Consistent Quantum Theory
Exercises for Ch. 2
(Version of 1 May 2003)

2.1 Given a wave packet $\psi(x)$, let

$$\phi(x) = \psi(x - x_0),$$

where x_0 is a positive constant.

a) Why is it reasonable to suppose that $\phi(x)$ represents a particle displaced by a distance x_0 to the right (i.e., to larger x) of the particle represented by $\psi(x)$?

b) Suppose that

$$\chi(x) = e^{ikx}\psi(x),$$

where k is a constant. Provide a similar physical interpretation of what $\chi(x)$ represents relative to $\psi(x)$. [Hint: Consider the Fourier transform.]

2.2 a) Suppose that

$$\phi(x) = c\psi(x/a),$$

where $a > 0$ is a constant. Find c so that $\|\phi\| = \|\psi\|$.

b) If the particle described by $\psi(x)$ is located between x_1 and x_2, what can you say about the location of the particle described by $\phi(x)$?

c) Suppose a physicist interprets $\psi(x)$ as representing a particle with momentum p within an uncertainty of Δp. What should be the corresponding interpretation of $\phi(x)$?

2.3 Let

$$\psi(x) = \begin{cases} 1 & \text{for } -\pi < x < \pi, \\ 0 & \text{elsewhere.} \end{cases}$$

a) For what values of k is the wave function

$$\phi_k(x) = \begin{cases} e^{ikx} & \text{for } -\pi < x < \pi, \\ 0 & \text{elsewhere.} \end{cases}$$

orthogonal to $\psi(x)$?

b) Same question, but with $\psi(x)$ defined by:

$$\psi(x) = \begin{cases} 1 & \text{for } 0 < x < 2\pi, \\ 0 & \text{elsewhere.} \end{cases}$$

2.4 Let $\psi(m, n)$ be the wave packet for a toy particle moving in two dimensions, where m and n are integers in the ranges

$$-M_a \le m \le M_b, \quad -N_a \le n \le N_b.$$

a) What is the inner product formula, the counterpart of (2.24)? How should the momentum wave packet $\hat{\psi}$, the counterpart of (2.29), be defined? What will be the counterpart of (2.31)?

b) For $M_a = M_b = N_a = N_b = 1$, construct a simple wave packet for which the momentum in the n (or y) direction is zero, whereas that in the m (or x) direction is not zero.