MAGSAC: marginalizing sample consensus – supplementary material

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1. Comparison with methods getting the ground truth noise level

In this section, we show that the proposed method, MAGSAC, leads to results superior to the competitor algorithms even when they get the ground truth noise level σ. To get the noise level for each image pair, first, we fit a model to the manually selected inliers provided in the datasets. Finally, the threshold was set, independently for each image pair, to the distance of the farthest inlier from the model which was fit to all of the inliers.

The results for the problems and datasets (same as what was used in the paper) are reported in Table 1. It can be seen that the methods having the ground truth noise level are more accurate than by using a fixed σ (i.e. the tests shown in the paper). However, MAGSAC is still superior in terms of geometric accuracy. The results of MAGSAC were copied from the paper.

2. Full derivation to get Eq. 3

Let $0 = \sigma_0 \leq D(\theta, p_1) = \sigma_1 < D(\theta, p_2) <= \sigma_2 .... < D(\theta, p_K) = \sigma_K < \sigma_{max} < D(\theta, p_{K+1}) < ... < D(\theta, p_{|\mathcal{P}|})$ and using the quality function of plain RANSAC $Q(\theta, \sigma, \mathcal{P}) = |I(\theta, \sigma, \mathcal{P})| = |I(\theta, \sigma_{i-1}, \mathcal{P})| = i - 1$ for all $\sigma \in [\sigma_{i-1}, \sigma_i)$. In this case quality function Q^* is derived as follows:

$$Q^*(\theta, \mathcal{P}) = \frac{1}{\sigma_{max}} \int_0^{\sigma_{max}} |I(\theta, \sigma, \mathcal{P})| \mathrm{d}\sigma$$

$$= \sum_{i=1}^{K} \int_{\sigma_{i-1}}^{\sigma_i} |I(\theta, \sigma_{i-1}, \mathcal{P})| \mathrm{d}\sigma + \int_{\sigma_K}^{\sigma_{max}} |I(\theta, \sigma_K, \mathcal{P})| \mathrm{d}\sigma$$

$$= \frac{1}{\sigma_{max}} \sum_{i=1}^{K} (i-1)(D(\theta, p_i) - D(\theta, p_{i-1}) + \frac{1}{\sigma_{max}} K(\sigma_{max} - D(\theta, p_K))$$

$$= K - \frac{1}{\sigma_{max}} \sum_{i=1}^{K} D(\theta, p_i) = \sum_{i=1}^{K} \left(1 - \frac{D(\theta, p_i)}{\sigma_{max}}\right).$$

Assuming the distribution of inliers and outliers to be uniform (inlier $\sim \mathcal{U}(0, \sigma)$; outlier $\sim \mathcal{U}(0, l)$) and using log-likelihood of model θ as its quality function Q, the likelihood becomes

$$L(\theta, \mathcal{P} | \sigma) = \left(\frac{1}{\sigma}\right)^{|I(\theta, \sigma, \mathcal{P})|} \left(\frac{1}{l}\right)^{|\mathcal{P}| - |I(\theta, \sigma, \mathcal{P})|}$$

and $Q(\theta, \sigma, \mathcal{P}) = \ln L(\theta, \mathcal{P} | \sigma) = |I(\theta, \sigma, \mathcal{P})| (\ln l - \ln \sigma) - |\mathcal{P}| \ln l$. The marginalized quality function Q^* is as follows:

$$Q^*(\theta, \mathcal{P}) =$$

$$\frac{1}{\sigma_{max}} \int_0^{\sigma_{max}} (|I(\theta, \sigma, \mathcal{P})| (\ln l - \ln \sigma) - |\mathcal{P}| \ln l) \mathrm{d}\sigma$$

$$= \frac{1}{\sigma_{max}} \sum_{i=1}^{K} \int_{\sigma_{i-1}}^{\sigma_i} (|I(\theta, \sigma_{i-1}, \mathcal{P})| (\ln l - \ln \sigma) - |\mathcal{P}| \ln l) \mathrm{d}\sigma$$

$$+ \frac{1}{\sigma_{max}} \int_{\sigma_K}^{\sigma_{max}} (|I(\theta, \sigma_K, \mathcal{P})| (\ln l - \ln \sigma) - |\mathcal{P}| \ln l) \mathrm{d}\sigma$$

$$= \frac{\ln l}{\sigma_{max}} \sum_{i=1}^{K} (i-1)(D(\theta, p_i) - D(\theta, p_{i-1}))$$

$$+ \frac{\ln l}{\sigma_{max}} K(\sigma_{max} - D(\theta, p_K))$$

$$- \frac{1}{\sigma_{max}} \sum_{i=1}^{K} (i-1) \int_{\sigma_{i-1}}^{\sigma_i} \ln \sigma \mathrm{d}\sigma$$

$$- \frac{1}{\sigma_{max}} K \int_{\sigma_K}^{\sigma_{max}} \ln \sigma \mathrm{d}\sigma - |\mathcal{P}| \ln l$$

$$= K \left(\ln \frac{l}{\sigma_{max}} + 1\right)$$

$$- \frac{1}{\sigma_{max}} \sum_{i=1}^{K} D(\theta, p_i) \left(1 + \ln \frac{l}{D(\theta, p_i)}\right) - |\mathcal{P}| \ln l.$$

3. Full derivation to get Eq. 8

For the stopping criterion of RANSAC with a minimal sample of size m and fixed σ, at least $k(\theta, \sigma, \mathcal{P})$ samples have to be drawn. Having the set of inliers $I(\theta, \sigma, \mathcal{P})$, k is calculated as follows:

$$k(\theta, \sigma, \mathcal{P}) = \frac{\ln(1-\mu)}{\ln(1 - \left(\frac{I(\theta,\sigma,\mathcal{P})}{|\mathcal{P}|}\right)^m)}.$$

Let $k^*(\theta, \mathcal{P})$ be the stopping criterion of MAGSAC and assume that $\sigma_1 = 0$, where k^* is k marginalized over σ as follows:

$$\begin{aligned} k^*(\theta, \mathcal{P}) &= \frac{1}{\sigma_{max}} \int_0^{\sigma_{max}} k(\theta, \sigma, \mathcal{P}) \mathrm{d}\sigma \\ = \frac{\ln(1-\mu)}{\sigma_{max}} (\sum_{i=1}^{K} \int_{\sigma_{i-1}}^{\sigma_i} & \frac{1}{\ln(1 - \left(\frac{I(\theta,\sigma,\mathcal{P})}{|\mathcal{P}|}\right)^m)} \mathrm{d}\sigma \\ &+ \int_{\sigma_K}^{\sigma_{max}} \frac{1}{\ln(1 - \left(\frac{I(\theta,\sigma,\mathcal{P})}{|\mathcal{P}|}\right)^m)} \mathrm{d}\sigma) \\ = \frac{\ln(1-\mu)}{\sigma_{max}} (\sum_{i=1}^{K} & \frac{\sigma_i - \sigma_{i-1}}{\ln(1 - \left(\frac{I(\theta,\sigma_{i-1},\mathcal{P})}{|\mathcal{P}|}\right)^m)} \\ &+ \frac{\sigma_{max} - \sigma_K}{\ln(1 - \left(\frac{I(\theta,\sigma_K,\mathcal{P})}{|\mathcal{P}|}\right)^m)}). \end{aligned}$$

			RANSAC	MSAC	LO-RANSAC	LO-MSAC	**MAGSAC**
kusvod2	**F**, 24	e_{avg}	0.87	0.79	0.67	0.54	**0.38**
		t	50	22	18	**15**	31
		s	1118	474	224	**150**	382
Adelaide	**F**, 19	e_{avg}	0.35	0.34	**0.28**	**0.28**	0.30
		t	529	454	423	**384**	939
		s	3 376	2 841	2 488	**2 231**	2 638
Multi-H	**F**, 4	e_{avg}	0.74	0.72	0.50	0.52	**0.47**
		t	45	**40**	79	78	467
		s	81	54	68	**52**	1 324
homogr	**H**, 16	e_{avg}	3.02	3.16	2.77	2.92	**1.37**
		t	23	**18**	40	38	131
		s	567	434	364	**313**	877
EVD	**H**, 15	e_{avg}	4.25	3.84	3.72	2.99	**1.76**
		t	292	251	172	**150**	162
		s	6 317	5 453	3 452	2 788	**2 239**
strecha	**E**, 467	e_{avg}	6.64	6.86	8.48	8.55	**6.51**
		t	9 781	9 039	3 745	3 726	**2 398**
		s	10 949	10 060	3 964	3 745	**2 183**
all		e_{avg}	2.65	2.62	2.74	2.62	**1.80**
		e_{med}	1.95	1.98	1.72	1.73	**0.92**
		t	1 787	1 637	746	732	**688**

Table 1: *Accuracy of robust estimators on two-view geometric estimation.* Fundamental matrix estimation (**F**) on `kusvod2` (24 pairs), `AdelaideRMF` (19 pairs) and `Multi-H` (4 pairs) datasets, homography estimation (**H**) on `homogr` (16 pairs) and `EVD` (15 pairs) datasets, and essential matrix estimation (**E**) on the `strecha` dataset (467 pairs). In total, the testing included 545 image pairs. The datasets, the problem, the number of the image pairs ($\#$) and the reported properties are shown in the first three columns. The other columns show the average results (100 runs on each image pair) of the competitor methods at 95% confidence. The mean geometric error (e_{avg}; in pixels) of the estimated model w.r.t. the manually selected inliers are written in each 1st row; the mean processing time (t, in milliseconds) and the required number of samples (s) are written in every 2nd and 3rd rows. The geometric error is the RMS Sampson distance for **F** and **E**, and the RMS re-projection error for **H** using the ground truth inlier set. The threshold was set for each image pair independently to the value which the manually selected inliers imply. For MAGSAC, $\sigma_{max} = 10$ pixels. The MAGSAC results are copied from the paper.