417.38	0.000	2.861	0.085	4.180	10	0	417.38
	417.38	***0.000***	***1.153***	***0.469***	***2.138***	***10***	***0***	***417.38***
	417.38	***0.000***	***0.909***	***0.228***	***1.233***	***10***	***0***	***417.38***
OPF								
CONOPT	423.75	0.000	0.445	0.122	0.671	10	0	423.75
	423.75	***0.000***	***0.220***	***0.627***	***0.453***	***10***	***0***	***423.75***
	423.75	***0.000***	***0.198***	***0.508***	***0.390***	***10***	***0***	***423.75***
IPOPT	423.76	0.000	2.718	0.073	4.150	10	0	423.75
	423.76	***0.000***	***1.143***	***0.441***	***2.355***	***10***	***0***	***423.75***
	423.76	***0.000***	***0.824***	***0.196***	***1.061***	***10***	***0***	***423.75***
OPFp								
CONOPT	423.75	0.000	0.226	0.087	0.343	10	0	423.75
	423.75	***0.000***	***0.136***	***0.375***	***0.219***	***10***	***0***	***423.75***
	423.75	***0.000***	***0.075***	***0.638***	***0.203***	***10***	***0***	***423.75***
IPOPT	423.76	0.000	2.348	0.178	6.022	10	0	423.75
	423.76	***0.000***	***0.920***	***0.446***	***1.904***	***10***	***0***	***423.75***
	423.76	***0.000***	***0.730***	***0.591***	***1.903***	***10***	***0***	***423.75***
OPFlin						NUMLP	MAXLPS	
GUROBI	420.88	0.003	0.278	0.171	0.046	50	0	
	420.88	0.003	0.280	0.185	0.044	50	0	
	420.88	0.003	0.288	0.171	0.043	50	0	
	420.87	***0.003***	***0.121***	***0.712***	***0.345***	***50***	***0***	
	420.87	***0.003***	***0.075***	***0.162***	***0.092***	***50***	***0***	
CPLEX	420.59	0.003	0.384	0.257	0.102	56	0	
	420.59	0.003	0.153	0.223	0.043	56	0	
	420.59	***0.003***	***0.183***	***0.221***	***0.227***	***56***	***0***	
	420.59	***0.003***	***0.111***	***0.261***	***0.156***	***56***	***0***	
BTHETA								
GUROBI	410.58	0.000	0.020	0.032	0.025	10	0	
	410.58	0.000	0.021	0.045	0.027	10	0	
	410.58	0.000	0.024	0.060	0.030	10	0	
	410.58	***0.000***	***0.008***	***0.413***	***0.017***	***10***	***0***	
	410.58	***0.000***	***0.006***	***0.092***	***0.006***	***10***	***0***	
CPLEX	410.58	0.000	0.019	0.031	0.024	10	0	
	410.58	0.000	0.047	0.049	0.063	10	0	
	410.58	***0.000***	***0.024***	***0.179***	***0.032***	***10***	***0***	
	410.58	***0.000***	***0.014***	***0.079***	***0.016***	***10***	***0***	

Note: For the CONOPT-OPF2, 3 of 10 instances declared local optimality with penalty variables in the objective function.

Table for 118 bus problem

OPF2	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPF
CONOPT	1807	0.049	8.441	0.103	14.227	9	9	1297
	1297		***3.317***	***0.366***	***5.975***	***10***	***9***	***1297***
	1297		***3.317***	***0.366***	***5.975***	***10***	***9***	***1297***
IPOPT	1956	0.038	10.906	0.113	20.327	10	10	1510
	1297		***3.310***	***0.378***	***6.099***	***10***	***10***	***1297***
			2.728	***0.359***	***4.805***	***10***	***10***	
OPF								
CONOPT	1467	0.056	6.204	0.335	25.053	10	3	1311
	1312	***0.000***	***1.462***	***0.475***	***3.120***	***10***	***0***	***1312***
	1312	***0.000***	***1.462***	***0.475***	***3.120***	***10***	***0***	***1312***
IPOPT	1312	0.000	9.076	0.037	10.764	10	0	1311
	1312	***0.000***	***1.538***	***0.475***	***3.417***	***10***	***0***	***1312***
	1312	***0.000***	***2.346***	***0.123***	***2.776***	***10***	***0***	***1312***
OPFp								
CONOPT	1326	0.012	1.911	0.232	5.990	10	1	1314
	1312	***0.000***	***0.785***	***0.642***	***2.137***	***10***	***1***	***1312***
	1312	***0.000***	***0.785***	***0.642***	***2.137***	***10***	***1***	***1312***
IPOPT	1367	0.038	5.022	0.204	13.977	10	1	1311
	1312	***0.000***	***0.743***	***0.614***	***1.981***	***10***	***1***	***1312***
	1312	***0.000***	***1.311***	***0.539***	***3.183***	***10***	***1***	***1312***
OPFlin						NUMLP	MAXLPS	
GUROBI	1293	0.006	1.890	0.410	0.150	86	0	
	1293	0.006	1.890	0.410	0.150	86	0	
	1293	0.006	1.950	0.410	0.160	86	0	
	1295	***0.006***	***0.617***	***0.423***	***1.043***	***86***	***9***	
	1294	***0.006***	***0.492***	***0.401***	***0.830***	***86***	***9***	
CPLEX	1295	0.006	1.890	0.430	0.170	82	0	
	1295	0.006	1.970	0.450	0.180	82	0	
	1295	***0.006***	***0.571***	***0.449***	***1.043***	***82***	***5***	
	1295	***0.006***	***0.619***	***0.449***	***1.047***	***82***	***5***	
BTHETA								
GUROBI	1261	0.000	0.090	0.030	0.100	10	0	
	1261	0.000	0.090	0.030	0.110	10	0	
	1261	0.000	0.090	0.030	0.110	10	0	
	1261	***0.000***	***0.038***	***0.061***	***0.044***	***10***	***0***	
	1261	***0.000***	***0.024***	***0.127***	***0.029***	***10***	***0***	
CPLEX	1261	0.000	0.130	0.010	0.140	10	0	
	1261	0.000	0.150	0.070	0.210	10	0	
	1261	***0.000***	***0.038***	***0.127***	***0.052***	***10***	***0***	
	1261	***0.000***	***0.038***	***0.127***	***0.052***	***10***	***0***	

Note: For the CONOPT-OPF2, 9 of 10 instances declared local optimality with penalty variables in the objective function. For the CONOPT-OPF, 3 of 10 instances declared local optimality with penalty variables in the objective function. For the CONOPT and IPOPT-OPFp, 1 of 10 instances declared local optimality with penalty variables in the objective function. For the IPOPT-OPF2, 10 of 10 instances declared local optimality with penalty variables in the objective function.

Table for the 300 bus problem

OPF2	OPTVAVG	OPTVCV	CPUAVG	CPUCV	CPUMAX	CONVRG	TEPSIN	MINOPT
CONOPT	7229	0.004	24.500	0.061	30.000	10	1	7197
	7197	***0.000***	***8.146***	***0.169***	***10.670***	***10***	***10***	***7197***
	7197	***0.000***	***8.146***	***0.169***	***10.670***	***10***	***10***	***7197***
IPOPT	7345	0.008	34.700	0.130	73.000	10	4	7197
	7197	***0.000***	***1.683***	***0.853***	***5.538***	***10***	***10***	***7197***
	7197	***0.000***	***1.683***	***0.853***	***5.538***	***10***	***10***	***7197***
OPF								
CONOPT	7488	0.000	6.600	0.086	11.100	10	0	7488
	7351	***0.000***	***2.724***	***0.338***	***4.165***	***10***	***10***	***7351***
	7351	***0.000***	***2.724***	***0.338***	***4.165***	***10***	***10***	***7351***
IPOPT	7488	0.000	16.000	0.209	34.500	10	0	7488
	7351	***0.000***	***3.418***	***0.561***	***6.833***	***10***	***10***	***7351***
	7351	***0.000***	***3.418***	***0.561***	***6.833***	***10***	***10***	***7351***
OPFp								
IPOPT	7776	0.027	8.300	0.091	13.100	10	2	7488
	8023	***0.180***	***4.647***	***0.881***	***14.290***	***10***	***10***	***7351***
	8023	***0.180***	***4.647***	***0.881***	***14.290***	***10***	***10***	***7351***
CONOPT	7731	0.010	8.700	0.227	18.300	10	5	7488
	7718	***0.053***	***5.070***	***0.517***	***9.735***	***10***	***10***	***7068***
	7718	***0.053***	***5.070***	***0.517***	***9.735***	***10***	***10***	***7351***
OPFlin						NUMLP	MAXLPS	
CPLEX	7528	0.012	9.900	0.338	0.400	100	10	
	7528	0.012	10.000	0.328	0.500	100	10	
	7068	***0.000***	***1.190***	***0.044***	***1.288***	***100***	***10***	
	7068	***0.000***	***1.190***	***0.044***	***1.288***	***100***	***10***	
GUROBI	7615	0.042	16.200	0.161	0.500	100	10	
	7615	0.042	16.400	0.146	0.500	100	10	
	7615	0.042	16.600	0.163	0.400	100	10	
	7615	0.042	16.600	0.148	0.400	100	10	
	7066	***0.000***	***1.593***	***0.074***	***1.802***	***100***	***10***	
	7068	***0.000***	***1.593***	***0.074***	***1.802***	***100***	***10***	
BTHETA								
GUROBI	7195	0.000	0.300	0.011	0.300	10	0	
	7195	0.000	0.300	0.012	0.300	10	0	
	7195	0.000	0.300	0.009	0.300	10	0	
	7195	0.000	0.300	0.034	0.400	10	0	
	7066	***0.000***	***0.174***	***0.020***	***0.181***	***10***	***0***	
	7066	***0.000***	***0.174***	***0.020***	***0.181***	***10***	***0***	
CPLEX	7195	0.000	0.300	0.027	0.400	10	0	
	7195	0.000	0.300	0.022	0.400	10	0	
	7066	***0.000***	***0.218***	***0.144***	***0.305***	***10***	***0***	
	7066	***0.000***	***0.218***	***0.144***	***0.305***	***10***	***0***	