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An integrated lot-sizing model for imperfect production with multiple disposals of defective items

Y.-L. Cheng[a,*], W.-T. Wang[a], C.-C. Wei[a], and K.-L. Lee[b]

a. *Department of Marketing and Distribution Management, Chien Hsin University of Science and Technology, Jungli 32097, Taiwan, R.O.C.*
b. *Department of Industrial Management, Chien Hsin University of Science and Technology, Jungli 32097, Taiwan, R.O.C.*

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Abstract. In this study, an optimal integrated vendor-buyer inventory model with defective items is proposed. Most researches on defective items assumed that an inspection process was carried out by the buyer. We consider that the vendor conducts the inspection process and disposes defective items in multiple batches. We prove that the function of annual cost is convex, and obtain closed-form expressions. A solution procedure is used to derive the optimal order quantity, the number of shipments, and the number of defective item disposals. Numerical examples are provided to illustrate our model. Setting the fraction of defective items to zero, the numerical examples indicate that the proposed model can result in the solutions to the existing models without considering defective items. Moreover, a sensitivity analysis is used to reveal the effects of cost parameters on the optimal solution. We show that when the disposal cost is relatively low, a multiple disposals strategy may perform better than a single disposal strategy.

1. Introduction

The problem of vendor-buyer integration is considered as the foundational research topic in supply chain management. The objective is to minimize the total system cost or maximize the entire system profit. One of the first integrated inventory models consisting of single vendor and single buyer was proposed by Goyal [1]. Banerjee [2] modified Goyal's [1] model considering a finite production rate for the vendor who followed a lot-for-lot shipment policy. Goyal [3] relaxed the lot-for-lot policy and proposed an integer multiple of equal-size vendor production quantity shipments. Lu [4] then generalized Goyal's [3] model by relaxing the assumption that the supplier could supply the retailer after completing a production batch. Goyal [5] developed a policy with which the shipment sizes increased by a fixed factor, which was equal to the production rate divided by the demand rate. Hill [6] proposed a generalized policy for shipment batches increasing by a geometric growth factor. Several researchers (e.g., [7-9]). proposed different batching and shipping policies for the integrated inventory models.

Since then, many extensions have been made to the basic integrated production inventory models. Huang [10] investigated the effect of quality on lot sizes for a single-vendor single-buyer inventory system. Nieuwenhuyse and Vandaele [11] proved that lot splitting policies would benefit both the supplier and the buyer. Ertogral et al. [12] developed an integrated vendor-buyer model under equal-size shipment and added transportation cost to the model.

*. *Corresponding author. Tel.: +886-3-4581196; Fax: +886-3-2503032*
E-mail addresses: jkcheng54@uch.edu.tw (Y.-L. Cheng); wangwt2006@gmail.com (W.-T. Wang); d887801@uch.edu.tw (C.-C. Wei); klee@uch.edu.tw (K.-L. Lee)

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Lin [13] proposed an integrated single-vendor single-buyer inventory model with backorder price discount and variable lead time. Sajadieh et al. [14] discussed an integrated model where the demand was dependent on the amount of goods displayed on the shelf. Sari et al. [15] considered an integrated vendor-buyer problem where the supplier offered temporary price discounts to the buyer during a sale period. Readers are referred to Glock [16] for a comprehensive review of integrated systems. Rad et al. [17] suggested a joint economic lot sizing model of a single-vendor single-buyer supply chain for items with imperfect quality and shortages under price-sensitive demand. Lee and Fu [18] analyzed an integrated production and delivery quantity model in a make-to-order producer-buyer supply chain. Sarakhsi et al. [19] studied a joint economic lot-sizing problem for a single-vendor single-buyer system where the demand was dependent on selling price. Some recent works on integrated vendor-buyer models have been done by many researchers, such as Lin [20], Yi & Sarker [21], Wee & Widyadana [22], Ouyang et al. [23], Giri & Sharma [24], and Chung et al. [25].

Rosenblatt and Lee [26] was one of the first models to consider imperfect production inventory models. Salameh and Jaber [27] presented a classic economic order quantity model in which the order lot contained a random proportion of imperfect quality items. Many extensions and modifications regarding quality issues have been researched recently. Wee et al. [28] and Eroglu and Ozdemir [29] extended Salameh and Jaber's [27] inventory model to consider shortages with complete backorders. Subsequently, Chang and Ho [30] revisited the inventory model of Wee et al. [28] and derived closed-form optimal solutions by applying Renewal Reward Theory. Hsu and Yu [31] studied a one-time-only discount policy for an economic order quantity model that considered imperfect quality items. Maddah et al. [32] presented an approach to avoid shortages during the screening period. Khan et al. [33] provided a detailed and complete review for economic order quantity models with imperfect quality items. Chang et al. [34] and Yassine et al. [35] developed an economic production quantity model for imperfect quality items. Rezaei and Davoodi [36,37] provided inventory models that considered supplier selection and imperfect quality. Moussawi-Haidar et al. [38] and Rezaei [39] investigated economic order quantity models where imperfect items were screened with sampling inspection plans. Hsu and Hsu [40] provided economic production quantity models to determine the optimal lot size and backorder quantity under imperfect productions. Taleizadeh et al. [41] proposed an extension to economic order quantity model with partial backordering and reparable products. Some other recent inventory models that considered imperfect quality items are Wahab et al. [42], Sarkar [43], Hsu & Hsu [44], Chang [45], Jaber et al. [46], Rezaei & Salimi [47], and Taleizadeh et al. [48].

This paper extends Salameh and Jaber [27] model to an integrated production inventory model with defective items. We assume that the integrated policy for the proposed model is accepted by the vendor and the buyer. To maintain a long-term cooperation, the vendor would deliver 100% good products to the buyer. In addition, as shown in Rezaei and Salimi [47], conducting the inspection process by the buyer may not be a cost-efficient strategy. We investigate that the products are screened by the vendor while some literature such as Huang [10] and Wu et al. [49] assumed that the buyer conducted inspection process whenever receiving products. We relax the assumption of defective items removed from stock after screening to allow the defective items be scrapped by multiple disposals during the production period.

The paper is organized into five sections. Section 1 provided an introduction. The notation and assumptions for this study are stated in Section 2. The model description, formulation, and solution procedure are presented in Section 3. Section 4 provides the numerical examples to illustrate the solution procedure for determining the optimum. Conclusions and suggestions for future research are given in Section 5.

2. Notation and assumptions

We summarize the notation used in this study as follows:

T	Order cycle time
P	Annual constant production rate
D	Annual constant demand rate
Q	Lot size of good quality items per shipment (decision variable)
n	Number of good quality items shipments per order cycle (decision variable)
n_M	Number of defective item disposals over a production period (decision variable)
β	Random fraction of defective items, a random variable with a known probability density function; $0 \leq \beta < 1$
S_V	Fixed production setup cost per production batch
S_B	Fixed ordering cost per order
h_V	Stock-holding cost for the vendor per unit per year
h_B	Stock-holding cost for the buyer per unit per year

u	Fixed cost of each disposal for defective items
g	Fixed cost of each shipment for good quality items
$TC_M(n,Q)$	Annual total relevant inventory cost for the vendor
$TC_R(n,Q)$	annual total relevant inventory cost for the buyer
$TC(N,Q)$	Annual total relevant inventory cost for both parties

We make the following assumptions for this study:

1. Consider single vendor and single buyer;
2. The demand rate is known and finite;
3. The production rate is known and finite;
4. All products are 100% inspected by the vendor. We assume that the inspection is included in production process. The vendor conducts multiple disposals of the defective items without salvage value;
5. The good items are delivered to the buyer while the defective ones are disposed from the vendor's stock. Thus, to meet the demand, it is constrained by: $(1-\beta)P > D$, i.e. $P > \frac{D}{1-\beta}$ (Taking the expectation, it yields $P > E(\frac{1}{1-\beta})D$);
6. The cost for facilitating multiple deliveries is the responsibility of the buyer;
7. We assume that the buyer's holding cost is greater than the vendor's holding cost, i.e. $h_B > h_V$;
8. Shortages are not allowed.

3. Model formulation

We consider a single-vendor single-buyer integrated model where the buyer's order quantity is manufactured and delivered in multiple equal-batch shipments. From the buyer's order quantity, the vendor plans their production batch and conducts the inspection process. During the production run, the defective items are scrapped by multiple disposals from the stock. The disposal cost for the defective items is paid by the vendor. The good quality products are spited into small lot sizes that are delivered to the buyer over an inventory cycle. The buyer incurs the shipment cost in the multiple deliveries.

Figure 1 depicts the behavior of inventory levels for the vendor and the buyer. Since the fraction of defective items is β (or the fraction of good quality items is $1-\beta$), the vendor would produce $\frac{nQ}{1-\beta}$ units to meet the demand of nQ units during an order cycle. The vendor delivers good items to the buyer who receives n shipments with each lot size Q. Since the demand rate is D, the shipment interval is $\frac{Q}{D}$

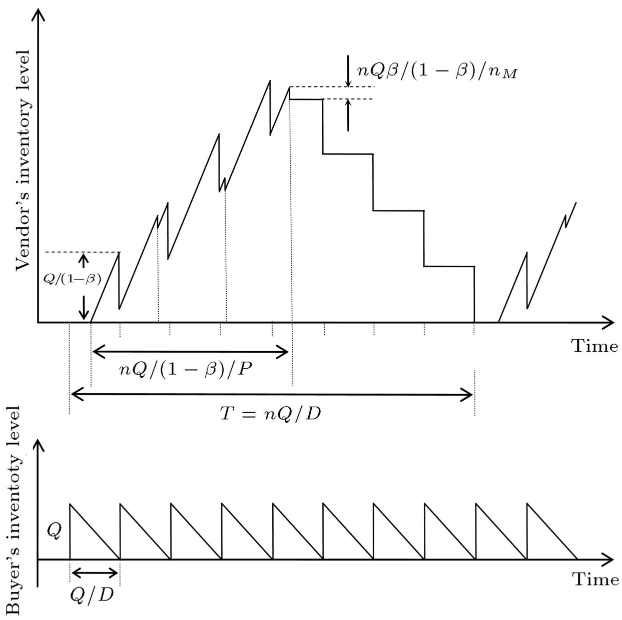

Figure 1. Vendor's and buyer's inventory levels with $n = 8$ and $n_M = 3$.

and the order cycle is $\frac{nQ}{D}$. During the production period, the $\frac{nQ\beta}{1-\beta}$ defective item units are scrapped with multiple disposals (or removals). Since the number of the disposals per cycle is n_M, the defective items unit in each disposal is $\frac{nQ\beta}{n_M(1-\beta)}$.

The total relevant inventory cost for an integrated inventory system includes the inventory costs from the vendor and the buyer. The vendor's cost comprises setup cost, holding cost, and disposal costs. The buyer's cost consists of ordering cost, holding cost, and shipment costs. Our purpose is to minimize the annual total relevant inventory cost for both parties, which contains the annual vendor's and buyer's costs. Subsequently, we derive different inventory costs for the vendor and the buyer.

3.1. Vendor's inventory holding cost:

Figure 2 shows that the vendor's holding inventory (time-weighted inventory) per order cycle can be derived by subtracting the shaded area from the bold area. The bold area includes the areas above interval T_1 and above interval T_2. Referring to Figure 2, we have:

$$T_1 = \frac{nQ}{(1-\beta)P}, \tag{1}$$

$$T_2 = \frac{nQ}{D} - \frac{nQ}{(1-\beta)P} - \left(\frac{Q}{D} - \frac{Q}{(1-\beta)P}\right)$$

$$= (n-1)Q\left(\frac{1}{D} - \frac{1}{(1-\beta)P}\right). \tag{2}$$

The bold area is derived as:

$$\frac{Eq.(5)}{T}h_V=\frac{\frac{nQ^2}{2D}\left[(n-1)+\frac{2D}{P(1-\beta)}-\frac{nD}{P(1-\beta)}+\frac{nD}{n_MP}\left(\frac{1}{(1-\beta)^2}-\frac{1}{(1-\beta)}\right)\right]}{\frac{nQ}{D}}h_V$$

$$=\frac{Q}{2}\left[(n-1)+\frac{2D}{P\left(1-\beta\right)}-\frac{nD}{P\left(1-\beta\right)}+\frac{nD}{n_MP}\left(\frac{1}{\left(1-\beta\right)^2}-\frac{1}{\left(1-\beta\right)}\right)\right]h_V. \tag{6}$$

Box I

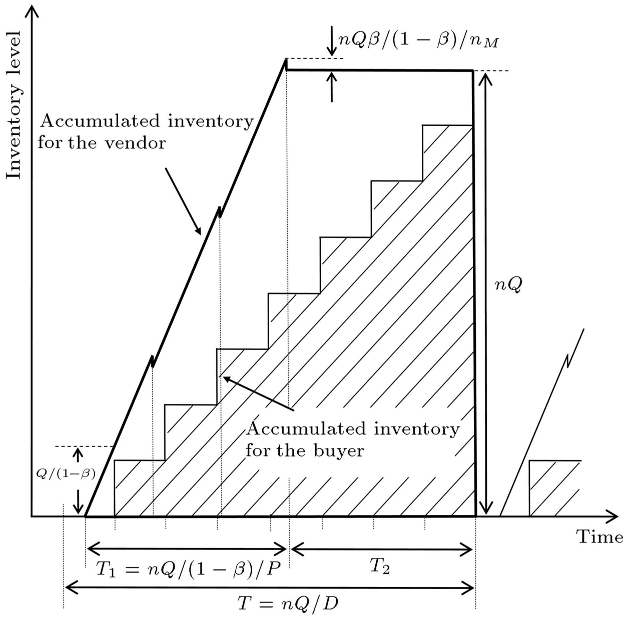

Figure 2. Vendor's accumulated inventory and buyer's accumulated inventory with $n = 8$ and $n_M = 3$.

$$\frac{1}{2}\frac{T_1}{n_M}\sum_{i=1}^{n_M}\left\{\left[(i-1)\frac{T_1}{n_M}P-(i-1)\frac{T_1}{n_M}P\beta\right]\right.$$
$$\left.+\left[i\frac{T_1}{n_M}P-(i-1)\frac{T_1}{n_M}P\beta\right]\right\}+nQT_2$$
$$=\frac{n^2Q^2\left(n_M\left(1-\beta\right)+\beta\right)}{2(1-\beta)^2n_MP}$$
$$+n\left(n-1\right)Q^2\left(\frac{1}{D}-\frac{1}{\left(1-\beta\right)P}\right). \tag{3}$$

Subsequently, the shaded area is calculated from:

$$\frac{Q}{D}\left\{Q+2Q+3Q+\cdots+\left(n-1\right)Q\right\}=\frac{Q}{D}\left(\sum_{i=1}^{n-1}iQ\right)$$
$$=\frac{n\left(n-1\right)Q^2}{2D}. \tag{4}$$

Subtracting shades area, Eq. (4), from bold area, Eq. (3), we yield the vendor's holding inventory per order cycle as:

$$\frac{n^2Q^2\left(n_M\left(1-\beta\right)+\beta\right)}{2(1-\beta)^2n_MP}+n\left(n-1\right)Q^2\left(\frac{1}{D}-\frac{1}{\left(1-\beta\right)P}\right)$$
$$-\frac{n\left(n-1\right)Q^2}{2D}=\frac{nQ^2}{2D}\left[(n-1)+\frac{2D}{P\left(1-\beta\right)}\right.$$
$$\left.-\frac{nD}{P\left(1-\beta\right)}+\frac{nD}{n_MP}\left(\frac{1}{\left(1-\beta\right)^2}-\frac{1}{\left(1-\beta\right)}\right)\right]. \tag{5}$$

Thus, we can obtain the vendor's inventory holding cost per year as shown in Box I.

3.2. Vendor's setup cost

Since the vendor has one production per cycle time and the setup cost per production is s_V, the vendor's setup cost per year is:

$$\frac{s_V}{nQ/D}=\frac{D}{nQ}s_V. \tag{7}$$

3.3. Vendor's disposal cost

The number of disposals of the defective items is n_M, the cost per disposal is u, and the cycle time is nQ/D. Therefore, the disposal cost per year is:

$$\frac{n_Mu}{nQ/D}=\frac{n_MD}{nQ}u. \tag{8}$$

Summing costs in Eqs. (6) to (8), we derive the annual total relevant inventory cost for the vendor:

$$TC_M\left(Q,n,n_M\right)=\frac{Q}{2}\left[(n-1)+\frac{2D}{P\left(1-\beta\right)}\right.$$
$$\left.-\frac{nD}{P\left(1-\beta\right)}+\frac{nD}{n_MP}\left(\frac{1}{\left(1-\beta\right)^2}-\frac{1}{\left(1-\beta\right)}\right)\right]h_V$$
$$+\frac{D}{nQ}s_V+\frac{n_MD}{nQ}u. \tag{9}$$

3.4. Buyer's inventory holding cost

Since the buyer's demand rate is constant and buyer's maximal stock level is Q, the buyer's average inventory is $Q/2$. Hence, the buyer's inventory holding cost per year is:

$$\frac{Q}{2}h_B. \tag{10}$$

3.5. Buyer's ordering cost

Because the buyer places an order per cycle and the cycle time is nQ/D, the buyer's ordering cost per year is:

$$\frac{D}{nQ}s_B. \tag{11}$$

3.6. Buyer's shipment cost

Since the buyer receives n shipments during a cycle time, the buyer's shipment cost per year is:

$$\frac{D}{Q}g. \tag{12}$$

Summing the costs in Eqs. (10) to (12), the annual total relevant inventory cost for the buyer is:

$$TC_R(Q,n)=\frac{Q}{2}h_B+\frac{D}{nQ}s_B+\frac{D}{Q}g. \tag{13}$$

Therefore, the annual total relevant cost for the vendor and the buyer is the sum of Eqs. (9) and (13):

$$\begin{aligned}TC(Q,n,n_M)&=TC_M(Q,n,n_M)+TC_R(Q,n)\\&=\frac{Q}{2}\left[(n-1)+\frac{2D}{P(1-\beta)}-\frac{nD}{P(1-\beta)}\right.\\&\left.+\frac{nD}{n_MP}\left(\frac{1}{(1-\beta)^2}-\frac{1}{(1-\beta)}\right)\right]h_V+\frac{D}{nQ}s_V\\&+\frac{n_MD}{nQ}u+\frac{Q}{2}h_B+\frac{D}{nQ}s_B+\frac{D}{Q}g.\end{aligned} \tag{14}$$

Since β is a random variable with a known probability density function, the expectation of $TC(Q,n,m_M)$ is derived as:

$$\begin{aligned}E(TC(Q,n,n_M))&=\frac{Q}{2}\left[(n-1)+\frac{2D}{P}E_a-\frac{nD}{P}E_a\right.\\&\left.+\frac{nD}{n_MP}(E_b-E_a)\right]h_V+\frac{D}{nQ}s_V+\frac{n_MD}{nQ}u+\frac{Q}{2}h_B\\&+\frac{D}{nQ}s_B+\frac{D}{Q}g,\end{aligned}$$

s.t.

$$Q>0, n>0 \text{ and } n_M>0, \tag{15}$$

where $E_a=E(1/(1-\beta))$ and $E_b=E(1/(1-\beta)^2)$.

Setting the number of defective item disposals (n_M) equal to one, our model is reduced to the model with one disposal strategy for defective items. Furthermore, setting the fraction of defective items (β) and the disposal cost (u) equal to zero, the model of Eq. (15) is same as the model discussed in [50].

If Q, n, and n_M are assumed to be continuous variables, the cost function of Eq. (15) is a convex function (see Appendix A for the proof). Since n and n_M are discrete variables, we develop the following approach to derive optimum. At a given set of n and n_M, we first derive the optimal solution to Q. For the given n and n_M, taking the first and second derivatives of $E(TC(Q,n,n_M))$ with respect to Q, we have:

$$\begin{aligned}\frac{\partial}{\partial Q}E(TC(Q|n,n_M))&=\frac{1}{2}\left(h_B+(n-1)h_V\right.\\&\left.+\frac{Dh_V(nE_b-(n+(n-2)n_M)E_a)}{n_MP}\right)\\&-\frac{D(s_B+s_V+n_Mu+ng)}{nQ^2},\end{aligned} \tag{16}$$

$$\begin{aligned}&\frac{\partial^2}{\partial Q^2}E(TC(Q|n,n_M))\\&=\frac{2D(s_B+s_V+n_Mu+ng)}{nQ^3}>0.\end{aligned} \tag{17}$$

From Eq. (17), it is clear that $E(TC(Q,n,n_M))$ is a convex function for $n>0$ and $n_M>0$. Setting Eq. (16) equal to zero, we obtain the Q value that minimizes cost function as shown in Box II.

Putting Eq. (18) into Eq. (15), the total cost function will be obtained by Eq. (19) as shown in Box III.

To minimize $E(TC(n,n_M))$ in Eq. (19), the algebraic calculation is simplified when we take the square of Eq. (19), since minimizing $E^2(TC(n,n_M))$ is same as minimizing $E(TC(n,n_M))$. We have Eq. (20) as shown in Box IV. Taking the first derivatives with respect to

$$Q(n,n_M)=\sqrt{\frac{2n_MPD(s_B+s_V+n_Mu+ng)}{n_MnPh_B+h_V\left[n^2D(E_b-E_a)+n_M(n(n-1)P-n(n-2)DE_a)\right]}}. \tag{18}$$

Box II

$$E\left(TC\left(n,n_M\right)\right)=$$
$$\frac{\left\{\sqrt{2}\sqrt{n_M nPh_B+h_V\left(n^2D\left(E_b-E_a\right)+n_M\left(n\left(n-1\right)P-n\left(n-2\right)DE_a\right)\right)}\times\sqrt{n_M PD\left(s_B+s_V+n_M u+ng\right)}\right\}}{n_M nP}. \tag{19}$$

Box III

$$E^2\left(TC\left(n,n_M\right)\right)=$$
$$\frac{\left\{\ 2\left[n_M Ph_B+h_V\left(nD\left(E_b-E_a\right)+n_M\left(\left(n-1\right)P-\left(n-2\right)DE_a\right)\right)\right]\times\left[D\left(s_B+s_V+n_M u+ng\right)\right]\ \right\}}{n_M nP}. \tag{20}$$

Box IV

n and n_M, we get:

$$\frac{\partial}{\partial n}E^2\left(TC\left(n,n_M\right)\right)=2Dgh_V-\frac{2D^2gh_V\left(\left(n_M+1\right)E_a-E_b\right)}{n_M P}-\frac{2D\left(2DE_a h_V+P(h_B-h_V)\right)(s_B+s_V+n_M u)}{n^2P}, \tag{21}$$

$$\frac{\partial}{\partial n_M}E^2\left(TC\left(n,n_M\right)\right)=\frac{2Du\left(P\left(h_B+\left(n-1\right)h_V\right)-\left(n-2\right)DE_a h_V\right)}{nP}-\frac{2D^2\left(E_b-E_a\right)\left(s_B+s_V+ng\right)h_V}{n_M^2 P}. \tag{22}$$

Let $n^{\#}$ and $n_M^{\#}$ be the solutions to $\frac{\partial}{\partial n}E^2(TC(n,n_M))=0$ and $\frac{\partial}{\partial n_M}E^2(TC(n,n_M))=0$, respectively; we have:

$$n^{\#}=\sqrt{\frac{n_M^{\#}\left(2DE_a h_V+P\left(h_B-h_V\right)\right)\left(s_B+s_V+n_M^{\#}u\right)}{gh_V\left(D\left(E_b-E_a\right)+n_M^{\#}\left(P-DE_a\right)\right)}}, \tag{23}$$

$$n_M^{\#}=\sqrt{\frac{n^{\#}Dh_V\left(E_b-E_a\right)\left(s_B+s_V+n^{\#}g\right)}{u\left(Ph_B+h_V\left(\left(n^{\#}-1\right)P-\left(n^{\#}-2\right)DE_a\right)\right)}}. \tag{24}$$

We can employ numerical search methods to solve Eqs. (23) and (24). Alternatively, applying mathematical software such as Mathematica or Maple, we derive the following closed-form expressions of $n_M^{\#}$ and $n^{\#}$. Substituting $n^{\#}$ in Eq. (24) into Eq. (23) yields:

$$n_M^{\#}=\sqrt{\frac{D\left(E_b-E_a\right)\left(s_B+s_V\right)}{\left(P-DE_a\right)u}}>0. \tag{25}$$

It can be seen that $n_M^{\#}$ decreases as u increases. Subsequently, substituting $n_M^{\#}$ in Eq. (25) into Eq. (23) gives:

$$n^{\#}=\sqrt{\frac{\left(2DE_a h_V+P\left(h_B-h_V\right)\right)\left(s_B+s_V\right)}{\left(P-DE_a\right)gh_V}}>0. \tag{26}$$

In Appendix B, we prove the convexity of $E^2(TC(n,n_M))$. It implies that $E(TC(n,n_M))$ is also a convex function because $E^2(TC(n,n_M))$ is derived from $E(TC(n,n_M))$. We demonstrate that $n_M^{\#}$ and $n^{\#}$ are, respectively, equal to n_{MC}^* in Eq. (A.7) and n_C^* in Eq. (A.10) since $E(TC(n,n_M))$ is the cost function with the optimal Q shown in Eq. (18). Due to the assumptions of $h_B>h_V$ and $P-DE_a>0$, Eqs. (25) and (26) show that $n^{\#}$ and $n_M^{\#}$ are positive numbers minimizing $E^2(TC(n,n_M))$ and $E(TC(n,n_M))$.

Since the optimal n and n_M should be integers, although $n^{\#}$ and $n_M^{\#}$ are positive and not always integer numbers, optimal n and n_M values should be the integers around $n^{\#}$ and $n_M^{\#}$. Let $n^{\#^-}(n_M^{\#-})$ and $n^{\#^+}(n_M^{\#+})$, respectively, denote the minimal integers less than and greater than $n^{\#}(n_M^{\#})$. Note that if $n^{\#}(n_M^{\#})$ is less than one, we let $n^{\#-}=n^{\#+}=1$ $(n_M^{\#-}=n_M^{\#+}=1)$ because zero is not feasible.

Let (n^*,n_M^*) be the optimal integer solution to $E(TC(n,n_M))$. If $E(TC(n,n_M))$ in Eq. (19) is minimized, we have:

$$E\left(TC\left(n^*, n_M^*\right)\right) = \text{Min}\Big\{E\left(TC\left(n^{\#-}, n_M^{\#-}\right)\right), E\left(TC\left(n^{\#-}, n_M^{\#+}\right)\right), E\left(TC\left(n^{\#+}, n_M^{\#-}\right)\right), E\left(TC\left(n^{\#+}, n_M^{\#+}\right)\right)\Big\}, \tag{27}$$

where:

$$\left(n^*, n_M^*\right) \in \left\{\left(n^{\#-}, n_M^{\#-}\right), \left(n^{\#-}, n_M^{\#+}\right), \left(n^{\#+}, n_M^{\#-}\right), \left(n^{\#+}, n_M^{\#+}\right)\right\}.$$

Substituting (n^*, n_M^*) for (n, n_M) in Eq. (18), the optimal shipment quantity is derived by Eq. (28) as shown in Box V.

The following solution procedure is provided to derive the optimal solution.

1. Compute $n_M^{\#}$ and $n^{\#}$ from Eqs. (25) and (26), respectively. The values of $n^{\#-}$, $n^{\#+}$, $n_M^{\#-}$, and $n_M^{\#+}$ are obtained. If $n_M^{\#}(n^{\#})$ is less than one, then $n_M^{\#-} = n_M^{\#+} = 1$ $(n^{\#-} = n^{\#+} = 1)$;
2. Derive the optimal solution to (n, n_M):

$$\left(n^*, n_M^*\right) \in \left\{\left(n^{\#-}, n_M^{\#-}\right), \left(n^{\#-}, n_M^{\#+}\right), \left(n^{\#+}, n_M^{\#-}\right), \left(n^{\#+}, n_M^{\#+}\right)\right\},$$

   which results in the lowest cost decided from Eq. (27);
3. From Eq. (18) or (28), the optimal shipment quantity is derived, i.e. $Q^* = Q(n^*, n_M^*)$.

4. Numerical example

To illustrate the proposed model, we provide the following examples:

Example 1. The imperfect fraction in each production batch follows a uniform distribution with the following probability density function:

$$f(\beta) = \begin{cases} 25, & 0 \leq \beta \leq 0.04, \\ 0, & \text{otherwise} \end{cases}$$

The parameter values are as follows:

- Production rate: $P = 48,000$ (unit/year)
- Demand rate: $D = 12,000$ (unit/year)
- Vendor's setup cost: $S_V = 500$ (\$/setup)
- Vendor's holding cost: $h_V = 10$ (\$/unit/year)
- Vendor's disposal cost: $u = 50$ (\$/disposal)
- Buyer's order cost: $S_B = 25$ (\$/order)
- Buyer's holding cost: $h_B = 12$ (\$/unit/year)
- Buyer's shipment cost: $g = 25$ (\$/shipment)

Compute the expectation values:

$$E_a = E\left(\frac{1}{1-\beta}\right) = 1.0206,$$

$$E_b = E\left(\frac{1}{(1-\beta)^2}\right) = 1.0417.$$

In Figure 3, we first illustrate that Eq. (19) is a convex function. The optimal solutions are derived through the following procedures.

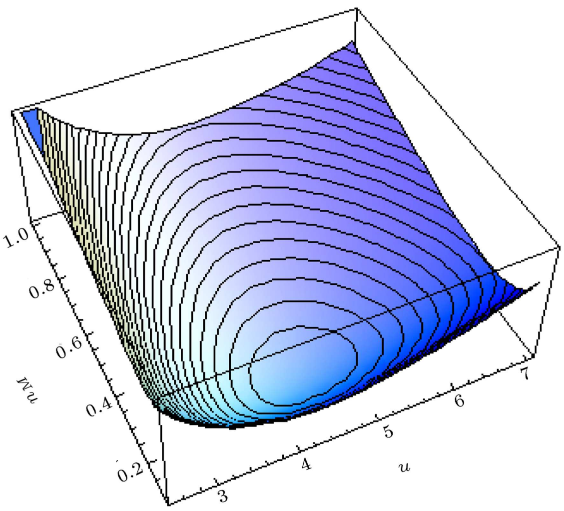

Figure 3. The graphic diagram for the integrated total relevant cost in Example 1.

$$Q\left(n^*, n_M^*\right) = \sqrt{\frac{2n_M^* PD\left(s_B + s_V + n_M^* u + n^* g\right)}{n_M^* n^* P h_B + h_V\left[n^{*2} D\left(E_b - E_a\right) + n_M^*\left(n^*\left(n^*-1\right)P - n^*\left(n^*-2\right)DE_a\right)\right]}}. \tag{28}$$

Box V

From Eqs. (25) and (26), we get:

$$n^{\#}=\sqrt{\frac{(2DE_a h_V+P(h_B-h_V))(s_B+s_V)}{(P-DE_a)\,gh_V}}=4.48,$$

$$n_M^{\#}=\sqrt{\frac{D\,(E_b-E_a)\,(s_B+s_V)}{(P-DE_a)\,u}}=0.27,$$

which give $n^{\#^-}=4$, $n^{\#^+}=5$, and $n_M^{\#^-}=n_M^{\#^+}=1$.

Using Eq. (19), we calculate:

$$E\left(TC\left(n^{\#-},n_M^{\#-}\right)\right)=E\left(TC\left(n^{\#-},n_M^{\#+}\right)\right)$$

$$=E\left(TC\left(4,1\right)\right)=\$12,259.2,$$

$$E\left(TC\left(n^{\#+},n_M^{\#-}\right)\right)=E\left(TC\left(n^{\#+},n_M^{\#+}\right)\right)$$

$$=E\left(TC\left(5,1\right)\right)=\$12,242.9.$$

Therefore, we have the optimal solution: $(n^*,n_M^*)=(5,1)$, which results in the lowest total relevant cost of $E(TC(n^*,n_M^*))=\$12,242.9$.

From Eq. (28), the optimal shipment size is $Q^*=Q(n^*,n_M^*)=274.44$.

As mentioned in Section 2, by assuming the fraction of defective items: $\beta=0$, the disposal cost: $u=0$, and the number of defective item disposals: $n_M=1$, the cost function of Eq. (15) reduces to the one without considering defective items. For the case of no defective items, we show how our model derives the optimum as follows.

Give the fraction of defective items: $\beta=0$ (i.e., $E_a=E_b=1$) and the disposal cost: $u=0$ for this example. We derive:

$$n^{\#}=\sqrt{\frac{(2DE_a h_V+P\,(h_B-h_V))\,(s_B+s_V)}{(P-DE_a)\,gh_V}}=4.43.$$

We subsequently set: $n_M^{\#}=1$. Then, we obtain: $n^{\#^-}=4$, $n^{\#^+}=5$, $n_M^{\#^-}=n_M^{\#^+}=1$. We calculate:

$$E\left(TC\left(n^{\#-},n_M^{\#-}\right)\right)=E\left(TC\left(n^{\#-},n_M^{\#+}\right)\right)$$

$$=E\left(TC\left(4,1\right)\right)=\$11,779.2,$$

$$E\left(TC\left(n^{\#+},n_M^{\#-}\right)\right)=E\left(TC\left(n^{\#+},n_M^{\#+}\right)\right)$$

$$=E\left(TC\left(5,1\right)\right)=\$11,783.0.$$

Therefore, we derive the optimal number of shipments: $n^*=4$, which results in the lowest total relevant cost \$11,779.2. The optimal shipment size is $Q^*=Q(4,1)=318.36$ and the optimal order size is $4\times 318.36=1,273.44$, which are the same as the results discussed in [50].

We subsequently discuss a policy for which the buyer optimizes the shipment lot size that is provided for the vendor. Using the optimal shipment size, the vendor then determines the optimal numbers of shipment and disposal. For distinction, the above-mentioned policy is called "non-integrated". In Eq. (15), we see that the vendor's and buyer's costs are:

$$\frac{Q}{2}\left[(n-1)+\frac{2D}{P}E_a-\frac{nD}{P}E_a+\frac{nD}{n_M P}\left(E_b-E_a\right)\right]h_V$$
$$+\frac{D}{nQ}s_V+\frac{n_M D}{nQ}u, \tag{29}$$

and:

$$\frac{Q}{2}h_B+\frac{D}{nQ}s_B+\frac{D}{Q}g, \tag{30}$$

respectively.

From Eq. (30), we derive the optimal shipment lot size:

$$\sqrt{\frac{2D\,(s_B+ng)}{nh_B}}. \tag{31}$$

Substituting Q by Eq. (31) into Eq. (29), we obtain the optimal (n,n_M), which minimizes the vendor's cost. The optimal solutions to the integrated and the non-integrated policies are shown in Table 1. It is shown that the two policies have the same optimal (n,n_M). With a lower optimal shipment size for the buyer, the non-integrated policy leads to a lower buyer's cost than the integrated policy does. This comparison reveals that the integrated policy results in a lower total inventory cost for both parties.

The effects of cost parameter values on the optimal policy for Example 1 are provided in Tables 2-7. Table 2 shows that the optimal number of shipments per order cycle slightly increases as the vendor's setup cost increases. When the vendor's setup cost significantly increases, the production lot size per cycle (i.e., nQ) will increase in this example. Table 3 indicates that optimal number of shipments is not sensitive to

Table 1. The results for Example 1 with integrated and non-integrated policies.

	(n^*,n_M^*)	Q^*	$E(TC_M(Q,n,n_M))$	$E(TC_R(Q))$	$E(TC(n^*,n_M^*))$
Integrated	(5, 1)	274.44	\$9,284.5	\$2,958.4	\$12,242.9
Non-integrated	(5, 1)	244.95	\$9,382.8	\$2,939.4	\$12,322.2

Table 2. The optimal solutions for Example 1 with different S_V values.

$P = 48,000$, $D = 12,000$, $h_V = 10$, $u = 50$, $S_B = 25$, $h_B = 12$, $g = 25$					
Vendor's setup cost (S_V)	**300**	**400**	**500**	**600**	**700**
(n^*, n_M^*)	(4,1)	(4,1)	(5, 1)	(5, 1)	(5,1)
Q^*	277.13	304.91	274.44	293.39	311.19
$E(TC(n^*, n_M^*))$	10283.9	\$11,314.8	\$12,242.9	\$13,088.2	\$13,882.2

Table 3. The optimal solutions for Example 1 with different S_B values.

$P = 48,000$, $D = 12,000$, $h_V = 10$, $u = 50$, $S_B = 25$, $h_B = 12$, $g = 25$					
Buyer's order cost (S_B)	**15**	**20**	**25**	**30**	**35**
(n^*, n_M^*)	(5, 1)	(5, 1)	(5, 1)	(5, 1)	(5, 1)
Q^*	272.48	273.46	274.44	275.42	276.40
$E(TC(n^*, n_M^*))$	\$12,155.2	\$12,199.1	\$12,242.9	\$12,286.6	\$12,330.1

Table 4. The optimal solutions for Example 1 with different h_V values.

$P = 48,000$, $D = 12,000$, $u = 50$, $S_V = 500$, $S_B = 25$ $h_B = 12$, $g = 25$					
Vendor's holding cost (h_V)	**8**	**9**	**10**	**11**	**12**
(n^*, n_M^*)	(6, 1)	(5, 1)	(5, 1)	(4, 1)	(4, 1)
Q^*	256.47	285.06	274.44	319.72	310.05
$E(TC(n^*, n_M^*))$	\$11,307.4	\$11,786.9	\$12,242.9	\$12,667.2	\$13,062.4

Table 5. The optimal solutions for Example 1 with different h_B values.

$P = 48,000$, $D = 12,000$, $u = 50$, $h_V = 10$, $u = 50$, $S_V = 500$ $S_B = 25$, $g = 25$					
Buyer's holding cost (h_B)	**10**	**11**	**12**	**13**	**14**
(n^*, n_M^*)	(4, 1)	(4, 1)	(5, 1)	(5, 1)	(5, 1)
Q^*	339.64	334.91	274.44	271.42	268.49
$E(TC(n^*, n_M^*))$	\$11,924.3	\$12,092.9	\$12,242.9	\$12,379.4	\$12,514.4

Table 6. The optimal solutions for Example 1 with different u values.

$P = 48,000$, $D = 12,000$, $h_V = 10$, $S_V = 500$ $S_B = 25$, $h_B = 12$, $g = 25$					
Vendor's disposal cost (u)	**0.1**	**1**	**50**	**100**	**200**
(n^*, n_M^*)	(5, 6)	(5, 2)	(5, 1)	(5, 1)	(5, 1)
Q^*	265.24	265.26	274.44	284.08	302.42
$E(TC(n^*, n_M^*))$	\$11,773.9	\$11,798.2	\$12,242.9	\$12,672.6	\$13,491.0

Table 7. The optimal solutions for Example 1 with different g values.

$P = 48,000$, $D = 12,000$, $h_V = 10$, $u = 50$, $S_V = 500$ $S_B = 25$, $h_B = 12$					
Buyer's shipment cost (g)	**5**	**15**	**25**	**35**	**45**
(n^*, n_M^*)	(10, 1)	(6, 1)	(5, 1)	(4, 1)	(4, 1)
Q^*	135.15	225.93	274.44	340.01	349.39
$E(TC(n^*, n_M^*))$	\$11,098.4	\$11,773.5	\$12,242.9	\$12,617.3	\$12,965.4

the buyer's ordering cost. It is obvious that the optimal number of shipments will increase when the buyer's ordering cost (the vendor's setup cost) increases. In Table 4, one can see that a larger vendor's holding cost results in a smaller optimal number of shipments. Table 5 reveals that the buyer would like a larger number of shipments when the buyer's holding cost is larger. From Table 6, it can be seen that when the disposal cost is low, the vendor will benefit from increasing the number of disposals. Table 7 concludes that the optimal number of shipments decreases as the buyer's shipment cost increases.

Example 2. The imperfect fraction in each production batch is same as that in Example 1. The parameter values are as follows:

- Production rate: $P = 3,200$ (unit/year)
- Demand rate: $D = 1,000$ (unit/year)
- Vendor's setup cost: $S_V = 400$ (\$/setup)
- Vendor's holding cost: $h_V = 4$ (\$/unit/year)
- Vendor's disposal cost: $u = 14$ (\$/disposal)
- Buyer's order cost: $S_B = 0$ (\$/order)
- Buyer's holding cost: $h_B = 5$ (\$/unit/year)
- Buyer's shipment cost: $g = 25$ (\$/shipment)

We have $E_a = 1.0206$ and $E_b = 1.0417$, like those in Example 1.

From Eqs. (25) and (26), we obtain: $n^{\#} = 4.57$ and $n_M^{\#} = 1.97$, which give $n^{\#^-} = 4$, $n^{\#^+} = 5$, $n_M^{\#^-} = 1$, and $n_M^{\#^+} = 2$.

From Eq. (19), we calculate:

$$E\left(TC\left(n^{\#-}, n_M^{\#-}\right)\right) = E\left(TC\left(4,1\right)\right) = \$1,909.4,$$

$$E\left(TC\left(n^{\#-}, n_M^{\#+}\right)\right) = E\left(TC\left(4,2\right)\right) = \$1,907.8,$$

$$E\left(TC\left(n^{\#+}, n_M^{\#-}\right)\right) = E\left(TC\left(5,1\right)\right) = \$1,908.1,$$

$$E\left(TC\left(n^{\#+}, n_M^{\#+}\right)\right) = E\left(TC\left(5,2\right)\right) = \$1,906.3.$$

Therefore, we derive the optimal solution: $(n^*, n_M^*) = (5,2)$, which results in the lowest total relevant cost of $E(TC(n^*, n_M^*)) = \$1,906.3$. From Eq. (28), the optimal shipment size is derived as: $Q^* = Q(n^*, n_M^*) = 110.58$.

We subsequently consider this example without defective items. Setting the fraction of defective items: $\beta = 0$ (i.e., $E_a = E_b = 1$) and the disposal cost: $u = 0$, we derive:

$$n^{\#} = \sqrt{\frac{\left(2DE_a h_V + P\left(h_B - h_V\right)\right)\left(s_B + s_V\right)}{\left(P - DE_a\right) g h_V}} = 4.51.$$

Set $n_M^{\#} = 1$. Therefore, we have: $n^{\#^-} = 4$, $n^{\#^+} = 5$, $n_M^{\#-} n_M^{\#+} = 1$. Then, we calculate:

$$E\left(TC\left(n^{\#-}, n_M^{\#-}\right)\right) = E\left(TC\left(n^{\#-}, n_M^{\#+}\right)\right)$$

$$= E\left(TC\left(4,1\right)\right) = \$1,903.94,$$

$$E\left(TC\left(n^{\#+}, n_M^{\#-}\right)\right) = E\left(TC\left(n^{\#+}, n_M^{\#+}\right)\right)$$

$$= E\left(TC\left(5,1\right)\right) = \$1,903.29.$$

The optimal number of shipments, which results in the lowest total relevant cost \$1,903.29, is $n^* = 5$. The optimal shipment size is $Q^* = Q(5,1) = 110.34$ and the optimal order size is $5 \times 110.34 = 551.70$. Our result is same as the result using the equal-size batch policy in Hill [7].

5. Conclusions

This study presents an integrated vendor-buyer inventory model with defective items. While some literature assumed products screened by the buyer and defective items removed after inspection process, our proposed model considered the vendor responsible to conduct quality inspection. A random fraction of defective items was produced by the vendor who implemented a 100% inspection to screen the defective units, which were scrapped by multiple disposals during the screening period. The multiple disposals strategy provides the vendor with a flexible treatment to scrap defective items. To minimize the integrated cost, the mathematical model was formulated to derive the closed-form optimal solutions to the order quantity, the number of shipments, and the number of defective item disposals. We proved that the function of annual cost was convex. The examples illustrated the solution procedure and compared our results with related models in the literature. The sensitivity analysis was provided to show how the cost parameters affected the optimal solution. If the disposal cost is relatively low, it may be better to scrap defective items by the multiple disposals strategy.

Future research directions are open to consider unequal-size shipment policies and to incorporate quantity discount or return policy. Moreover, our study can be extended to models in which the screening rate is different from the production rate, and the defective items are reworked during the same production cycle.

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Appendix A

When Q, n, and n_M are continuous variables, we prove that Eq. (15) is a convex function.

Taking the first partial derivatives of $E(TC(Q, n, n_M))$ with respect to Q, n, and n_M, we have:

$$\frac{\partial}{\partial Q}E\left(TC\left(Q,n,n_M\right)\right)=\frac{1}{2}\Bigg(h_B+\left(n-1\right)h_V+\frac{Dh_V\left(nE_b-\left(n+\left(n-2\right)n_M\right)E_a\right)}{n_M P}\Bigg)-\frac{D\left(s_B+s_V+n_M u+ng\right)}{nQ^2},\tag{A.1}$$

$$\frac{\partial}{\partial n}E\left(TC\left(Q,n,n_M\right)\right)=\frac{Q}{2}\Bigg(h_V-\frac{Dh_V\left(\left(n_M+1\right)E_a-E_b\right)}{n_M P}\Bigg)-\frac{D\left(s_B+s_V+n_M u\right)}{n^2 Q},\tag{A.2}$$

$$\frac{\partial}{\partial n_M}E\left(TC\left(Q,n,n_M\right)\right)=\frac{Du}{nQ}-\frac{nQD\left(E_b-E_a\right)h_V}{2n_M^2 P}.\tag{A.3}$$

$$Q_C^* = \sqrt{\frac{2n_{MC}^* PD\left(s_B + s_V + n_{MC}^* u + n_C^* g\right)}{n_C^* n_{MC}^* Ph_B + h_V\left[{n_C^*}^2 D\left(E_b - E_a\right) + n_{MC}^*\left(n_C^*\left(n_C^* - 1\right)P - n_C^*\left(n_C^* - 2\right)DE_a\right)\right]}}. \tag{A.4}$$

$$n_C^* = \frac{1}{Q_C^*}\sqrt{\frac{2n_{MC}^* PD\left(s_B + s_V + n_{MC}^* u\right)}{h_V\left[D\left(E_b - E_a\right) + n_{MC}^*\left(P - DE_a\right)\right]}}, \tag{A.5}$$

$$n_{MC}^* = n_C^* Q_C^* \sqrt{\frac{h_V\left(E_b - E_a\right)}{2Pu}}. \tag{A.6}$$

Box A.I

Setting Q_C^*, n_C^*, and n_{MC}^* as the solutions with which the first partial derivatives in Eqs. (A.1)-(A.3) are equal to zero, respectively, we obtain Eqs. (A.4) to (A.6) as shown in Box A.I. Substituting n_C^* in Eq. (A.5) into Eq. (A.6), we get:

$$n_{MC}^* = \sqrt{\frac{D\left(E_b - E_a\right)\left(s_B + s_V\right)}{\left(P - DE_a\right)u}}. \tag{A.7}$$

Putting Eq. (A.7) into Eq. (A.5), we have:

$$n_C^* = \frac{1}{Q_C^*}\sqrt{\frac{2PD\left(s_B + s_V\right)}{h_V\left(P - DE_a\right)}}. \tag{A.8}$$

Replacing n_C^* and n_{MC}^* in Eq. (A.4) with Eqs. (A.7) and (A.8) gives:

$$Q_C^* = \sqrt{\frac{2PDg}{2DE_a h_V + P\left(h_B - h_V\right)}}. \tag{A.9}$$

Substituting Q_C^* in Eq. (A.9) into Eq. (A.8), n_C^* becomes:

$$n_C^* = \sqrt{\frac{\left(2DE_a h_V + P\left(h_B - h_V\right)\right)\left(s_B + s_V\right)}{\left(P - DE_a\right)gh_V}}. \tag{A.10}$$

We subsequently take the second partial derivatives for $E(TC(Q, n, n_M))$:

$$\frac{\partial^2}{\partial Q^2}E\left(TC\left(Q, n, n_M\right)\right) = \frac{2D\left(s_B + s_V + n_M u + ng\right)}{nQ^3} > 0, \tag{A.11}$$

$$\frac{\partial^2}{\partial Q \partial n}E\left(TC\left(Q, n, n_M\right)\right) = \frac{\left(D\left(E_b - E_a - n_M E_a\right) + n_M P\right)h_V}{2n_M P} + \frac{D\left(s_B + s_V + n_M u\right)}{n^2 Q^2}, \tag{A.12}$$

$$\frac{\partial^2}{\partial Q \partial n_M}E\left(TC\left(Q, n, n_M\right)\right) = -\frac{nD\left(E_b - E_a\right)h_V}{2n_M^2 P} - \frac{Du}{nQ^2}, \tag{A.13}$$

$$\frac{\partial^2}{\partial n \partial Q}E\left(TC\left(Q, n, n_M\right)\right) = \frac{\left(D\left(E_b - E_a - n_M E_a\right) + n_M P\right)h_V}{2n_M P} + \frac{D\left(s_B + s_V + n_M u\right)}{n^2 Q^2}, \tag{A.14}$$

$$\frac{\partial^2}{\partial n^2}E\left(TC\left(Q, n, n_M\right)\right) = \frac{2D\left(s_B + s_V + n_M u\right)}{n^3 Q} > 0, \tag{A.15}$$

$$\frac{\partial^2}{\partial n \partial n_M}E\left(TC\left(Q, n, n_M\right)\right) = -\frac{QD\left(E_b - E_a\right)h_V}{2n_M^2 P} - \frac{Du}{n^2 Q}, \tag{A.16}$$

$$\frac{\partial^2}{\partial n_M \partial Q}E\left(TC\left(Q, n, n_M\right)\right) = -\frac{nD\left(E_b - E_a\right)h_V}{2n_M^2 P} - \frac{Du}{nQ^2}, \tag{A.17}$$

$$\frac{\partial^2}{\partial n_M \partial n}E\left(TC\left(Q, n, n_M\right)\right) = -\frac{QD\left(E_b - E_a\right)h_V}{2n_M^2 P} - \frac{Du}{n^2 Q}, \tag{A.18}$$

$$\frac{\partial^2}{\partial n_M^2}E\left(TC\left(Q,n,n_M\right)\right)=\frac{nQD\left(E_b-E_a\right)h_V}{n_M^3P}>0. \tag{A.19}$$

If Q_C^*, n_C^*, and n_{MC}^* are the optimal solutions of $E(TC(Q,n,n_M))$, the following conditions are satisfied:

$$|H_1|=h_{11}>0,\quad |H_2|=h_{11}h_{22}-h_{12}h_{21}>0,$$

$$|H_3|=\begin{vmatrix} h_{11} & h_{12} & h_{13}\\ h_{21} & h_{22} & h_{23}\\ h_{31} & h_{32} & h_{33}\end{vmatrix}>0, \tag{A.20}$$

where h_{ij} $(i,j=1,2,3)$ are the second partial derivatives, i.e. $h_{11}=\frac{\partial^2}{\partial Q^2}E\left(TC\left(Q,n,n_M\right)\right)$, $h_{12}=\partial^2/\partial Q\partial nE\left(TC\left(Q,n,n_M\right)\right)$ and $h_{13}=\partial^2/\partial Q\partial n_ME\left(TC\left(Q,n,n_M\right)\right)$. From Eq. (A.11), it is obvious that $|H_1|=h_{11}>0$.

To further simplify expressions, let:

$$a_1=2DE_ah_V+P\left(h_B-h_V\right)>0, \tag{A.21}$$

$$a_2=s_B+s_V>0, \tag{A.22}$$

$$a_3=P-DE_a>0, \tag{A.23}$$

$$a_4=E_b-E_a\geq 0. \tag{A.24}$$

Because $P>E(1/(1-\beta))D=E_aD$ (see the assumptions in Section 2) and $E_b=E(1/(1-\beta)^2)\geq E(1/(1-\beta))=E_a$, it implies $a_3>0$ and $a_4\geq 0$. To prove $|H_2|=h_{11}h_{22}-h_{12}h_{21}>0$ on Q_C^*, n_C^*, and n_{MC}^*, we rewrite the related equations by using a_i shown in Eqs. (A.21)-(A.24). We have:

$$Q_C^*=\sqrt{\frac{2PDg}{a_1}}, \tag{A.25}$$

$$n_C^*=\sqrt{\frac{a_1a_2}{a_3gh_V}}, \tag{A.26}$$

$$n_{MC}^*=\sqrt{\frac{Da_4a_2}{a_3u}}, \tag{A.27}$$

$$h_{11}=\frac{\partial^2}{\partial Q^2}E\left(TC\left(Q,n,n_M\right)\right)=\frac{2D\left(a_2+n_Mu+ng\right)}{nQ^3}>0, \tag{A.28}$$

$$h_{12}=\frac{\partial^2}{\partial Q\partial n}E\left(TC\left(Q,n,n_M\right)\right)=\frac{\left(D\left(a_4-n_ME_a\right)+n_MP\right)h_V}{2n_MP}+\frac{D\left(a_2+n_Mu\right)}{n^2Q^2}, \tag{A.29}$$

$$h_{21}=\frac{\partial^2}{\partial n\partial Q}E\left(TC\left(Q,n,n_M\right)\right)=\frac{\left(D\left(a_4-n_ME_a\right)+n_MP\right)h_V}{2n_MP}+\frac{D\left(a_2+n_Mu\right)}{n^2Q^2}, \tag{A.30}$$

$$h_{22}=\frac{\partial^2}{\partial n^2}E\left(TC\left(Q,n,n_M\right)\right)=\frac{2D\left(a_2+n_Mu\right)}{n^3Q}. \tag{A.31}$$

We apply Eqs. (A.28)-(A.31) to calculate $|H_2|=h_{11}h_{22}-h_{12}h_{21}$.

Then, substituting Eqs. (A.25)-(A.27) for Q, n, and n_M in $|H_2|$, we take $|H_2|>0$. Applying mathematical software for simplification of equations, we can obtain $|H_3|>0$.

Appendix B

Taking the second partial derivatives of $E^2(TC(n,n_M))$ with respect to n and n_M, we have:

$$\frac{\partial^2}{\partial n^2}E^2\left(TC\left(n,n_M\right)\right)=\frac{4D\left(2DE_ah_V+P\left(h_B-h_V\right)\right)\left(s_B+s_V+n_Mu\right)}{n^3P}, \tag{B.1}$$

$$\frac{\partial^2}{\partial n_M^2}E^2\left(TC\left(n,n_M\right)\right)=\frac{4D^2h_V\left(E_b-E_a\right)\left(s_B+s_V+ng\right)}{n_M^3P}, \tag{B.2}$$

$$\frac{\partial^2}{\partial n\partial n_M}E^2\left(TC\left(n,n_M\right)\right)=-\left(\frac{2D^2gh_V\left(E_b-E_a\right)}{n_M^2P}+\frac{2Du\left(2DE_ah_V+P\left(h_B-h_V\right)\right)}{n^2P}\right). \tag{B.3}$$

Letting:

$$|H|=\frac{\partial^2}{\partial n^2}E\left(TC\left(n,n_M\right)\right)\cdot\frac{\partial^2}{\partial n_M^2}E\left(TC\left(n,n_M\right)\right)-\left(\frac{\partial^2}{\partial n\partial n_M}E\left(TC\left(n,n_M\right)\right)\right)^2,$$

we get:

$$|H|=$$

$$\frac{4D\left(2DE_ah_V+P\left(h_B-h_V\right)\right)\left(s_B+s_V+n_Mu\right)}{n^3P}$$

$$\frac{\partial^2}{\partial n^2}E^2\left(TC\left(n,n_M\right)\right)=\frac{4Da_1\left(a_2+n_M u\right)}{n^3P},\tag{B.5}$$

$$\frac{\partial^2}{\partial n\partial n_M}E^2\left(TC\left(n,n_M\right)\right)=-\left(\frac{2D^2gh_Va_4}{n_M^2P}+\frac{2Dua_1}{n^2P}\right),\tag{B.6}$$

$$\frac{\partial^2}{\partial n_M^2}E^2\left(TC\left(n,n_M\right)\right)=\frac{4D^2h_Va_4\left(a_2+ng\right)}{n_M^3P},\tag{B.7}$$

$$|H|=\frac{4D^2\left(4n_MnDh_Va_1a_4\left(a_2+n_Mu\right)\left(a_2+ng\right)-\left(n_M^2ua_1+n^2Dgh_Va_4\right)^2\right)}{n_M^4n^4P^2}.\tag{B.8}$$

Box B.I

$$|H|\Big|_{n=n^{\#},n_M=n_M^{\#}}=\frac{16Dgh_V^2u^2a_3^2\left(Da_4\sqrt{a_1a_2a_3gh_V}+a_3\sqrt{\frac{Da_2a_4}{ua_3}}\left(ga_1+\sqrt{a_1a_2a_3gh_V}\right)\right)}{a_1a_2^2a_4P^2}>0.\tag{B.11}$$

Box B.II

$$\times\frac{4D^2h_V\left(E_b-E_a\right)\left(s_B+s_V+ng\right)}{n_M^3P}$$
$$-\left(\frac{2D^2gh_V\left(E_b-E_a\right)}{n_M^2P}\right.$$
$$\left.+\frac{2Du\left(2DE_ah_V+P\left(h_B-h_V\right)\right)}{n^2P}\right)^2.\tag{B.4}$$

Substituting a_i in Eqs. (A.21)-(A.24) into the above equations, we have Eqs. (B.5) to (B.8) as shown in Box B.I.

Subsequently, we can rewrite $n_M^{\#}$ in Eq. (25) and $n^{\#}$ in Eq. (26) as:

$$n_M^{\#}=\sqrt{\frac{Da_2a_4}{ua_3}},\tag{B.9}$$

$$n^{\#}=\sqrt{\frac{a_1a_2}{gh_Va_3}}.\tag{B.10}$$

Substituting Eqs. (B.9) and (B.10) for n_M and n in Eq. (B.8), we obtain Eq. (B.11) as shown in Box B.II. Therefore, $n_M^{\#}$ in Eq. (25) and $n^{\#}$ in Eq. (26) are the optimal solutions that minimize $E^2(TC(n,n_M))$.

Biographies

Yung-Lung Cheng received his PhD degree from the Department of Industrial and Systems Engineering at Chung Yuan Christian University in Taiwan. His works have been published in Renewable and Sustainable Energy Reviews, Applied Mathematics and Computation, etc. His research interests include production/inventory control and supply chain management.

Wan-Tsu Wang received his PhD degree from the Department of Industrial and Systems Engineering at Chung Yuan Christian University in Taiwan. His works have been published in Decision Support Systems, OMEGA, European Journal of Operational Research, etc. His research interests include production/inventory control and supply chain management.

Chun-Chin Wei received his PhD degree from the Department of Industrial Engineering and Engineering Management at Tsing Hua University in Taiwan. His works have been published in various international journals. His research interests include inventory control, decision-making, and supply chain management.

Kuo-Liang Lee is an Assistant Professor in the Department of Industrial Engineering and Management at Ching Yun University in Taiwan. He received his doctoral degree in Industrial Engineering and Manufacturing from Cleveland State University in USA. His research interests include quality management, service quality, and Six Sigma management. He is also a

Six Sigma consultant and an ISO 9001 Lead Auditor. There are many firms that have benefited from his help.

Dr. Lee is an Associate Professor and the Chairman of the Department of Industrial Management at Chien Hsin University of Science and Technology in Taiwan. He received his PhD degree in Industrial Engineering and Manufacturing from Cleveland State University in USA. His research interests include quality management, service quality, and Six Sigma management and Lean Six Sigma. He is also a Six Sigma consultant and an ISO 9001:2015 Lead Auditor. There are many firms that have benefited from his help.