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Fuzzy spaces topology change and BH thermodynamics

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Abstract. What is the ultimate fate of something that falls into a black hole? From this question arises one of the most intricate problems of modern theoretical physics: the black hole information loss paradox. Bekenstein and Hawking have been shown that the entropy in a black hole is proportional to the surface area of its event horizon, which should be quantized in a multiple of the Planck area. This led G.'t Hooft and L. Susskind to propose the holographic principle which states that all the information inside the black hole can be stored on its event horizon. From this results, one may think if the solution to the information paradox could lies in the quantum properties of the black hole horizon. One way to quantize the event horizon is to see it as a fuzzy sphere, which posses a closed relation with Hopf algebras. This relation makes possible a topology change process where a fuzzy sphere splits in two others. In this work it will be shown that, if one quantize the black hole event horizon as a fuzzy sphere taking into account its quantum symmetry properties, a topology change process to black holes can be defined without break unitarity or locality, and we can obtain a possible solution to the information paradox. Moreover, we show that this model can explain the origin of the black hole entropy, and why black holes obey a generalized second law of thermodynamics.

1. Introduction

Black hole (BH) thermodynamics has been discussed in the context of topology change, as conceived for some classes of quantum spaces, called fuzzy spheres [1]. It has been argued that a model based on the topology change of fuzzy manifolds can be used to shed some light on the origin of the BH entropy, including why BH evaporation process obeys the Generalized Second Law of Thermodynamics(GSL). Moreover, it has been suggested that topology change for fuzzy spaces can relieve the information problem.

Fuzzy spheres consist in one of the most simplest example of noncommutative spaces, and appear as vacuum solutions in Euclidean gravity [3]. It is obtained when we quantize the usual sphere S^2 replacing the commutative algebra of functions on this manifold by the noncommutative algebra of matrices. The use of fuzzy spheres in the context of BH physics is mostly motivated by the Bekenstein's limit [4]. Since fuzzy spheres, are obtained from quantization of a compact space, they are described by finite dimensional matrices, in a way that the number of independent states defined on the fuzzy sphere is limited, and the entropy associated with these states is finite, in agreement with the Bekenstein's limit [4]. Another important feature of fuzzy spheres is its close relationship with Hopf algebras, which allow us to define a linear operation (the coproduct of a Hopf algebra) on a fuzzy sphere S_F^2 and compose two fuzzy spheres preserving algebraic properties intact. This operation produces a topology change process where a fuzzy sphere splits into two others [5], and can be used as a good mathematical model to BH topology change [1, 2].

In this paper, we will revise the main results of the references [1, 2], where the BH thermodynamics is put in connection with topology change of fuzzy manifolds. It will be shown that a model based on the topology change of fuzzy manifolds can be used to shed some light on BH evaporation.

2. Fuzzy spaces topology change and BH thermodynamics

To describe the fuzzy sphere topology change, we have that under the quantization procedure, functions defined on S^2 are replaced by matrices on S_F^2 [3]. In this way, let a matrix $\hat{M}$ describing a wave function on S_F^2, the Hopf coproduct $\Delta : S_F^2(j) \to S_F^2(K) \otimes S_F^2(L)$ acts on $\hat{M}$ as

$$\Delta_{(K,L)}(\hat{M}) = \sum_{\mu_1,\mu_2,m_1,m_2} C_{K,L,J;\mu_1,\mu_2} C_{K,L,J;m_1,m_2} \times M_{\mu_1+\mu_2,m_1+m_2} e^{\mu_1 m_1}(K) \otimes e^{\mu_2 m_2}(L) \ , \quad (1)$$

where C's are the Clebsh-Gordan coefficients and $e^{\mu_i m_j}$'s are basis for a matrix space defined on the fuzzy sphere [5].

The meaning of the eq. (1) is that a wavefunction $\hat{M} \in S_F^2(J)$ splits into a superposition of wavefunctions on $S_F^2(K) \otimes S_F^2(L)$. In this way, the information in $\hat{M}$ is divided between two regions of the spacetime, i.e, the two fuzzy spheres with spins K and L respectively. In this way, the basic assumptions of this article are introduced as follows: (i) If one use the fuzzy sphere Hilbert space as the ones of the BH, the maximum of information about the BH that an outside observer can obtain would be encoded in wave functions defined on the fuzzy sphere Hilbert space. (ii) We shall find out, through the Hopf coproduct Δ, a topology change process for the BH. In this process the information about the BH initial state, will be divided into two spacetime regions. One of them is a fuzzy sphere with spin K, which we shall consider as the original world and name it "the main world". The other one is a fuzzy sphere with spin L which we shall name "the baby world". (iii) Since the baby world arises in the BH interior, an observer in the main world can not access the degrees of freedom there. In this way, from his standpoint, the BH will appear to evolve from a pure to a mixed state described by a density matrix $\hat{\rho}$. It enable us to define an entropy, measured by the observer in the main world, associated to the BH horizon. The coproduct Δ is an unitary operation, in a way that information is globally preserved in the fuzzy BH topology change.

In order to analyze how the topology change process drives the BH evaporation, as seen by an observer in our world, we will investigate how the fuzzy topology change drives the BH area transitions. We will admit that the selection rules for the BH area transitions are the ones for the topology change. These rules are obtained from the eq. (1), when one traces over the degrees of freedom in the baby universe. The probability amplitude for a BH n-steps transition can be obtained from eq. (1). In ref. [1], it has been shown that the splitting process (1) obeys cluster decomposition (locality). Then, we have that different steps $J \to J - 1/2$ (the lowest step in the BH evaporation) are independent events. In this way, the probability amplitude of n steps occur in the black evaporation process is given by the product of the probability amplitudes of each one of this steps occurring by itself. It is easy to show that the probability amplitude for n steps in the BH evaporation is given by [2]

$$a_{Jn} = \frac{2J+1}{2J-n+1} \ . \quad (2)$$

In order to analyze the BH area transitions, we will introduce a canonical ensemble in which our system (the BH) can occupy different area microstates. The idea of using these types of ensembles goes back to Krasnov [6]. In this framework, the probability amplitude for the BH evaporate is given by $a_{Jn} = e^{-\beta\delta A_{Jn}}$, where β is a temperature-like parameter dual to the BH area. If one identifies this probability amplitude with the expression given by the equation (2), we will have that the value of the BH area in the J-state will be written as

$$A_J = \beta^{-1} l_p^2 \ln(2J+1) \ . \tag{3}$$

The logarithmic dependence of the BH area spectrum on J, in the expression (3), tell us that the decrease in the horizon area is continuous at large values of J, and discrete to small values of J, when the BH approaches the Planck scale.

In this way, if one models a BH horizon by a fuzzy sphere and consider its quantum symmetry properties, a topological change process which can be used to solve the black hole information loss paradox is obtained. In this process a BH event horizon, modeled by a fuzzy sphere with spin J , splits into two others. The fuzzy sphere splitting can be used to describe a black hole evaporation process in which information about the black hole initial state is divided between two topologically disconnected regions: the main and the baby world. Nor unitarity or locality is broken in the evolution of the whole system. On the other hand, an observer in the main world sees the topology change process occurs in a non-unitary way, due to the impossibility of access the degrees of freedom in the baby world. It is possible, but not necessary, that information returns via quantum gravity tunneling at the final stages of BH evaporation. In this point, the baby universe ceases to exist, and the BH evolution as seen by an observer outside the BH is unitary. In fact, as we shall see in the next section, due to the discreteness of BH area spectrum (3), we have that the BH emission spectrum must have some deviations from the strictly thermality when the BH approaches the Planck scale. This point has been also discussed in the reference [7], where the discreteness of the BH area spectrum in the Planck regime has been put into connection with the obedience to a Generalized Uncertainty Principle(GUP) by the radiation emitted by the BH.

3. Entropy emitted during the evaporation process

The entropy of a system measures one's lack of information about its actual internal configuration. Suppose that everything we know about the internal configuration of the system is that it may be found in any of a number of states, with probability p_n for the nth state. Then the entropy associated with the system is given by Shannon's well-known relation $S = -\sum p_n ln p_n$.

The probability for a BH to emit a specific quantum should be given by the expression (2), in which we must yet include a gray-body factor Γ (representing a scattering of the quantum off the spacetime curvature surround the BH). Thus, the probability p_n to the BH goes n steps down in the area ladder is proportional to $\Gamma(n)\, e^{-\frac{\delta A_{Jn}}{4}}$. Moreover, the discrete area spectrum (3) implies a discrete line emission from a quantum BH. For a Schwarzschild black hole, the radiation emitted will be at frequencies given by $\omega_{Jn} = \frac{1}{2\sqrt{\pi}}(\sqrt{ln(2J+1)} - \sqrt{ln(2J+1-n)})$.

To gain some insight into the physical problem, we shall consider a simple toy model suggested by Hod [8]. To begin with, it is well known that, for massless field, $\Gamma(M\omega)$ approaches 0 in the low-frequency limit, and has a high-frequency limit of 1. A rough approximation of this effect can be archived by introducing a low frequency cutoff at some $\omega = \omega_c$. That is, $\Gamma(\bar{\omega}) = 0$ for $\bar{\omega} < \bar{\omega}_c$, and $\Gamma(\bar{\omega}) = 1$ otherwise, where $\bar{\omega} = M\omega$.

The ratio $R = \mid \dot{S}_{rad}/\dot{S}_{BH} \mid$ of entropy emission rate from the quantum BH to the rate of BH entropy decrease is given by:

$$R = \left| \frac{\sum_{i=1}^{N_s}\sum_{n=1}^{2J} C\Gamma(n) e^{-\frac{\delta A_{Jn}}{4}} ln\left[C\Gamma(n)e^{-\frac{\delta A_{Jn}}{4}}\right]}{\sum_{i=1}^{N_s}\sum_{n=1}^{2J} C\Gamma(n) e^{-\frac{\delta A_{Jn}}{4}} \quad \frac{\delta A_{Jn}}{4}\Big)} \right| \ , \tag{4}$$

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where C is a normalization factor, defined by the normalization condition:

$$\sum_{i=1}^{N_s}\sum_{n=1}^{2J} C\Gamma(n)e^{-\frac{\delta A_{Jn}}{4}} = 1. \tag{5}$$

For the effective number of particle species emitted (N_s), we will take into account the various massless modes emitted. We will consider

$$N_s = \begin{cases} 2J+1 & \text{for} \quad 2J+1 < 112\ , \\ 112 & \text{for} \quad 2J+1 \geq 112\ . \end{cases}$$

In this way N_s is upper limited by the number of modes of massless particles in nature which make the dominant contribution to the black-hole spectrum (the 1/2, 3/2, 5/2 neutrino modes, the 1 and 2 photon modes, and the 2 and 3 graviton modes [8]), and by the size of the fuzzy sphere Hilbert space.

At this point it would be interesting to discuss the possible ways a BH could end. Specifically, if the BH will evaporate completely or give rise to a remnant. As we have said, due to the form of the spectrum (3), some deviations from the thermal behavior must occur in the final stages of BH evaporation. In the reference [7], these deviations have been related with a GUP.

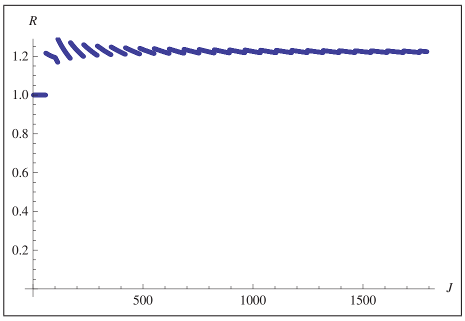
Some authors have argued that that the total BH evaporation is prevented by the GUP in exactly the same way that the Uncertainty Principle prevents the hydrogen atom from total collapse. In this way, a BH remnant must appear in the final of BH evaporation [9]. However, the remnant hypothesis suffers from some troubles. The main problem is, since the initial BH has been arbitrarily massive, the remnant must be capable of carrying an arbitrarily large amount of information. In this way, there must be an infinity of distinct, long-lived, species of remnant, with mass comparable with Planck mass, for each BH which could possibly be formed. As has been discussed in [10], it seems hard to reconcile the remnant infinite degeneracy with causality and unitarity.

On the other hand, as have been emphasized by [11], these problems are alleviated in topology change case, as the baby universes do not manifest themselves directly in the parent universe, and a finite number of long lived (but not eternal) remnants can arise from small BHs in the final stages of BH evaporation. In this context, the information stored in the remnant could return to the external universe in an unitary process. In this way, in our work, we shall consider that BH will evaporate completely.

In the fig. 3, we have plotted down R taking $\bar{\omega}_c \simeq 0.2$ (the location of the peak in the total power spectrum [8]). With this frequency cutoff, the minimal non-null value to the quantum number J, in order to have $\Gamma \neq 0$, is $J = 6.0$. In this point, the BH must evaporate completely. From the graphic for R, we have that the non-unitary evolution of the BH geometry in the main world, due to the topology change process, imposes obedience to a "second law of thermodynamics" on the BH evolution process, since R is ever larger than (or equal) to unity. The value of R approaches the value of 1.3 at the large J limit in agreement with known semiclassical results [12]

It is important to notice that the entropy emitted from the BH decreases as the area spacing increases. The striking consequence of this is the possibility that, since the BH radiation becomes less and less entropic as the evaporation process takes place, some information about the black hole initial state could leak out from its interior and be accessible to an observer in our universe, where we can perform measures. The possibility of information leakage from a BH with a discrete area spectrum is already pointed out by Hod [8].

Concerning this point, information could leak out from the BH during the final stages of evaporation through quantum tunneling. In fact, in a famous article, Parikh and Wilczek [13], by analyzing Hawking radiation as tunneling, have been shown that the radiation spectrum



cannot be strictly thermal because of the BH back reaction. This result has been confirmed recently in the reference [14], where the authors have found the existence of correlations among Hawking radiations which are described as hidden messengers in BH evaporation, in a way that information can be recovered during the emission process. Other interesting approach is due to Corda, who interpreted BH quasi-normal models in terms of quantum levels and the Hawking quanta in terms of "jumps" like the levels [15], and recently found a time dependent Schrodinger equation for the system composed by Hawking radiation and BH quasi-normal models which permits pure states to evolve in pure states [16].

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