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Measuring Inconsistency over Sequences of Business Rule Cases

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Abstract

We investigate inconsistency and culpability measures for multisets of business rule bases. As companies might encounter thousands of rule bases daily, studying not only individual rule bases separately, but rather also their interrelations, becomes necessary. As current works on inconsistency measurement focus on assessing individual rule bases, we therefore present an extension of those works in the domain of business rules management. We show how arbitrary culpability measures (for single rule bases) can be automatically transformed for multisets, propose new rationality postulates for this setting, and investigate the complexity of central aspects regarding multi-rule base inconsistency measurement.

1 Introduction

In the context of Business Process Management, *business rules* are commonly used to govern company processes (Graham 2007). To this aim, business rules are modelled to capture (legal) regulations as a declarative business logic. For example, consider the set of business rules in Figure 1 with the intuitive meaning that we have two rules stating that 1) platinum customers are credit worthy, and 2) customers with a mental condition are not credit worthy. Given a new process instance (denoted as a *case*), case-dependent facts can be evaluated against the set of business rules for reasoning at run-time. For example, in Figure 1, the facts from a new loan application can be evaluated against the rule set to reason about the case. Note that "facts" as discussed in this work refer to static "case attributes" (e.g., the case-dependent (customer) data on the loan application).

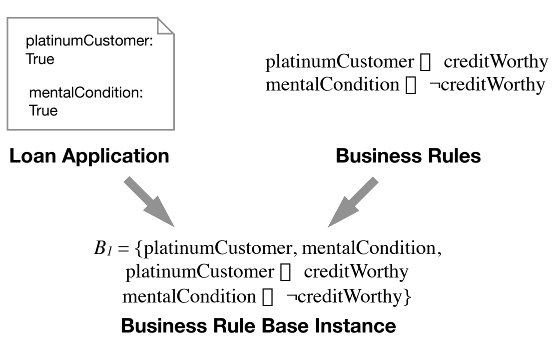

Figure 1: Exemplary business rule base instance $\mathcal{B}_1$.

The observant reader might have noticed, that the shown example yields an inconsistency, i.e., the contradictory conclusions $creditWorthy, \neg creditWorthy$. In fact, this is a current problem for companies, which can result from modelling errors in the business rules, or unexpected (case-dependent) facts. This problem has widely been acknowledged and has been addressed by a series of recent works, cf. e.g. (Corea, Deisen, and Delfmann 2019; Di Ciccio et al. 2017; Corea and Thimm 2020).

While existing results allow to handle inconsistencies in a *single* business rule base instance, in practice, companies often face thousands of such instances daily. For example, the retailer Zalando reported that 37 million cases were executed in the first quarter of 2020 alone[1]. As we will show in this work, considering not only single rule bases individually, but rather the entirety of all cases and their *interrelations*, can yield valuable insights, especially in regard to inconsistency resolution. For example, consider the following rule set, and assume there were four customer cases (with respective case-dependent facts), yielding the set of business rule cases $\mathcal{M}_1$ shown in Figure 2.

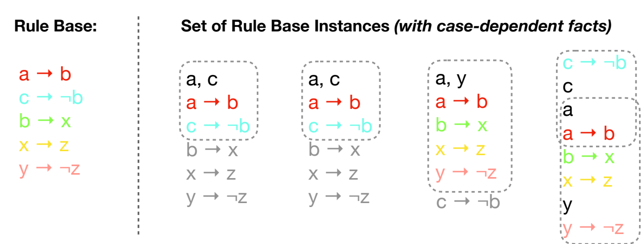

Figure 2: Exemplary rule base instances, constructed over a seq. of case-dependent facts.

When auditing such an overview of rule base instances, it is important for the company to identify which specific rules were responsible for these inconsistencies from a global perspective, as pin-pointing the culprits of inconsistency is necessary for determining suitable resolution and re-modelling strategies. Here, new methods are needed that support companies in assessing which individual rules are highly problematic over all cases. For instance, in Figure 2, the rule

[1]https://zln.do/2SFRnfC

$a \to b$ is part of all inconsistencies (over all cases) and can therefore be seen as highly problematic. In this work, we therefore introduce novel means for an element-based assessment of inconsistency over a set of rule base instances by extending results from the field of *inconsistency measurement* (Thimm 2019). Here, our contribution is as follows:

We present a novel approach to transform arbitrary inconsistency measures for an application in a multiset of business rule bases and propose postulates that should be satisfied by respective measures for this use-case (Section 3). Here, we also conduct initial experiments with real-life data sets. Furthermore, we examine the complexity of central aspects regarding inconsistency measurement in multisets of business rule bases (Section 4). We present preliminaries in Section 2 and conclude in Section 5. Proofs for technical results are provided in a supplementary document[2].

2 Preliminaries

Business Rule Bases. In this work, we consider a basic (monotonic) logic programming language to formalise business rule bases. A *(business) rule base* is then constructed over a finite set $\mathcal{A}$ of atoms, with $\mathcal{L}$ being the corresponding set of literals, with a rule base $\mathcal{B}$ being a set of rules r of the form

$$r: \quad l_1, \ldots, l_m \to l_0. \tag{1}$$

with every $l_i \in \mathcal{L}$. Let $\mathbb{B}$ denote all such rule bases. Also, we denote $head(r) = l_0$ and $body(r) = \{l_1, \ldots, l_m\}$. If $body(r) = \emptyset$, r is called a *fact*. For a rule base $\mathcal{B}$, we denote $\mathcal{F}(\mathcal{B}) \subseteq \mathcal{B}$ as the facts in $\mathcal{B}$ and $\mathcal{R}(\mathcal{B}) \subseteq \mathcal{B}$ as the rules in $\mathcal{B}$.

Example 1. *We recall the business rule base $\mathcal{B}_1$. Then we have*

$$\mathcal{F}(\mathcal{B}_1) = \{mentalCondition, platinumCustomer\}$$
$$\mathcal{R}(\mathcal{B}_1) = \{platinumCustomer \to creditWorthy, \; mentalCondition \to \neg creditWorthy\}.$$

A set of literals M is called *closed* w.r.t. $\mathcal{B}$ if it holds that for every rule of the form 1: if $l_1, \ldots, l_m \in M$ then $l_0 \in M$. The *minimal model* of a rule base $\mathcal{B}$ is the smallest closed set of literals (w.r.t. set inclusion). A set M of literals is called consistent if it does not contain both a and $\neg a$ for an atom a. We say a rule base $\mathcal{B}$ is consistent if its minimal model is consistent. If $\mathcal{B}$ is not consistent, we say $\mathcal{B}$ is inconsistent, denoted as $\mathcal{B} \models \perp$.

To assess inconsistency, the field of *inconsistency measurement* has evolved, which studies quantitative measures to assess the severity of inconsistency (Grant and Martinez 2018; Thimm 2019). An inconsistency measure is a function $\mathcal{I} : \mathbb{B} \to \mathbb{R}^{\infty}_{\geq 0}$, where a higher value $\mathcal{I}(\mathcal{B})$ reflects a higher degree, or severity, of inconsistency. A basic inconsistency measure is the $\mathcal{I}_{\mathsf{MI}}$ inconsistency measure, which counts the number of minimal inconsistent subsets MI of a rule base $\mathcal{B}$, defined via

$$\mathsf{MI}(\mathcal{B}) = \{M \subseteq \mathcal{B} \mid M \models \perp, \forall M' \subset M : M' \not\models \perp\}.$$

[2] https://bit.ly/3qIikOw

For example, in $\mathcal{B}_1$, there is one minimal inconsistent subset, consequently, $\mathcal{I}_{\mathsf{MI}}(\mathcal{B}_1) = 1$.

As the concept of a "severity" of inconsistency is not easily characterisable, numerous inconsistency measures have been proposed, see (Thimm 2019) for an overview. To guide the development of inconsistency measures, various rationality postulates have been proposed, cf. (Thimm 2017) for an overview. For example, a widely agreed upon property is that of *consistency*, which states that an inconsistency measure should return a value of 0 w.r.t. a rule base $\mathcal{B}$ iff $\mathcal{B}$ is consistent. Various other postulates exist and we will revisit some of them later when introducing measures for multisets of rule bases.

Measuring Inconsistency in Multisets of Business Rule Bases. In this work, we are not only interested in measuring inconsistency in single business rule bases, but rather in a series of corresponding business rule base instances. As motivated in the introduction, companies currently apply a set of business rules in order to assess a stream of (case-dependent) fact sets. Therefore, given a stream of fact sets $f = \mathcal{F}_1, ..., \mathcal{F}_n$, we consider multisets of business rule bases which are constructed by matching the individual fact sets in f to a shared rule set $\mathcal{R}$. To clarify, a multiset of rule bases is an n-tuple $\mathcal{M} = (\{\mathcal{F}_1 \cup \mathcal{R}\}, ..., \{\mathcal{F}_n \cup \mathcal{R}\}) = (\mathcal{B}_1, ..., \mathcal{B}_n)$. Let $\mathbb{M}$ denote all such multisets.

3 Culpability Measures for Multisets of Business Rule Bases

In the field of inconsistency measurement, a culpability measure $\mathcal{C}$ (Daniel 2009) is a function that assigns a non-negative numerical value to *elements* of a rule base. This quantitative assessment is also referred to as an *inconsistency value* (Hunter and Konieczny 2010). Again, the intuition is that a higher inconsistency value reflects a higher blame that the specific element carries in the context of the overall inconsistency. Such measures are useful to identify the culprits of inconsistency and therefore represent a good candidate for an application in assessing highly problematic rules in a multiset of rule bases. In this section, we therefore investigate culpability measures for sequences of business rule bases.

3.1 Baseline Approach and Basic Properties

Given a multiset of business rule bases $\mathcal{M}$, let $\mathcal{R}(\mathcal{M})$ denote the shared rule set of the respective business rule bases in $\mathcal{M}$. Furthermore, let $\mathbb{R}_{\mathbb{M}}$ denote the set of all possible rules that can appear in these shared rule sets. Then, a culpability measure for a multiset of rule bases (short: multi-$\mathcal{B}$ measure) is defined as follows.

Definition 1 (Multi-$\mathcal{B}$ Culpability Measure)**.** *A culpability measure for a multiset of rule bases is a function $\mathcal{C}^m : \mathbb{M} \times \mathbb{R}_{\mathbb{M}} \to \mathbb{R}^{\infty}_{\geq 0}$.*

To derive concrete multi-$\mathcal{B}$ measures, we propose to transform culpability measures (for single rule bases) for a multiset use-case. We denote such transformed measures as Σ-induced ("sum"-induced) culpability measures.

Definition 2 (Σ-induced multi-$\mathcal{B}$ culpability measure)**.** *Given a culpability measure $\mathcal{C}$, a multiset of rule-bases $\mathcal{M}$*

and a rule $r \in \mathcal{R}(\mathcal{M})$, a Σ-induced multi-$\mathcal{B}$ culpability measure $m^{\Sigma}_{\mathcal{C}}$ is defined as $m^{\Sigma}_{\mathcal{C}} : \mathbb{M} \times \mathbb{R}_{\mathbb{M}} \rightarrow \mathbb{R}^{\infty}_{\geq 0}$ with $m^{\Sigma}_{\mathcal{C}}(\mathcal{M}, r) = \sum_{B \in \mathcal{M}} \mathcal{C}(B, r)$.

The main idea of the proposed approach is that arbitrary existing culpability measures can be transformed for a multiset use-case. Here, it is however important that desirable properties of the existing measures are preserved during this transformation. We therefore propose the following rationality postulates by adapting postulates for traditional culpability measures (Hunter and Konieczny 2010). For that, we consider a multiset of business rules $\mathcal{M}$ and a rule $r \in \mathcal{R}(\mathcal{M})$. Also, we define a rule $r \in \mathcal{R}(\mathcal{M})$ as a *free formula* if $r \notin M, \forall M \in \bigcup_{b \in \mathcal{M}} \mathsf{MI}(b)$. We denote the set of all free formulas of $\mathcal{R}(\mathcal{M})$ as $\mathsf{Free}(\mathcal{M})$. We then propose the following postulates.

Rule Symmetry **(RS)** $\mathcal{C}^m(\mathcal{M}, r) = \mathcal{C}^m((\mathcal{B}_1, ...\mathcal{B}_n), r)$, for any permutation of the order of $\mathcal{B}_1$ to $\mathcal{B}_n$.

Rule Minimality **(RM)** if $r \in \mathsf{Free}(\mathcal{M})$, then $\mathcal{C}^m(\mathcal{M}, r) = 0$.

The first postulate states that the order of rule bases in the multiset should not affect the inconsistency value of an individual rule. The second postulate is adapted from the postulate MIN[3] and states that the inconsistency value of a rule is zero if this rule is a free formula w.r.t. the multiset of business rule bases.

Proposition 1. *Any Σ-induced multi-$\mathcal{B}$ culpability measure satisfies RS. Given a culpability measure $\mathcal{C}$ satisfying MIN, any Σ-induced multi-$\mathcal{B}$ culpability measure (via $\mathcal{C}$) satisfies RM.*

Given a multiset of business rules $\mathcal{M}$ and a multi-$\mathcal{B}$ culpability measure $\mathcal{C}^m$, we consider all rules of $\mathcal{R}(\mathcal{M})$ as a vector $(r_1, ...r_n)$, and denote $V^{\mathcal{C}^m}(\mathcal{M})$ as the vector of corresponding multi-$\mathcal{B}$ culpability values of all rules in $\mathcal{R}(\mathcal{M})$ w.r.t. $\mathcal{C}^m$, i.e., $V^{\mathcal{C}^m}(\mathcal{M}) = (\mathcal{C}^m(\mathcal{M}, r_1), ..., \mathcal{C}^m(\mathcal{M}, r_n))$. Next, let $\hat{V}^{\mathcal{C}^m}(\mathcal{M}) = max_{r \in \mathcal{R}(\mathcal{M})}(\mathcal{C}^m(\mathcal{M}, r))$ denote the largest multi-$\mathcal{B}$ culpability value w.r.t. $\mathcal{C}^m$ for all rules. Last, we denote adding a rule r to the shared rule set $\mathcal{R}(\mathcal{M})$ of a multiset $\mathcal{M}$ as $\mathcal{M} \cup \{r\}$ by a slight missuse of notation, i.e., given $\mathcal{M} = (\mathcal{B}_1, ..., \mathcal{B}_n), \mathcal{M} \cup \{r\} = (\mathcal{B}_1 \cup \{r\}, ..., \mathcal{B}_n \cup \{r\})$. This allows to adapt some further desirable properties for multi-$\mathcal{B}$ measures.

Multiset Consistency **(CO)** $\hat{V}^{\mathcal{C}^m}(\mathcal{M}) = 0$ iff $\nexists \mathcal{B} \in \mathcal{M} : \mathcal{B} \models \perp$.

Multiset Monotony **(MO)** Let a multiset of business rule bases $\mathcal{M}$ and a rule r, $\hat{V}^{\mathcal{C}^m}(\mathcal{M} \cup \{r\}) \geq \hat{V}^{\mathcal{C}^m}(\mathcal{M})$

Multiset Free formula independence **(IN)** If a rule r is a free formula of $(\mathcal{M} \cup \{r\})$, then $\hat{V}^{\mathcal{C}^m}(\mathcal{M} \cup r) = \hat{V}^{\mathcal{C}^m}(\mathcal{M})$

The first property states that the largest multi-$\mathcal{B}$ culpability value for a rule can only be zero if all business rule bases of the multiset are consistent. The second property demands that adding a rule to the shared rule set can only increase the culpability values. Similar to this property, the third postulate demands that adding a free formula to the shared rule set does not alter the culpability values.

[3]Let a rule $r \in \mathcal{B}$, if $r \notin M, \forall M \in \mathsf{MI}(\mathcal{B})$, then the inconsistency value of r is zero.

3.2 Initial Measures and Future Steps

Various culpability measures have been proposed (cf. e.g. (McAreavey, Liu, and Miller 2014)) and could therefore be transformed for a multiset use-case via Σ-induction. As this work is an initial "applications" investigation, we will however leave a detailed discussion of concrete multi-$\mathcal{B}$ culpability measures for future work. Still, we will present some initial baseline measures to showcase the proposed approach of Σ-induction.

Two baseline culpability measures proposed in (Hunter, Konieczny, and others 2008) are the $\mathcal{C}_D$ and $\mathcal{C}_{\#}$ measures.

Definition 3. *Let a rule base $\mathcal{B}$ and a rule $r \in \mathcal{B}$, then*

- $\mathcal{C}_D(\mathcal{B}, r) = \begin{cases} 1 & \text{if } \exists M \in \mathsf{MI}(\mathcal{B}) : r \in M \\ 0 & \text{otherwise} \end{cases}$
- $\mathcal{C}_{\#}(\mathcal{B}, r) = |\{M \in \mathsf{MI}(\mathcal{B}) \mid r \in M\}|$

Using Σ-induction, we can use these baseline culpability measures to entail the multi-rb culpability measures $m^{\Sigma}_{\mathcal{C}_D}$ and $m^{\Sigma}_{\mathcal{C}_{\#}}$.

Example 2. *We recall the multiset of rule bases $\mathcal{M}_1$ from Figure 2. For the shown rule $a \rightarrow b$, we have that*

$$m^{\Sigma}_{\mathcal{C}_D}(\mathcal{M}_1, a \rightarrow b) = 4$$
$$m^{\Sigma}_{\mathcal{C}_{\#}}(\mathcal{M}_1, a \rightarrow b) = 5$$

We see that the Σ-induced measures are able to compute the desired assessment of culpability over a series of corresponding rule base instances. Also, we see that the desirable properties of the original measures are transfered via our proposed transformation approach, shown in Table 1.

$\mathcal{C}^m$	RS	RM	CO	MO	IN
$m^{\Sigma}_{\mathcal{C}_D}$	✓	✓	✓	✓	✓
$m^{\Sigma}_{\mathcal{C}_{\#}}$	✓	✓	✓	✓	✓

Table 1: Compliance with rationality postulates of the investigated measures.

It thus seems that compliance management and auditing is an interesting application domain for the field of inconsistency measurement. Here, the initial approach proposed in this work can be used to transform existing culpability measures for a multi-set use-case. Future work should further focus on other means to support companies in analyzing process data.

On a last note, we would like to mention that besides designing specific culpability measures, an important approach in element-based analysis is also to decompose the assessment of inconsistency measures (in order to derive corresponding culpability measures) by means of Shapley inconsistency values (cf. (Hunter and Konieczny 2010)). Given an inconsistency measure $\mathcal{I}$ and a rule base $\mathcal{B}$, the intuition is that the overall blame mass $\mathcal{I}(\mathcal{B})$ is distributed amongst all elements in $\mathcal{B}$, by applying results from game theory.

The advantage of this approach is that arbitrary inconsistency measures can be applied to derive a corresponding element-based assessment. Thus, the Shapley inconsistency value S^I can also be used to Σ-induce $m^{\Sigma}_{S^I}$. Here, regarding the relation of inconsisteny measures and the corresponding $\Sigma-$induced Shapley inconsistency values for multi-rb analysis, we propose the following postulates.

***Distribution* (DIS)** $\sum_{\alpha \in \mathcal{R}(\mathcal{M})} m^{\Sigma}_{S*^I}(\mathcal{M}, \alpha) = m^{\Sigma}_{I}(\mathcal{M})$

***Upper Bound* (UB)** $\hat{V}^{S*^I}(\mathcal{M}) \leq m^{\Sigma}_{I}(\mathcal{M})$

The first postulate states that the sum adjusted multi-rb Shapley inconsistency values over all rules is equal to the overall blame mass of the original multi-rb inconsistency measure $\mathcal{I}$ (used as a parameter to derive the corresponding Shapley values). Also, the second property states that the adjusted multi-rb Shapley inconsistency values for an individual element cannot be greater than the overall assessment of the original multi-rb inconsistency measure $\mathcal{I}$.

Proposition 2. *Given an inconsistency measure I and a Shapley inconsistency value S^I, any multi-rb measure $m^{\Sigma}_{S^I}$ satsisfies DIS and UB.*

3.3 Motivational Example: BPI Dataset 2020

To evaluate the plausibility of applying our approach in practice, we conducted experiments with the real-life data set of the *Business Process Intelligence (BPI) challenge 2020*[4] (log of a travel expense claim process with 10,500 cases). From the log, we mined a shared rule base R (using the tool from (Di Ciccio et al. 2017)). Then, for all cases $C_1, ..., C_n$, the individual case-dependent fact inputs (case-properties, such as "travel expense cost") $F_1, ..., F_n$ were extracted from the log. We then constructed a multiset of rule bases $B_1, ..., B_n$, where every $B_i = (R, F_i)$. We then computed inconsistencies over $B_1, ..., B_n$.

A central assumption of our approach is that a global perspective over all cases should be considered as opposed to viewing cases individually. Interestingly, this was confirmed by our experiments: For every individual rule base instance, we computed the $\mathcal{C}_{\#}$ values for all rules and then ranked all rules by this value (rank 1 meaning that this rule is the most problematic element, and so on). Figure 3 shows the distribution of all assigned ranks for the rules for the BPI'20 data set over all cases. For readability, rules that did not participate in any inconsistencies are omitted.

While there were some rules that had the same rank in all cases (e.g. r2), there were many rules where the respective local rankings had a large variability over the cases (e.g. r3). This shows that regarding cases individually is not sufficient, but rather, the interrelations of all cases must be considered in the scope of auditing.

4 Complexity Analysis

Especially when wanting to apply results from KRR in practice, the computational complexity is important in regard to feasibility. In the following, we therefore investigate initial complexity aspects related to measuring inconsistency over a sequence of business rule base cases.

[4]https://data.4tu.nl/search?q=bpi+challenge

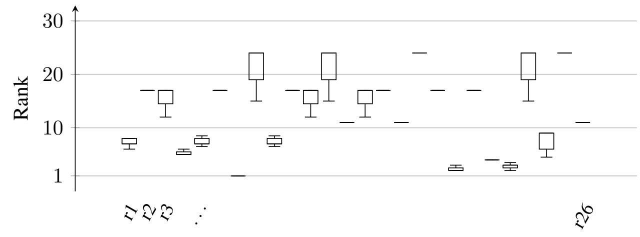

Figure 3: Rank distribution for the individual rules of the BPI'20 rule set over all cases.

We assume familiarity with basic concepts of computational complexity and basic complexity classes such as P and NP, see (Papadimitriou 1994) for an introduction. We first observe that the satisfiability problem for our formalism of business rules bases is tractable (note that similar observations have been made before on similar formalisms, see e. g., (Dantsin et al. 1997)).

Proposition 3. *Let $\mathcal{B}$ be a rule base. The problem of deciding whether $\mathcal{B}$ is consistent can be solved in pol. time.*

Then, the complexity of deciding whether a certain rule is contributing to the overall inconsistency is as follows.

Proposition 4. *Let $\mathcal{M}$ be a multiset of rule bases with $\mathcal{M} = (\mathcal{B}_1, ..., \mathcal{B}_n) = (\{\mathcal{F}_1 \cup \mathcal{R}\}, ..., \{\mathcal{F}_n \cup \mathcal{R}\})$ and let $r \in \mathcal{B}$. The problem of deciding whether there is a $i \in \{1, \ldots, n\}$ and $M \in$ MI$(\mathcal{B}_i)$ s. t. $r \in M$ is NP-complete.*

The following two results deal with the computational complexity of computing the baseline measure $\mathcal{C}_{\#}$.

Proposition 5. *Let $\mathcal{B}$ be a rule base and $M \subseteq \mathcal{B}$. The problem of deciding whether $M \in$ MI$(\mathcal{B})$ can be solved in polynomial time.*

For our final result note that #P is the complexity class of counting problems where the problem of deciding whether an element has to be counted is in P, cf. (Valiant 1979).

Proposition 6. *Let $\mathcal{M}$ be a multiset of rule bases with $\mathcal{M} = (\mathcal{B}_1, ..., \mathcal{B}_n) = (\{\mathcal{F}_1 \cup \mathcal{R}\}, ..., \{\mathcal{F}_n \cup \mathcal{R}\})$ and let $r \in \mathcal{R}$. The problem of determining $|\{M \in$ MI$(\mathcal{B}_i) \mid i \in \{1, \ldots, n\} r \in M\}|$ is #P-complete.*

5 Conclusion

In this work, we have shown how arbitrary culpability measures (for single rule bases) can be automatically transformed into multi-$\mathcal{B}$ measures while maintaining desirable properties. This is highly needed in practice, as companies are often faced with thousands of rule bases daily, and thus need means to assess inconsistency from a global perspective. As a main takeaway, our initial experiment results indicate that the interrelations of individual cases need to be considered for business rules management, and current means did not suffice to support companies in this aim. It thus seems that the field of business rules management, respectively auditing company processes, is an interesting application domain for KRR and bares many opportunities for future work.

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