SKYRMIONS WITH VECTOR MESONS: SINGLE SKYRMION AND BARYONIC MATTER

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SEMINAR @ BLTP, JINR

KNU 경북대학교

CONTENTS

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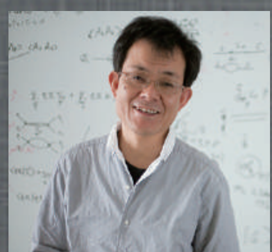


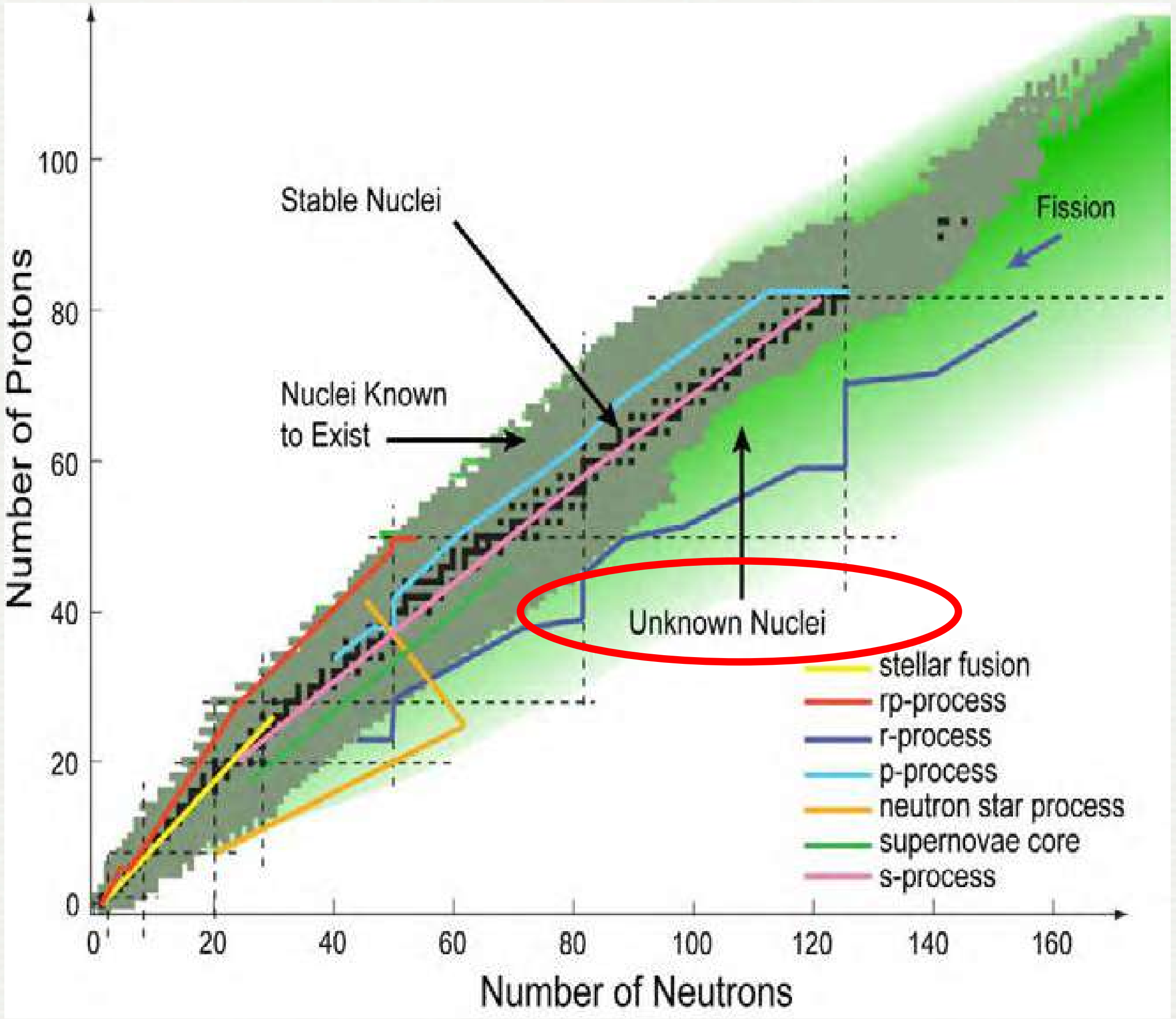
References:

Y.-L. Ma, Y. Oh, G.-S. Yang, M. Harada, H.K. Lee, B.-Y. Park, and M. Rho, *Hidden local symmetry and infinite tower of vector mesons for baryons*, **Phys. Rev. D86, 074025 (2012)**

Y.-L. Ma, G.-S. Yang, Y. Oh, and M. Harada, *Skyrmions with vector mesons in the hidden local symmetry approach*, **Phys. Rev. D87, 034023 (2013)**

Y.-L. Ma, M. Harada, H.K. Lee, Y. Oh, B.-Y. Park, and M. Rho, *Dense baryonic matter in hidden local symmetry approach: Half-Skyrmions and nucleon mass*, **arXiv:1304.5638 (submitted to Phys. Rev. D)**

MOTIVATION



Nuclear Physics → Hadron Physics → Nuclear Physics

IBS & RAON


Fushun
Liaoyang
Liaoning
North Korea
Pyongyang
East Sea
Chuncheon
South Korea
Japan
Seoul
Incheon
Suwon
Cheongju
Daejeon
Daegu
Ulsan
Jeonju
Gwangju
Busan
Yeosu
Mokpo
Tongyeong
Goheung
Jindo
Jeju
Seogwipo
Matsue
Tottori
Kanazawa
Fukui
Gifu
Nagoya
Tsu
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Matsuyama
Tokushima
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Fukuoka
Oita
Saga
Nagasaki
Kumamoto
Miyazaki
Kagoshima

The 7th BLTP JINR-APCTP Joint Workshop

"Modern problems in nuclear and elementary particle physics",

July 14-19, 2013

Russia, Irkutsk Region, Bolshiye Koty

July 17

9.00-9.40 Yongseok Oh (Kyngpook National University)
Skyrmions with vector mesons in hidden local symmetry

9.40-10.20 Byung-Yoon Park (Chungnam National University)
Dense Baryonic Matter in Hidden Local Symmetry

10.20-10.50 Coffee

SKYRME MODEL

1960s: T.H.R. Skyrme

Baryons are topological solitons within a nonlinear theory of pions.

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{1}{32e^2} \mathrm{Tr}\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

f_π : pion decay constant

e : Skyrme parameter

Topological soliton

winding number = baryon number

$$B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \mathrm{Tr}\left(U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U\right)$$

T.H.R. Skyrme: Proc. Roy. Soc. (London) 260, 127 (1961), Nucl. Phys. 31, 556 (1962)

REVIVAL

In large N_c, QCD ~ effective field theory of mesons and baryons may emerge as solitons in this theory.

E. Witten, 1980s

HEDGEHOG SOLUTION

$$U = \exp\left(iF(r)\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\right)$$

$R \sim 1$ fm

$$M_{\rm sol} \sim 146|B|\left(\frac{f_\pi}{2e}\right) \sim 1.2 \text{ GeV}$$

for $B = 1$

$$M_{\rm sol} \sim 1.23 \times 12\pi^2|B| > 12\pi^2|B|$$

in the Skyrme unit: $f_\pi/(2e)$

Bogomolny bound

13년 6월 26일 수요일

BARYON MASSES

- To give correct quantum numbers
  - SU(2) collective coordinate quantization

$$U(t) = A(t)U_0 A^\dagger(t)$$

  - Mass formula: infinite tower of $I = J$

$$M = M_{\rm sol} + \frac{1}{2\mathcal{I}} I(I+1) \qquad \mathcal{I}: \text{ moment of inertia}$$

$$M_N = M_{\rm sol} + \frac{3}{8\mathcal{I}}, \qquad M_\Delta = M_{\rm sol} + \frac{15}{8\mathcal{I}}$$

  - Adjust f_π and e to reproduce the nucleon and Delta masses

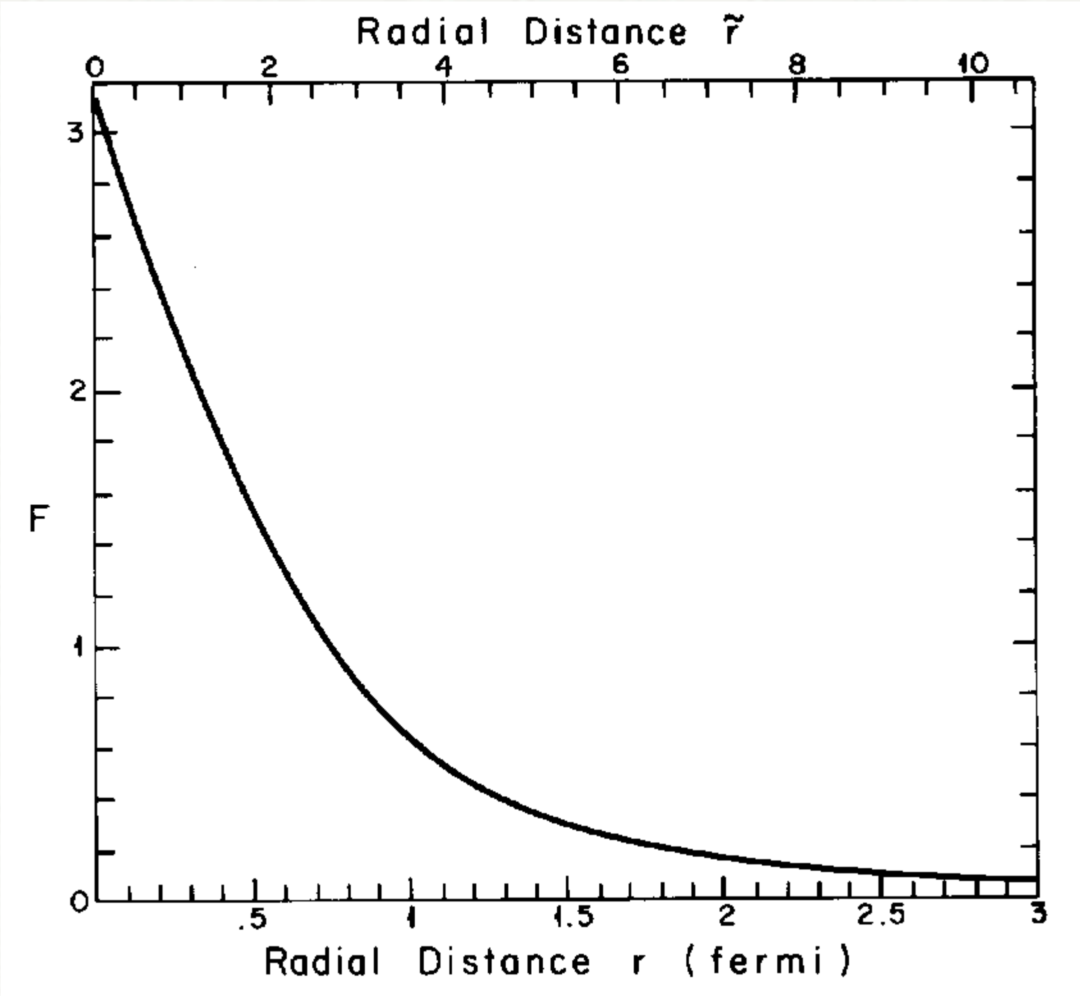
$$f_\pi = 64.5 \text{ MeV}, e = 5.45$$

Empirically, $f_\pi = 93 \text{ MeV}, e = 5.85(?)$

Skyrme model: results

- Best-fitted results

Quantity	Prediction	Expt
M_N	input	939 MeV
M_Δ	input	1232 MeV
$\langle r^2 \rangle^{1/2}_{I=0}$	0.59 fm	0.72 fm
$\langle r^2 \rangle^{1/2}_{M,I=0}$	0.92 fm	0.81 fm
μ_p	1.87	2.79
μ_n	-1.31	-1.91
$\lvert\mu_p/\mu_n\rvert$	1.43	1.46



G.S. Adkins, C.R. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983)
A.D. Jackson and M. Rho, Phys. Rev. Lett. 51, 751 (1983)

Skyrme model for Nuclear Physics

Single Baryon

Improvement of the model

- more degrees of freedom (mesons)
- $1/N_c$ corrections
- ChPT

Extension to other hadrons

- SU(3) extension to hyperons
- Heavy-quark baryons
- Hypernuclei & Exotic baryons

Nuclear Matter

Topics

- Properties of single baryon
- Equation of State
- Phase transition
- Application to nuclei

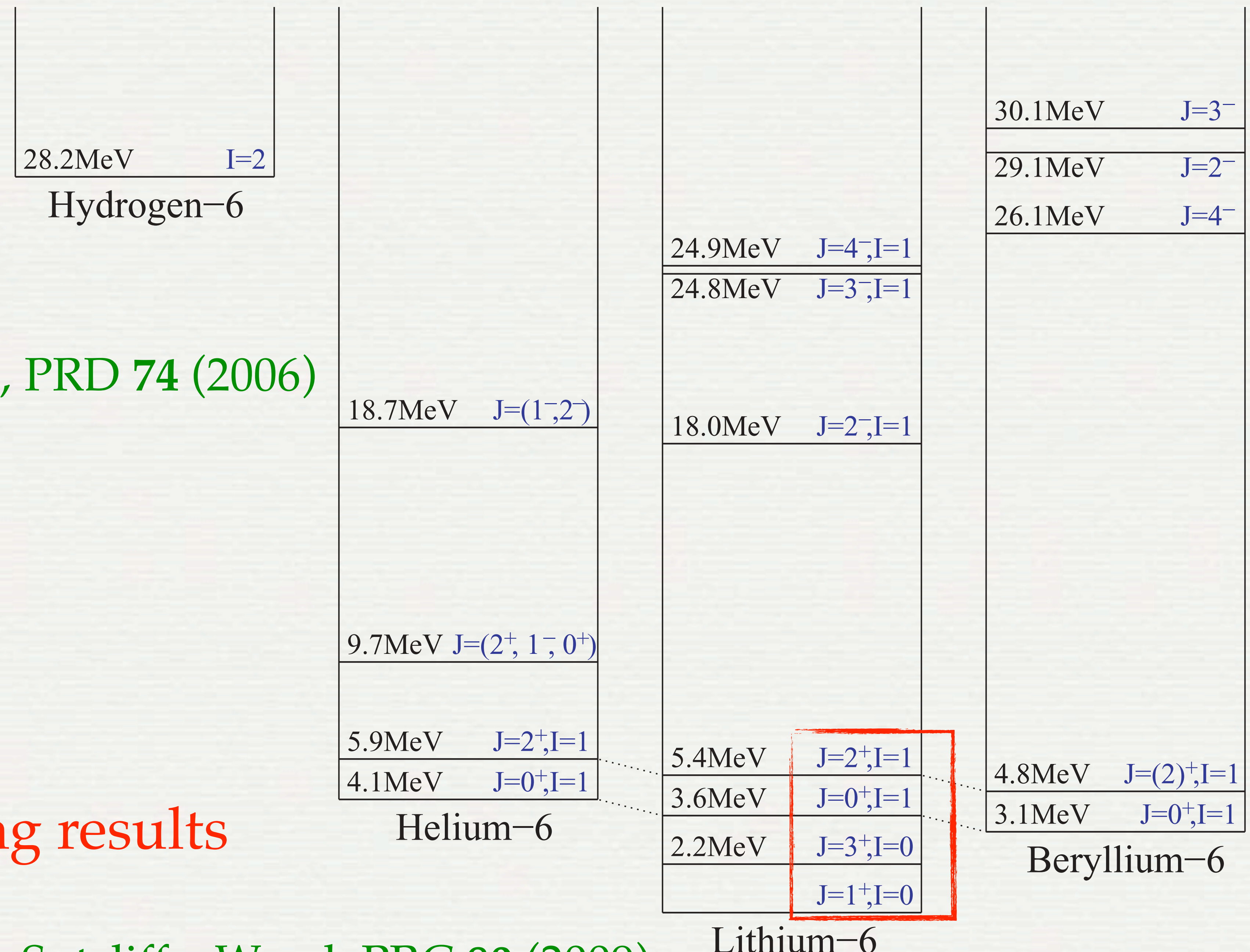
Approaches

- Modified Effective Lagrangian
- Skyrmion Crystal
- Winding number n solutions

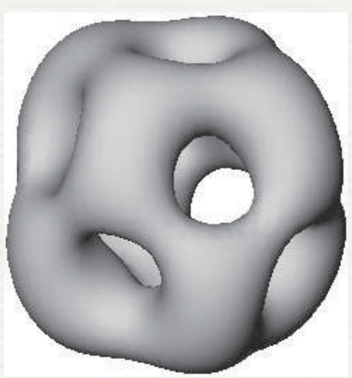
Still there are many works to do

Nuclei

Light nuclei in the Skyrme model (e.g., mass number 6)



Manton, Wood, PRD **74** (2006)



encouraging results

Battye, Manton, Sutcliffe, Wood, PRC **80** (2009)

NUCLEI

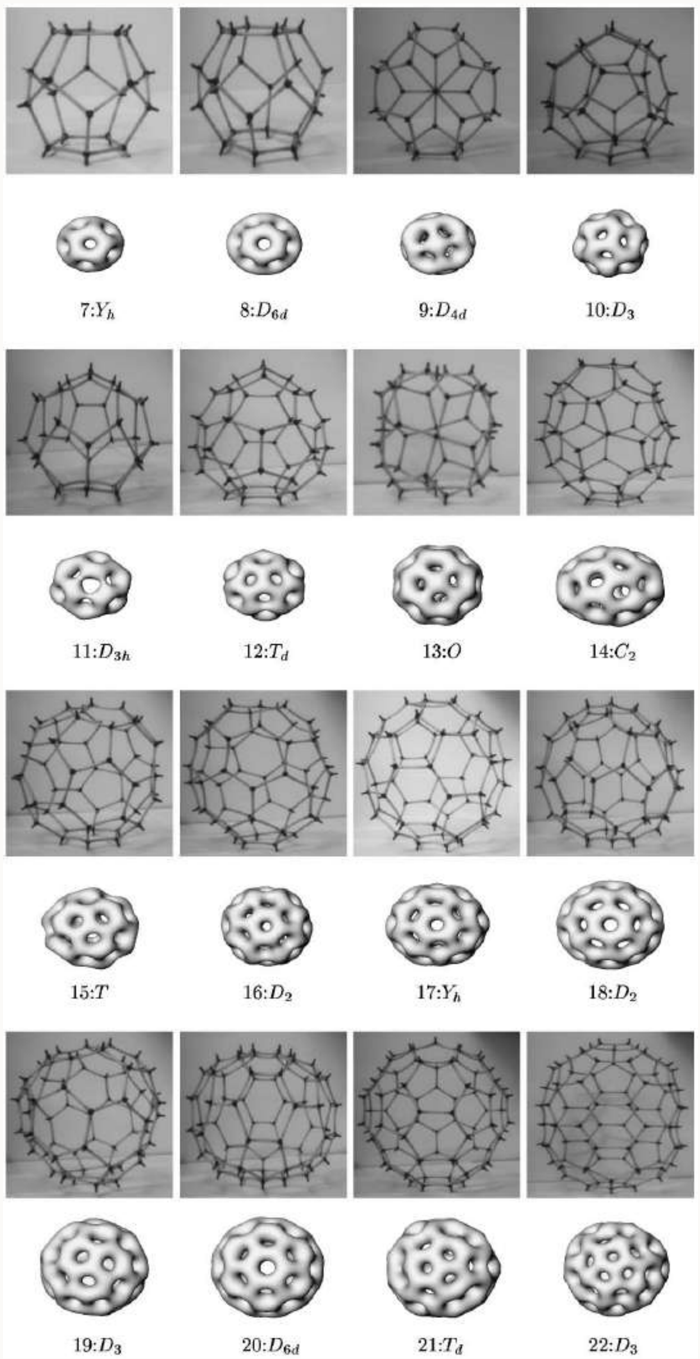
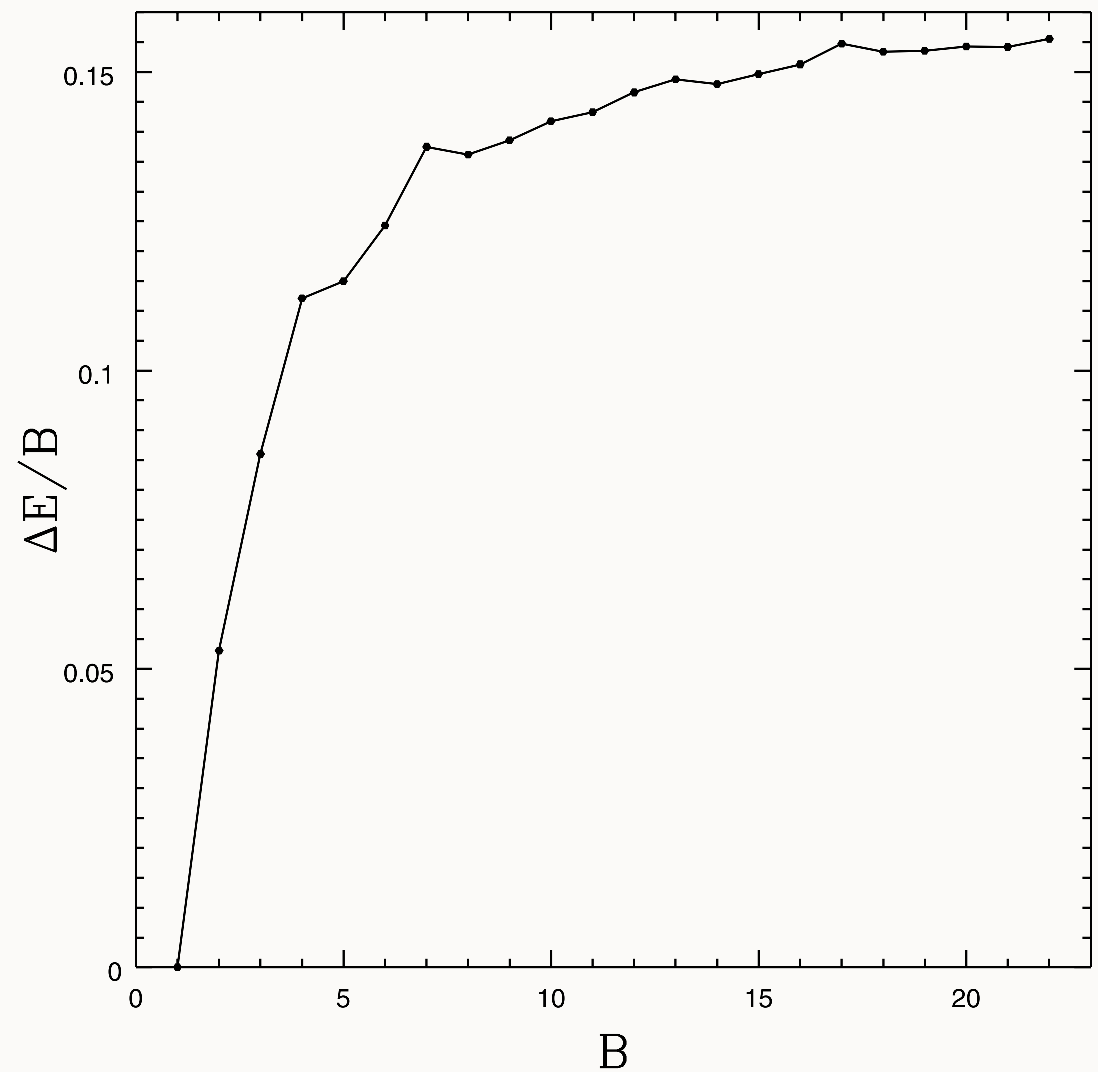


FIG. 1. The baryon density isosurfaces for the solutions which we have identified as the minima for $7 \leq B \leq 22$, and the associated polyhedral models. The isosurfaces correspond to $\mathcal{B} = 0.035$ and are presented to scale, whereas the polyhedra are not to scale.

multi--baryon-number Skyrmion



Battye, Sutcliffe, PRL **86** (2001) 3989

Single Baryon (with Pion Mass)

- Adding the pion-mass term

Adkins, Nappi, NPB **233** (1984)

$$\mathcal{L}_{\text{pion}} = \frac{1}{2} m_\pi^2 f_\pi^2 \left(\text{Tr}(U) - 2\right)$$

Quantity	Prediction (massless pion)	Prediction (massive pion)	Expt
M_N	input	input	939 MeV
M_Δ	input	input	1232 MeV
f_π	64.5 MeV	54 MeV	93 MeV
$\langle r^2 \rangle_{I=0}^{1/2}$	0.59 fm	0.68 fm	0.72 fm
$\langle r^2 \rangle_{I=1}^{1/2}$	∞	1.04 fm	0.88 fm
$\langle r^2 \rangle_{M,I=0}^{1/2}$	0.92 fm	0.95 fm	0.81 fm
$\langle r^2 \rangle_{M,I=1}^{1/2}$	∞	1.04 fm	0.80 fm
μ_p	1.87	1.97	2.79
μ_n	-1.31	-1.24	-1.91
$\lvert\mu_p/\mu_n\rvert$	1.43	1.59	1.46

Why vector mesons?

- Witten: QCD ~ weakly interacting mesons in large N_c
  - The lightest meson is the pion
  - The next low-lying mesons are vector mesons (ω and ρ)

- Stability of the soliton

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Skyrme terms

  - Without the Skyrme term, the soliton collapses. Derrick's Theorem
  - Vector mesons can stabilize the soliton without the Skyrme term.

Early Attempts to include VM

Including ω meson

$$\mathcal{L} = \mathcal{L}_{\text{pion}} + \mathcal{L}_{\omega} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{pion}} = \frac{f_\pi^2}{4}\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{f_\pi^2}{2} m_\pi^2 \left(\text{Tr}(U) - 2\right),$$

$$\mathcal{L}_{\omega} = \frac{m_\omega^2}{2}\omega_\mu \omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}, \qquad \boxed{\mathcal{L}_{\text{int}} = \beta \omega_\mu B^\mu}$$

G.S. Adkins and C.R. Nappi, Phys. Lett. B137, 251 (1984)

Including ρ meson

$$\mathcal{L} = \mathcal{L}_{\text{pion}} + \mathcal{L}_{\rho} + \mathcal{L}_{\text{int}}$$

$$\boxed{\mathcal{L}_{\text{int}} = \alpha \text{Tr}(\rho_{\mu\nu}\partial^\mu U^\dagger U \partial^\nu U^\dagger)}$$ $\rho\pi\pi$ interaction

G.S. Adkins, Phys. Rev. D33, 193 (1986)

Early Attempts: results

Quantity	Skyrme (massive pion)	ω	ρ	Expt
M_N	input	input	input	939 MeV
M_Δ	input	input	input	1232 MeV
f_π	54 MeV	62 MeV	52.4 MeV	93 MeV
$\langle r^2 \rangle^{1/2}_{I=0}$	0.68 fm	0.74 fm	0.70 fm	0.72 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	1.04 fm	1.06 fm	1.08 fm	0.88 fm
$\langle r^2 \rangle^{1/2}_{M,I=0}$	0.95 fm	0.92 fm	0.98 fm	0.81 fm
$\langle r^2 \rangle^{1/2}_{M,I=1}$	1.04 fm	1.02 fm	1.06 fm	0.80 fm
μ_p	1.97	2.34	2.16	2.79
μ_n	-1.24	-1.46	-1.38	-1.91
$\lvert\mu_p/\mu_n\rvert$	1.59	1.60	1.56	1.46
$\mu_{I=0}$	0.365	0.44	0.39	0.44
$\mu_{I=1}$	1.605	1.9	1.77	2.35

SU(3) EXTENSION

- Can we describe hyperons in the Skyrme model?
- Direct extension: SU(3) collective coordinate quantization
- New approaches
  - exact diagonalization methods
    Yabu, Ando, NPB **301** (1988)
    Weigel et al., PRD **42** (1990)
  - bound state model
    Callan, Klebanov, NPB **262** (1985)

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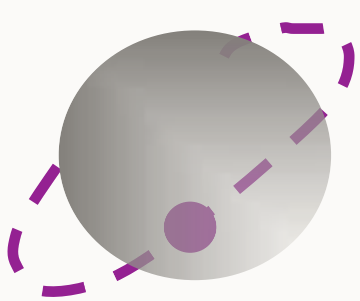
BOUND STATE MODEL

- Starting point: flavor SU(3) symmetry is badly broken
  - treat light flavors and strangeness on a different footing

$$\mathrm{SU}(3) \to \mathrm{SU}(2) \times \mathrm{U}(1)$$

  - Lagrangian $\mathcal{L} = \mathcal{L}_{\mathrm{SU}(2)} + \mathcal{L}_{K/K^*}$
  - The soliton provides a background potential that traps K/K* (or heavy) mesons



Callan, Klebanov, NPB **262** (1985)

BOUND STATE MODEL

- Anomalous Lagrangian
  - Pushes up the state of $S = +1$ states to the continuum → no bound state
  - Pulls down the state of $S = -1$ states below the threshold → makes bound states → description of hyperons
- Renders two bound states with $S = -1$
  - the lowest state: p-wave → gives (+) parity $\Lambda(1116)$

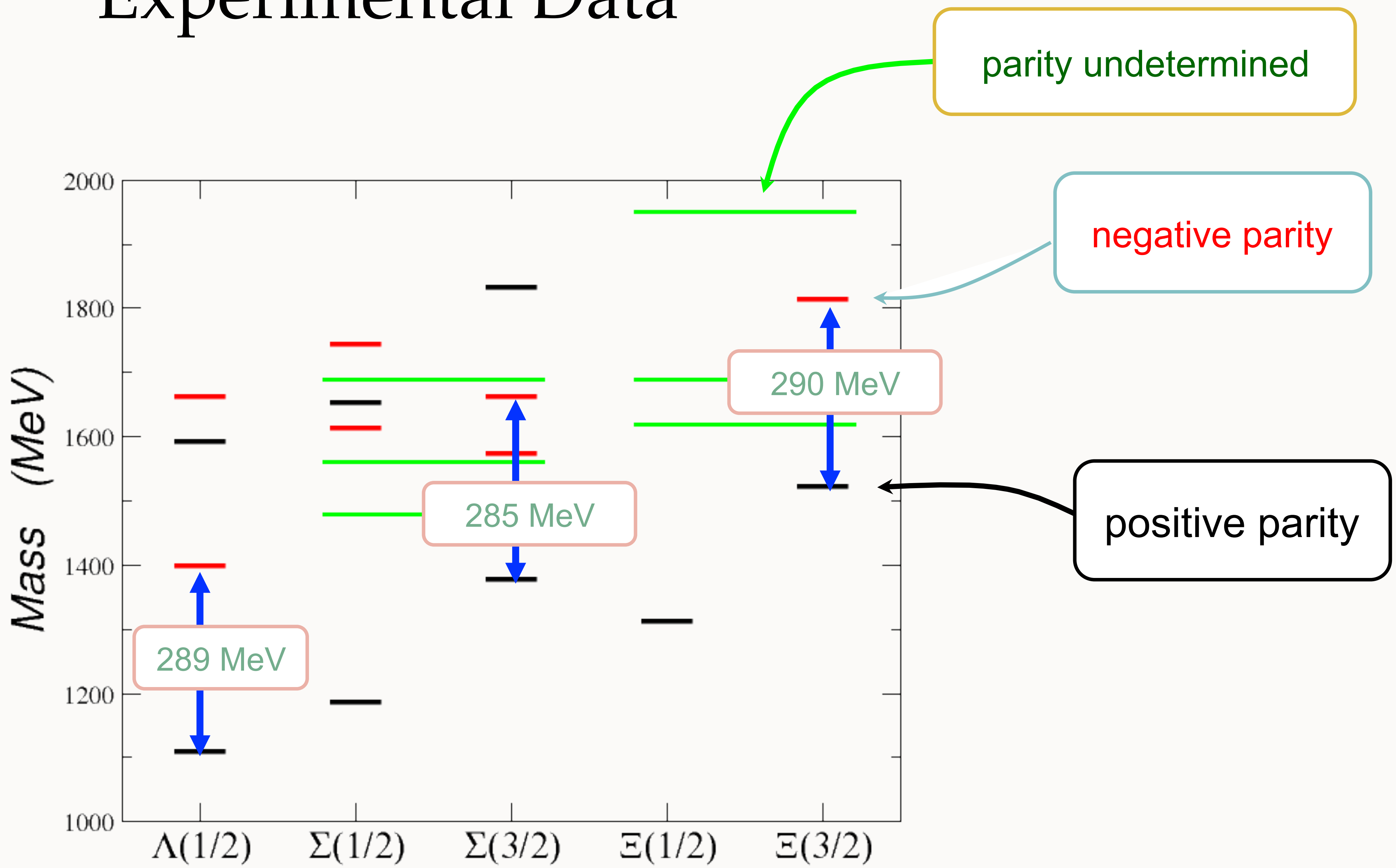
270 MeV energy difference

  - excited state: s-wave → gives (–) parity $\Lambda(1405)$

after quantization

BOUND STATE MODEL

Experimental Data



BOUND STATE MODEL

- Mass sum rules
  - modification to GMO and equal spacing rule

$$3\Lambda + \Sigma - 2(N + \Xi) = \Sigma^* - \Delta - (\Omega - \Xi^*)$$

$$(\Omega - \Xi^*) - (\Xi^* - \Sigma^*) = (\Xi^* - \Sigma^*) - (\Sigma^* - \Delta$$

  - hyperfine relation

$$\Sigma^* - \Sigma + \frac{3}{2}(\Sigma - \Lambda) = \Delta - N$$

  - The same relations hold for

$$\Lambda(1/2^-), \Sigma(1/2^-), \Sigma(3/2^-), \Xi(1/2^+), \Xi(3/2^+), \Omega(3/2^-)$$

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BOUND STATE MODEL

- Best-fitted results based on the derived mass formula

Particle	Prediction (MeV)	Expt
N	939*	N(939)
Δ	1232*	$\Delta(1232)$
$\Lambda(1/2^+)$	1116*	$\Lambda(1116)$
$\Lambda(1/2^-)$	1405*	$\Lambda(1405)$
$\Sigma(1/2^+)$	1164	$\Sigma(1193)$
$\Sigma(3/2^+)$	1385	$\Sigma(1385)$
$\Sigma(1/2^-)$	1475	$\Sigma(1480)$?
$\Sigma(3/2^-)$	1663	$\Sigma(1670)$
$\Xi(1/2^+)$	1318*	$\Xi(1318)$
$\Xi(3/2^+)$	1539	$\Xi(1530)$
$\Xi(1/2^-)$	1658 (1660)	$\Xi(1690)$?
$\Xi(1/2^-)$	1616 (1614)	$\Xi(1620)$?
$\Xi(3/2^-)$	1820	$\Xi(1820)$
$\Xi(1/2^+)$	1932	$\Xi(1950)$?
$\Xi(3/2^+)$	2120*	$\Xi(2120)$
$\Omega(3/2^+)$	1694	$\Omega(1672)$
$\Omega(1/2^-)$	1837	
$\Omega(3/2^-)$	1978	
$\Omega(1/2^+)$	2140	
$\Omega(3/2^+)$	2282	$\Omega(2250)$?
$\Omega(3/2^-)$	2604	

Recently confirmed by COSY
PRL ***96*** *(2006)*

BaBar : the spin-parity of $\Xi(1690)$ is $1/2^-$
PRD ***78*** *(2008)*
NRQM predicts $1/2^+$

puzzle in QM

Unique prediction of this model.
The $\Xi(1620)$ should be there.
still one-star resonance

Ω's would be discovered in future.

YO, PRD **75** (2007)

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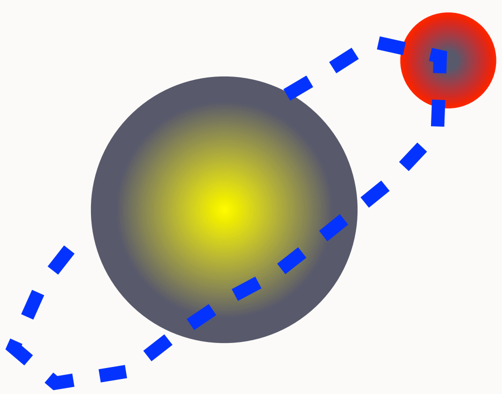
13년 6월 26일 수요일

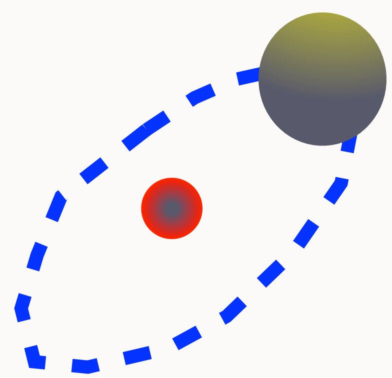
HEAVY QUARK BARYONS

- Replace the strangeness by the heavy-flavor
- $m_D/m_\pi \gg m_K/m_\pi$
- A dog wagging a tail?

large N_c vs. large m_Q



$(m_Q < M_{\rm sol})$



$(m_Q > M_{\rm sol})$

The two approaches converge only when both $N_c \to \infty$ and $m_Q \to \infty$

Heavy quark symmetry

HEAVY QUARK BARYONS

Soliton-fixed frame

Heavy-meson-fixed frame

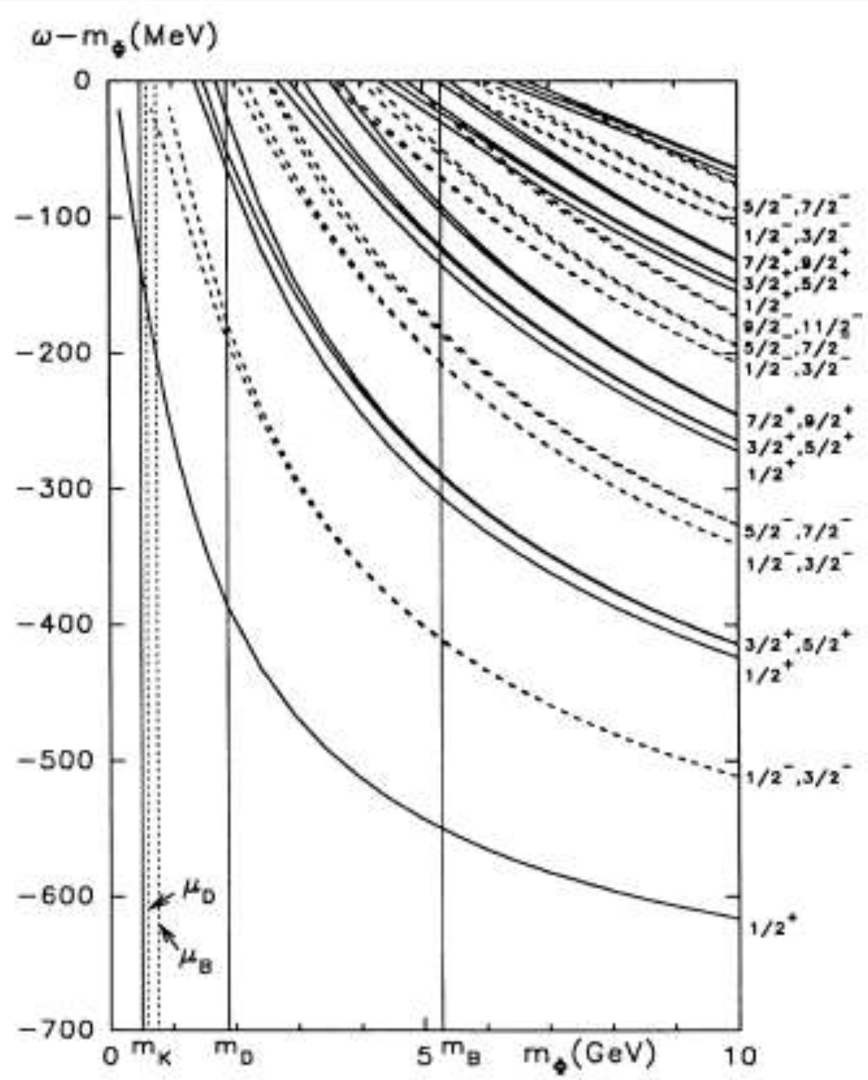


FIG. 4. Binding energies $\omega - m_\Phi$ of the bound states with k^π as functions of the heavy-meson mass. Solid (dashed) lines denote the positive (negative) parity states.

YO, B.Y. Park PRD **51** (1995)

Table 2. Numerical results on the bound states. Energies are given in MeV unit

(n, k_ℓ^π)	Set I	Set II	Set III	Set IV	Exp.
$(0, 0^+)$	−287	−461	−366	−588	−610
$(1, 0^+)$	−12	−62	−15	−79	–
$(0, 1^-)$	−89	−196	−113	−250	−320
$(0, 1^+)^a$	−17	−54	−21	−69	–

[a] Bound state of soliton to antiflavored heavy meson

300 MeV

YO, B.Y. Park ZPA **359** (1997)

fewer bound states

13년 6월 26일 수요일

Vector Mesons

- Systematic way to include vector mesons
  - Massive Yang-Mills approach — Syracuse group
  - Hidden Local Symmetry — Nagoya group
  - Equivalence of the two approaches

- Skyrmions in the HLS
  - ρ meson stabilized model

    Y. Igarashi et al., Nucl. Phys. B259, 721 (1985)
  - ρ and ω meson stabilized model

    U.-G. Meissner, N. Kaiser, and W. Weise, Nucl. Phys. A466, 685 (1987)
  - ρ, ω and a_1 meson stabilized model

    N. Kaiser and U.-G. Meissner, Nucl. Phys. A519, 671 (1990)

    L. Zhang and N.C. Mukhopadhyay, Phys. Rev. D50, 4668 (1994)

Recent Works for Skyrmions with Vector Mesons

Holographic QCD: infinite tower of vector mesons
Solitons in hQCD

D.K. Hong, M. Rho, H.-U. Yee, and P. Yi, Phys. Rev. D76, 061901 (2007); JHEP 0709, 063 (2007)
H. Hata, T. Sakai, S. Sugimoto, and S. Tamato, Prog. Theor. Phys. 117, 1157 (2007)

HLS Lagrangian
O(p^4) terms: M. Tanabashi, Phys. Lett. B316, 534 (1993)
O(p^4) terms & hQCD: M. Harada and K. Yamawaki, Phys. Rep. 381, 1 (2003)

Skyrmions in HLS with ρ meson up to O(p^4) terms with hQCD
K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D75, 086003 (2007)
K. Nawa, A. Hosaka, and H. Suganuma, Phys. Rev. D79, 126005 (2009)

Earlier works

O(p^2) Lagrangian with HLS

$$\mathcal{L}_\sigma = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad \text{with } U = \xi_L^\dagger \xi_R$$

Hidden Symmetry

$$\xi_{L,R}(x) \to h(x)\xi_{L,R}(x), \quad h \in \mathrm{SU}(2)$$

$$V_\mu(x) \to i h(x)\partial_\mu h^\dagger(x) + h(x)V_\mu(x)h^\dagger(x)$$

Covariant derivative: $D_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} - i V_\mu \xi_{L,R}$

$$\hat{\alpha}_{\mu\parallel} = \frac{1}{2i}(D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R^\dagger)$$

$$\hat{\alpha}_{\mu\perp} = \frac{1}{2i}(D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger)$$

Unitary gauge: $\xi_L^\dagger = \xi_R = \xi$

HLS Lagrangian

$$\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{\mathrm{kin}}$$

$$\mathcal{L}_A = f_\pi^2\,\mathrm{Tr}(\hat{\alpha}_{\mu\perp}^2) = \mathcal{L}_\sigma, \quad \mathcal{L}_V = f_\pi^2\,\mathrm{Tr}(\hat{\alpha}_{\mu\parallel}^2)$$

$$\mathcal{L}_{\mathrm{kin}} = -\frac{1}{2g^2}\,\mathrm{Tr}(F_{\mu\nu}^2)$$

$$m_V^2 = a g^2 f_\pi^2$$

$$g_{\rho\pi\pi} = \frac{1}{2} a g$$

$a = 2$ gives KSRF relation and the universality of ρ coupling

M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. 164, 217 (1988)

ρ meson and the Skyrme term

As $a \to \infty$, i.e., as $m_V \to \infty$

$$\mathcal{L}_V \propto (\alpha_{\mu\|} - V_\mu)^2 = 0$$

where $\alpha_{\mu\|} = \dfrac{1}{2i}(\partial_\mu \xi_L \xi_L^\dagger + \partial_\mu \xi_R \xi_R^\dagger)$

$\Rightarrow$

$$\mathcal{L}_{\rm kin} \to \frac{1}{32 g^2} \mathrm{Tr}\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2 = \mathcal{L}_{\rm Skyrme}$$

Skyrmion in the HLS with the ρ meson

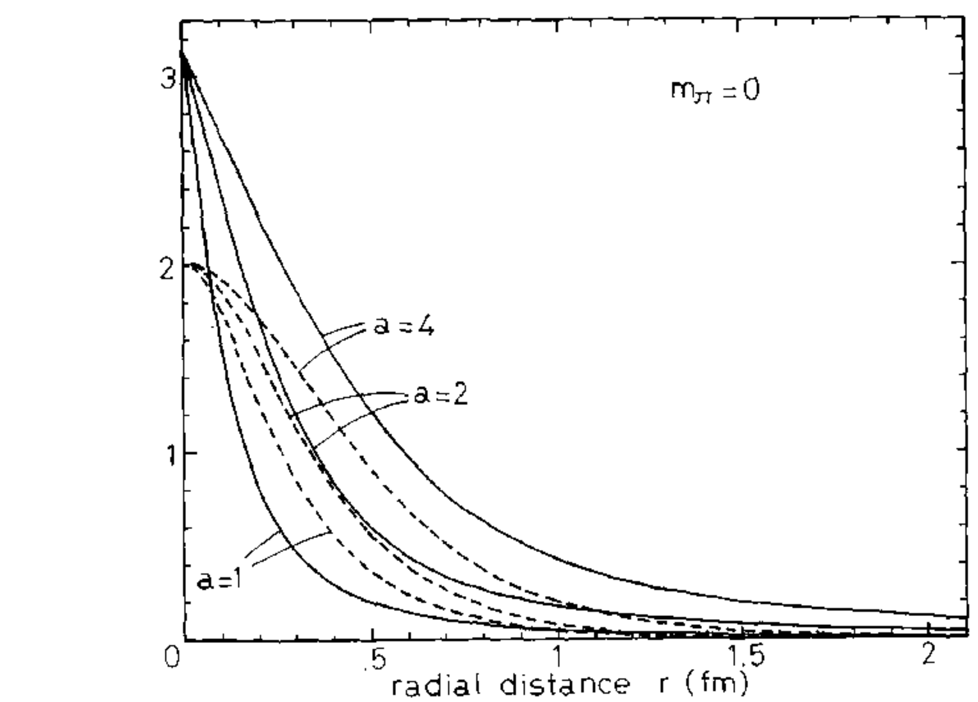


Fig. 1. $n = 1$ solutions of $F(r)$ and $G(r)$ (dashed curves) for $m_\pi = 0$ (chiral limit) and $a = 1$, 2 and 4 fixing $a g^2 f_\pi^2 = m_\rho^2$.

$$M_{\rm sol} = (667 \sim 1575)\ \mathrm{MeV}$$

for $1 \le a \le 4$

$$M_{\rm sol} = 1045\ \mathrm{MeV} \qquad \text{for } a = 2$$

Y. Igarashi, M. Johmura, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Nucl. Phys. B259, 721 (1985)

ρ and ω mesons

ω meson: introduced through HGS like the ρ meson

Anomalous Lagrangian: source of the ω meson

$$\begin{aligned}\mathcal{L}_{\rm an} &= \frac{3}{8} g N_c (c_1 - c_2 - c_3)\omega_\mu B^\mu \\ &- \frac{g^3 N_c}{32\pi^2}(c_1 + c_2)\varepsilon^{\mu\nu\alpha\beta}\omega_\mu \,\mathrm{tr}\,(a_\nu \bar{\rho}_\alpha \bar{\rho}_\beta) \\ &- \frac{g N_c}{8\pi^2} c_3 \varepsilon^{\mu\nu\alpha\beta}\left\{ -\omega_\mu \,\mathrm{tr}\,(a_\nu v_\alpha v_\beta) + \frac{ig}{4}\partial_\mu \omega_\nu \,\mathrm{tr}\,(a_\alpha \rho_\beta - \rho_\alpha a_\beta) - \frac{ig}{4}\omega_\mu \,\mathrm{tr}\,(\rho_{\nu\alpha} a_\beta)\right\},\end{aligned}$$

Determination of parameters

T. Fujiwara, T. Kugo, H. Terao, S. Uehara, K. Yamawaki, Prog. Theor. Phys., 73, 926 (1985)

Minimal model: $c_1 = \frac{2}{3}, c_2 = -\frac{2}{3}, c_3 = 0$ — *$\omega^\mu B_\mu$ term only*

Vector Dominance: $c_1 = 1, c_2 = 0, c_3 = 1$ — *No $\omega\pi^3$ term*

Or fit them to known phenomenology

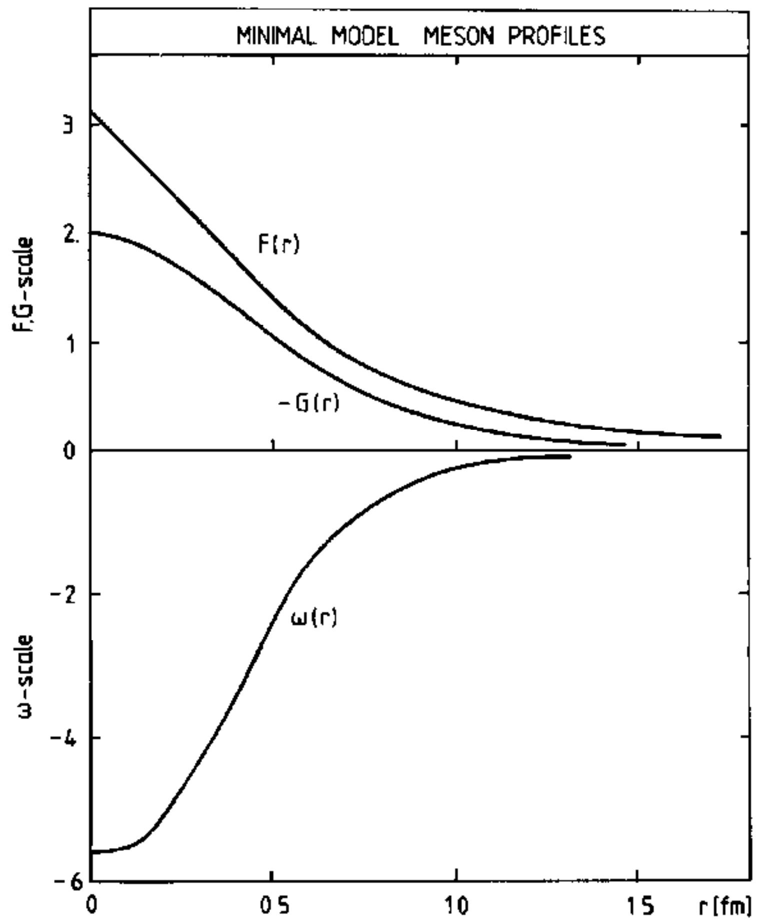
See, for example, P. Jain, U.-G. Meissner, N. Kaiser, H. Weigel, N.C. Mukhopadhyay, etc

U.-G. Meissner, N. Kaiser and W. Weise, Nucl. Phys. A466, 685 (1987)

ρ and ω mesons

minimal model results with a = 2, f_π *= 93 MeV, g = 5.85*

M_{sol} = *1475* MeV



U.-G. Meissner et al. / Nucleons as Skyrme solitons

TABLE 1

Properties of the Skyrme soliton resulting from the lagrangians (2.11) or (2.19) with π, ρ and ω mesons

	Minimal model	Complete model	Following ref. [17])
M_H [MeV]	1474	1465	1057
r_H [fm]	0.50	0.48	0.27

For comparison, the results of the model of ref. [17]) including pions and ρ mesons are also given. The parameters used are $m_\pi = 139$ MeV, $f_\pi = 93$ MeV, and $g = 5.85$. Here M_H is the static soliton mass, and r_H the baryonic r.m.s. radius.

U.-G. Meissner, N. Kaiser and W. Weise, Nucl. Phys. A466, 685 (1987)

ρ and ω mesons

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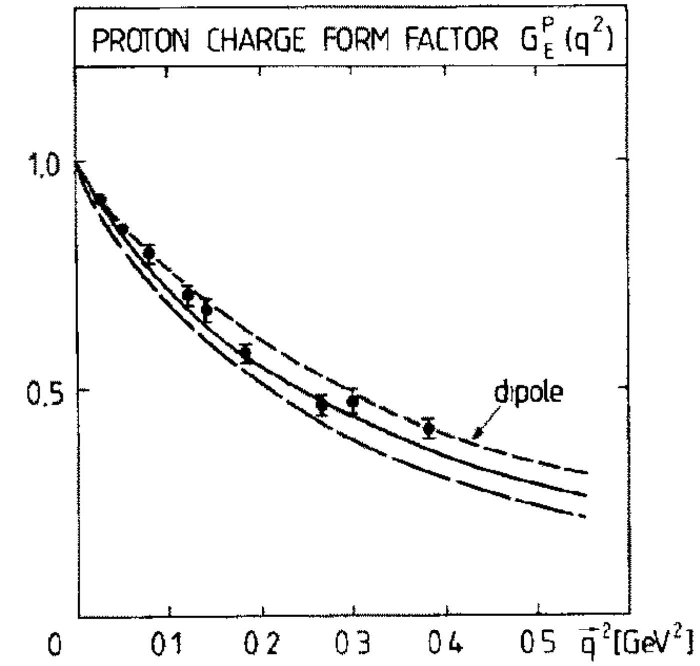
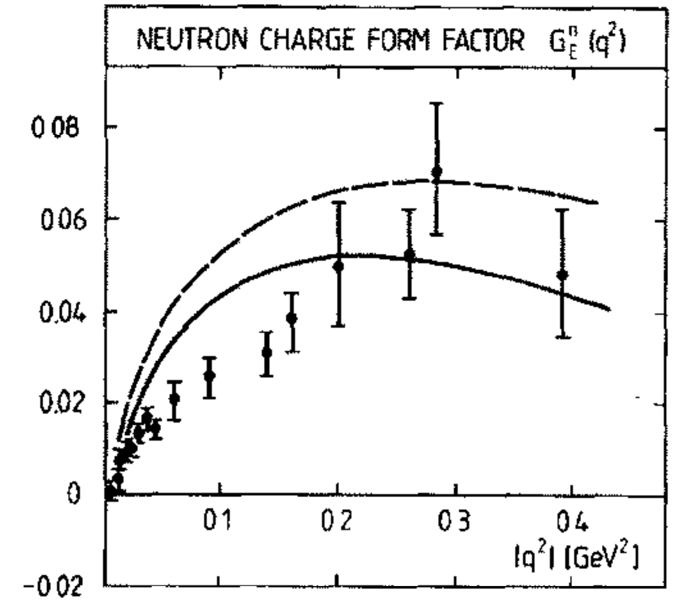
TABLE 2

Baryon properties; parameters as in table 1

	Minimal model	Complete model	Experiment
Θ [fm]	0.82	0.68	
$M_\Delta - M_N$ [MeV]	359	437	293
M_N [MeV]	1564	1575	939
$r_H \equiv \langle r_B^2 \rangle^{1/2}$ [fm]	0.50	0.48	
$\langle r_E^2 \rangle_p^{1/2}$ [fm]	0.92	0.98	0.86 ± 0.01
$\langle r_E^2 \rangle_n$ [fm^2]	-0.22	-0.25	-0.119 ± 0.004
$\langle r_M^2 \rangle_p^{1/2}$ [fm]	0.84	0.94	0.86 ± 0.06
$\langle r_M^2 \rangle_n^{1/2}$ [fm]	0.85	0.93	0.88 ± 0.07
μ_p [n.m.]	3.36	2.77	2.79
μ_n [n.m.]	-2.57	-1.84	-1.91
$\|\mu_p/\mu_n\|$	1.31	1.51	1.46





U.-G. Meissner, N. Kaiser and W. Weise, Nucl. Phys. A466, 685 (1987)

ρ, ω, and a_1 mesons

- Axial vector meson

$$U(x) = \xi_L^\dagger(x)\xi_M(x)\xi_R(x)$$

N. Kaiser and U.-G. Meissner, Nucl. Phys. A519, 671 (1990)
L. Zhang and N.C. Mukhopadhyay, Phys. Rev. D50, 4668 (1994)

- 14 anomalous terms

cf. 6 independent terms in the $\pi\rho\omega$ system

- Hard to control the parameters

Results with $a = 2$, $f_\pi = 93$ MeV, $g = g_\omega/1.5 = 5.85$, $m_V = 770$ MeV

$M_{sol} = 1002$ MeV

H. Forkel et al. / Skyrmions with vector mesons

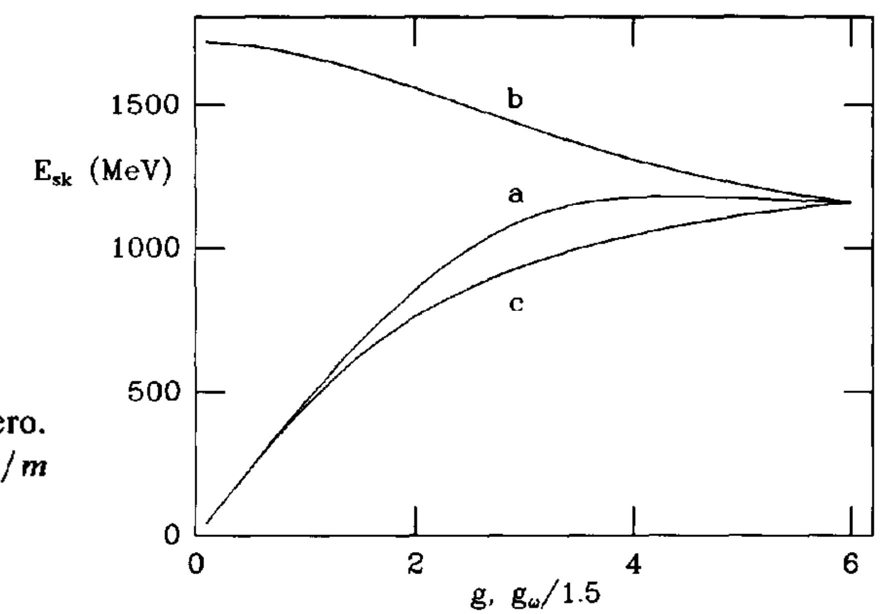


Fig. 1. The behaviour of the skyrmion energy as the vector meson couplings and the masses go to zero. (a) $g = g_\omega/1.5 \to 0$, (b) $g \to 0$, $g_\omega/1.5 = 5.85$ (fixed), (c) $g_\omega \to 0$, $g = 5.85$ (fixed). In all cases the ratios g/m and $g_\omega/1.5m$ are kept constant at 5.85/770 MeV.

H. Forkel, A.D. Jackson, and C. Weiss, Nucl. Phys. A526, 453 (1991)

ρ, ω, and a_1 mesons

The results are sensitive to the parameters.

TABLE VI. Nucleon observables in the $\pi\rho\omega a_1(f_1)$ chiral soliton model *without* the ϕ decay constraints (with all the energies in MeV). Here h_2 is calculated through Eq. (59) with $S_\omega > 0$.

Model	M_H	g_A	$g_{\pi NN}$	$\sigma_{\pi N}$
(1) $h_1 = 0.10$, $c_i' = 0, i = 2, \ldots, 6, 8, Z = 0.9$	1403	0.70	10.56	29.8
(2) $h_1 = -0.10$, $c_i' = 0, i = 2, \ldots, 6, 8, Z = 1.0$	1578	1.00	16.97	50.7
(3) $h_1 = -0.30$, $c_i' = 0, i = 2, \ldots, 6, 8, Z = 1.0$	1725	1.25	23.19	70.6
(4) $h_1 = 0.10$, $c_2' = -0.0020, c_8' = -0.13$, $c_i' = 0, i = 3, \ldots, 6, Z = 1.0$	1503	0.85	13.77	38.1
(5) $h_1 = 0.10, c_2' = -0.012$, $c_3' = 0.29, c_4' = -0.42, c_5' = 0.13$, $c_6' = -0.015, c_8' = -0.021, Z = 1.0$	1579	1.12	19.01	58.7
(6) $h_1 = 0.51, c_2' = -0.019$, $c_3' = -0.0022, c_4' = -0.029, c_5' = 0.53$, $c_6' = -1.2, c_8' = -0.094, Z = 1.0$	1379	0.90	13.30	43.5
The $\pi\rho\omega$ model[a]	1462	0.91	14.28	41.6
Expt.	939 ±0	1.26 ±0.01	13.45 ±0.05	45 ±10

[a]Reference [6].

TABLE III. Nucleon observables in the $\pi\rho\omega a_1(f_1)$ chiral soliton model with the ϕ decay constraints [Eq. (66)] put in (with all the energies in MeV).

Model	M_H	g_A	$g_{\pi NN}$	$\sigma_{\pi N}$
(Set A) $c_2' = c_3' = c_4' = c_5'$, $c_6' = c_8' = 0, Z = 0$	704	0.18	1.38	3.4
(Set B) $c_2' \approx 0.053, c_3' \approx -0.029$, $c_4' \approx 0.071, c_5' \approx 0.72$, $c_6' \approx -0.42, c_8' \approx -0.24$, $Z = 0.4$	1070	0.59	6.74	23.0
The $\pi\rho\omega$ model[a]	1462	0.91	14.28	41.6
Expt.	939 ±0	1.26 ±0.01	13.45 ±0.05	45 ±10

[a]Reference [6].

L. Zhang and N.C. Mukhopadhyay, Phys. Rev. D50, 4668 (1994)

Summary of the earlier works

1. ***a* dependence**

   - ambiguity in the value of *a* results in a large uncertainty in the soliton mass
   (in free space, $a \sim 2$ and at high temperature/density $a \sim 1$)

2. **Higher order terms**

   - $O(p^4)$ etc are at $O(N_c)$ like the $O(p^2)$ terms
   - More complicated form of the Lagrangian
   - Uncontrollably large number of low energy constants

   E.g. 6 anomalous terms for the ω meson at $O(p^2)$
   14 anomalous terms for the axial vector mesons at $O(p^2)$

3. **In this work,**

   - $O(p^4)$ with ρ and ω mesons
   - Fix the couplings by using hQCD

HLS Lagrangian up to O(p⁴)

$$\mathcal{L}_{\rm HGS} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\rm anom}$$

$$\mathcal{L}_{(2)} = f_\pi^2 \,\mathrm{Tr}\,(\hat{a}_{\perp\mu}\hat{a}_\perp^\mu) + a f_\pi^2 \,\mathrm{Tr}\left(\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu\right) - \frac{1}{2g^2}\mathrm{Tr}\,(V_{\mu\nu}V^{\mu\nu}),$$

$$\mathcal{L}_{(4)} = \mathcal{L}_{(4)y} + \mathcal{L}_{(4)z},$$

where

$$\begin{aligned}
\mathcal{L}_{(4)y} = {} & y_1 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\perp\nu}\hat{\alpha}_\perp^\nu\right] + y_2\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu\right] + y_3\mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\nu\right] + y_4\mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right] \\
& + y_5\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\parallel^\nu\right] + y_6\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right] + y_7\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\nu\hat{\alpha}_\parallel^\mu\right] \\
& + y_8\left\{\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\perp\nu}\hat{\alpha}_\parallel^\nu\right] + \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\perp^\nu\hat{\alpha}_\parallel^\mu\right]\right\} + y_9\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\parallel^\nu\right], \\
\mathcal{L}_{(4)z} = {} & i z_4 \mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu\right] + i z_5 \mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_\parallel^\mu\hat{\alpha}_\parallel^\nu\right].
\end{aligned}$$

$$\mathcal{L}_{\rm anom} = \frac{N_c}{16\pi^2}\sum_{i=1}^{3} c_i \mathcal{L}_i,$$

17 terms

where

$$\begin{aligned}
\mathcal{L}_1 &= i\,\mathrm{Tr}\left[\hat{\alpha}_{\rm L}^3\hat{\alpha}_{\rm R} - \hat{\alpha}_{\rm R}^3\hat{\alpha}_{\rm L}\right], \\
\mathcal{L}_2 &= i\,\mathrm{Tr}\left[\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R}\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R}\right], \\
\mathcal{L}_3 &= \mathrm{Tr}\left[F_{\rm V}\left(\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R} - \hat{\alpha}_{\rm R}\hat{\alpha}_{\rm L}\right)\right],
\end{aligned}$$

in the 1-form notation with

$$\begin{aligned}
\hat{\alpha}_L &= \hat{\alpha}_\parallel - \hat{\alpha}_\perp, \\
\hat{\alpha}_R &= \hat{\alpha}_\parallel + \hat{\alpha}_\perp, \\
F_V &= dV - iV^2.
\end{aligned}$$

M. Harada and K. Yamawaki, Phys. Rep. 381, 1 (2003)

HLS & hQCD

1. ***5d action***

$$S_5 = S_5^{\rm DBI} + S_5^{\rm CS}$$

$$S_5^{\rm DBI} = N_c G_{\rm YM} \int d^4x dz \Big\{ -\frac{1}{2} K_1(z) {\rm Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] + K_2(z) M_{KK}^2 \,{\rm Tr}[\mathcal{F}_{\mu z}\mathcal{F}^{\mu z}] \Big\},$$

$$S_5^{\rm CS} = \frac{N_c}{24\pi^2} \int_{M^4 \times R} w_5(A).$$

$$w_5(A) = {\rm Tr}\Big[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5 \Big].$$

2. *induce the HLS Lagrangian from S_5: integrate out the higher modes*

$$\begin{aligned} A_\mu(x, z) \to A_\mu^{\rm integ}(x, z) &= \hat{\alpha}_{\mu\perp}(x)\psi_0(z) + [\hat{\alpha}_{\mu\parallel}(x) + V_\mu(x)] \\ &\quad + \hat{\alpha}_{\mu\parallel}(x)\psi_1(z), \end{aligned}$$

Determination of couplings

$$
\begin{aligned}
f_\pi^2 &= N_c G_{\rm YM} M_{KK}^2 \int dz K_2(z) \left[\dot{\psi}_0(z)\right]^2, \\
a f_\pi^2 &= N_c G_{\rm YM} M_{KK}^2 \lambda_1 \langle \psi_1^2 \rangle, \\
\frac{1}{g^2} &= N_c G_{\rm YM} \langle \psi_1^2 \rangle, \\
y_1 &= -y_2 = -N_c G_{\rm YM} \left\langle \left(1+\psi_1-\psi_0^2\right)^2 \right\rangle, \\
y_3 &= -y_4 = -N_c G_{\rm YM} \left\langle \psi_1^2 \left(1+\psi_1\right)^2 \right\rangle, \\
y_5 &= 2y_8 = -y_9 = -2N_c G_{\rm YM} \left\langle \psi_1^2 \psi_0^2 \right\rangle, \\
y_6 &= -\left(y_5 + y_7\right), \\
y_7 &= 2N_c G_{\rm YM} \left\langle \psi_1 \left(1+\psi_1\right)\left(1+\psi_1-\psi_0^2\right) \right\rangle, \\
z_4 &= 2N_c G_{\rm YM} \left\langle \psi_1 \left(1+\psi_1-\psi_0^2\right) \right\rangle, \\
z_5 &= -2N_c G_{\rm YM} \left\langle \psi_1^2 \left(1+\psi_1\right) \right\rangle, \\
c_1 &= \left\langle\!\left\langle \dot{\psi}_0 \psi_1 \left(\frac{1}{2}\psi_0^2 + \frac{1}{6}\psi_1^2 - \frac{1}{2}\right) \right\rangle\!\right\rangle, \\
c_2 &= \left\langle\!\left\langle \dot{\psi}_0 \psi_1 \left(-\frac{1}{2}\psi_0^2 + \frac{1}{6}\psi_1^2 + \frac{1}{2}\psi_1 + \frac{1}{2}\right) \right\rangle\!\right\rangle, \\
c_3 &= \left\langle\!\left\langle \frac{1}{2}\dot{\psi}_0 \psi_1^2 \right\rangle\!\right\rangle,
\end{aligned}
$$

a is still undetermined

where λ_1 is the smallest (non-zero) eigenvalue of the eigenvalue equation given in Eq. (34), and $\langle\,\rangle$ and $\langle\langle\,\rangle\rangle$ are defined as

$$
\begin{aligned}
\langle A \rangle &\equiv \int_{-\infty}^{\infty} dz K_1(z) A(z), \\
\langle\langle A \rangle\rangle &\equiv \int_{-\infty}^{\infty} dz A(z) \qquad (36)
\end{aligned}
$$

$K_1(z),\ K_2(z)$: metric functions

$K_1(z) = K^{-1/3}(z),\ K_2(z) = K(z)$

with $K(z) = 1 + z^2$

in the Sakai-Sugimoto model

Two parameters

KK MASS

'T HOOFT COUPLING

$m_\rho = 776$ MeV

$f_\pi = 92.4$ MeV

TABLE I. Low energy constants of the HLS Lagrangian at $O(p^4)$ with $a = 2$.

Model	y_1	y_3	y_5	y_6	z_4	z_5	c_1	c_2	c_3
SS model	−0.001096	−0.002830	−0.015917	+0.013712	0.010795	−0.007325	+0.381653	−0.129602	0.767374
BPS model	−0.071910	−0.153511	−0.012286	−0.196545	0.090338	−0.130778	−0.206992	+3.031734	1.470210

Comparison with the Skyrme Lagrangian

Original Skyrme Lagrangian

$$\mathcal{L}_{\rm Sk} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{Tr}\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2, \tag{55}$$

After integrating out VM in HLS

$$\begin{aligned}\mathcal{L}_{\rm ChPT} = {} & f_\pi^2\,\mathrm{Tr}\left[\alpha_{\perp\mu}\alpha_\perp^\mu\right] \\ & + \left(\frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4}\right)\mathrm{Tr}\left[\alpha_{\perp\mu}, \alpha_{\perp\nu}\right]^2 \\ & + \frac{y_1 + y_2}{4}\,\mathrm{Tr}\left\{\alpha_{\perp\mu}, \alpha_{\perp\nu}\right\}^2,\end{aligned} \tag{56}$$

$$\frac{1}{2e^2} = \frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4}.$$

$$e \simeq 7.31$$

in the SS model

13년 6월 26일 수요일

Three models

- HLS(π, ρ, ω) model:
  full O(p^4) Lagrangian with hWZ terms
- HLS(π, ρ) model:
  without hQZ terms, the ω meson decouples
- HLS(π) model:
  integrates out VMs
  same as the Skyrme Lagrangian but e is fixed by the HLS

Soliton Wave Functions

Classical Solution

$$\xi(\boldsymbol{r}) = \exp\left[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\frac{F(r)}{2}\right]$$

$$\omega_\mu = W(r)\ \delta_{0\mu},$$

$$\rho_0 = 0, \quad \boldsymbol{\rho} = \frac{G(r)}{gr}\left(\hat{\boldsymbol{r}}\times\boldsymbol{\tau}\right)$$

Boundary Conditions

$$\begin{aligned} F(0) &= \pi, & F(\infty) &= 0, \\ G(0) &= -2, & G(\infty) &= 0, \\ W'(0) &= 0, & W(\infty) &= 0. \end{aligned}$$

FOR B=1 SOLITON

Collective Quantization

$$\begin{aligned} \xi(\boldsymbol{r}) &\to \xi(\boldsymbol{r},t) = A(t)\,\xi(\boldsymbol{r})A^\dagger(t), \\ V_\mu(\boldsymbol{r}) &\to V_\mu(\boldsymbol{r},t) = A(t)\,V_\mu(\boldsymbol{r})A^\dagger(t), \end{aligned}$$

$$i\boldsymbol{\tau}\cdot\boldsymbol{\Omega} \equiv A^\dagger(t)\partial_0 A(t).$$

$$\rho^0(\boldsymbol{r},t) = A(t)\frac{2}{g}\left[\boldsymbol{\tau}\cdot\boldsymbol{\Omega}\,\xi_1(r) + \hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{r}}\,\boldsymbol{\Omega}\cdot\hat{\boldsymbol{r}}\,\xi_2(r)\right]A^\dagger(t),$$

$$\omega^i(\boldsymbol{r},t) = \frac{\varphi(r)}{r}\left(\boldsymbol{\Omega}\times\hat{\boldsymbol{r}}\right)^i, \qquad (21)$$

Boundary Conditions

$$\begin{aligned} \xi_1'(0) &= \xi_1(\infty) = 0, \\ \xi_2'(0) &= \xi_2(\infty) = 0, \\ \varphi(0) &= \varphi(\infty) = 0, \end{aligned}$$

$$M_{baryon}(I,J) = M_{sol} + \frac{I^2}{2\mathcal{I}} = M_{sol} + \frac{J^2}{2\mathcal{I}}$$

Soliton mass

$$M_{\rm sol} = 4\pi \int dr \left[M_{(2)}(r) + M_{(4)}(r) + M_{\rm anom}(r) \right], \tag{A1}$$

where $M_{(2)}$, $M_{(4)}$, and $M_{\rm anom}$ are from $\mathcal{L}_{(2)}$, $\mathcal{L}_{(4)y} + \mathcal{L}_{(4)z}$, and $\mathcal{L}_{\rm anom}$, respectively. Their explicit forms are

$$M_{(2)}(r) = \frac{f_\pi^2}{2}\left(F'^2 r^2 + 2\sin^2 F\right) - \frac{a g^2 f_\pi^2}{2} W^2 r^2 + a f_\pi^2 \left(G + 2\sin^2\frac{F}{2}\right)^2 - \frac{W'^2 r^2}{2} + \frac{G'^2}{g^2} + \frac{G^2}{2g^2 r^2}(G+2)^2, \tag{A2}$$

$$\begin{aligned}
M_{(4)}(r) = {} & -y_1 \frac{r^2}{8}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right)^2 - y_2 \frac{r^2}{8} F'^2 \left(F'^2 - \frac{4}{r^2}\sin^2 F\right) - y_3 \frac{r^2}{2}\left[\frac{g^2 W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right]^2 \\
& - y_4 \frac{g^2 W^2 r^2}{2}\left\{\frac{g^2 W^2}{4} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right\} + \frac{y_5}{4}\left(r^2 F'^2 + 2\sin^2 F\right)\left[\frac{g^2 W^2}{2} - \frac{1}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right] \\
& + \left(y_8 - \frac{y_7}{2}\right)\frac{\sin^2 F}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2 + y_9\left\{\frac{g^2 W^2 r^2}{8}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right) + \frac{F'^2}{4}\left(G + 2\sin^2\frac{F}{2}\right)^2\right\} \\
& + z_4\left\{G'F'\sin F + \frac{\sin^2 F}{2r^2} G(G+2)\right\} + \frac{z_5}{2r^2} G(G+2)\left(G + 2\sin^2\frac{F}{2}\right)^2,
\end{aligned} \tag{A3}$$

$$\begin{aligned}
M_{\rm anom}(r) = {} & \alpha_1 F' W \sin^2 F + \alpha_2 W F' \left(G + 2\sin^2\frac{F}{2}\right)^2 \\
& - \alpha_3\left\{G(G+2) W F' + 2\sin F\left[W G' - W'\left(G + 2\sin^2\frac{F}{2}\right)\right]\right\},
\end{aligned} \tag{A4}$$

where

$$\alpha_1 = \frac{3gN_c}{16\pi^2}(c_1 - c_2), \qquad \alpha_2 = \frac{gN_c}{16\pi^2}(c_1 + c_2), \qquad \alpha_3 = \frac{gN_c}{16\pi^2} c_3. \tag{A5}$$

Moment of Inertia

$$L = -M_{\rm sol} + \mathcal{I}\,{\rm Tr}(\dot{A}\dot{A}^\dagger), \qquad \mathcal{I} = 4\pi \int dr[\mathcal{I}_{(2)}(r) + \mathcal{I}_{(4)}(r) + \mathcal{I}_{\rm anom}(r)].$$

$$\mathcal{I}_2(r) = \frac{2}{3}f_\pi^2 r^2 \sin^2 F + \frac{1}{3}af_\pi^2 r^2\left[(\xi_1+\xi_2)^2 + 2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2\right] - \frac{1}{6}ag^2 f_\pi^2 \varphi^2 - \frac{1}{6}\left(\varphi'^2 + \frac{2\varphi^2}{r^2}\right)$$
$$+ \frac{r^2}{3g^2}(3\xi_1'^2 + 2\xi_1'\xi_2' + \xi_2'^2) + \frac{4}{3g^2}G^2(\xi_1 - 1)(\xi_1+\xi_2-1) + \frac{2}{3g^2}(G^2+2G+2)\xi_2^2.$$

$$\mathcal{I}_{(4)} = \sum_i y_i \mathcal{I}_{y_i} + \sum_i z_i \mathcal{I}_{z_i}.$$

$$\mathcal{I}_{y_1}(r) = -\frac{1}{3}r^2\sin^2 F\left(F'^2 + \frac{2}{r^2}\sin^2 F\right),$$

$$\mathcal{I}_{y_2}(r) = \frac{1}{3}r^2 \sin^2 F F'^2,$$

$$\mathcal{I}_{y_3}(r) = -\frac{1}{12}g^2\varphi^2\left[g^2W^2 - \frac{4}{r^2}\left(G + 2\sin^2\frac{F}{2}\right)^2\right] + \frac{2}{3}g^2W\varphi\left(G+2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right)$$
$$+\left[\frac{1}{2}r^2g^2W^2 - \frac{1}{3}\left(G+2\sin^2\frac{F}{2}\right)^2\right]\left[(\xi_1+\xi_2)^2 + 2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2\right],$$

$$\mathcal{I}_{y_4}(r) = \frac{r^2}{2}g^2W^2\left[(\xi_1+\xi_2)^2 + 2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2\right] - \frac{1}{12}g^2W\varphi\left[g^2W\varphi - 8\left(G+2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right)\right]$$
$$+\frac{1}{3}\left(G+2\sin^2\frac{F}{2}\right)^2\left[\frac{g^2\varphi^2}{r^2} + (\xi_1+\xi_2)^2\right],$$

$$\mathcal{I}_{y_5}(r) = \frac{1}{6}\sin^2 F\left[r^2g^2W^2 - 2\left(G+2\sin^2\frac{F}{2}\right)^2\right] - \frac{r^2}{12}\left(F'^2 + \frac{2}{r^2}\sin^2 F\right)\left[2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2 + (\xi_1+\xi_2)^2 - \frac{g^2\varphi^2}{2r^2}\right],$$

$$\mathcal{I}_{y_6}(r) = \frac{1}{6}\sin^2 F\left(rgW - \frac{g\varphi}{2r}\right)^2,$$

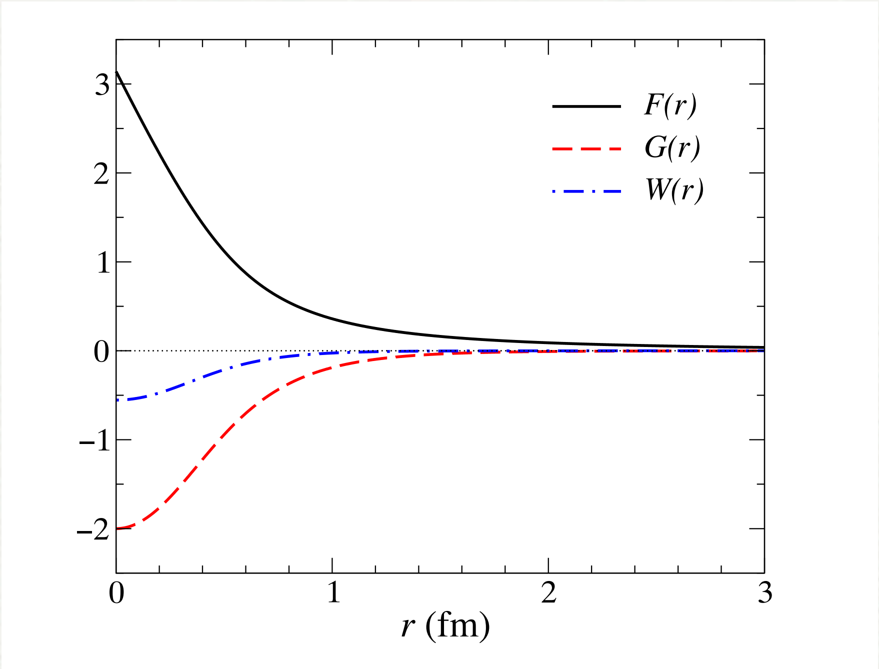
$$\mathcal{I}_{y_7}(r) = \frac{1}{6}\sin^2 F\left[\left(rgW - \frac{g\varphi}{2r}\right)^2 + 4\left(G+2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right)\right],$$

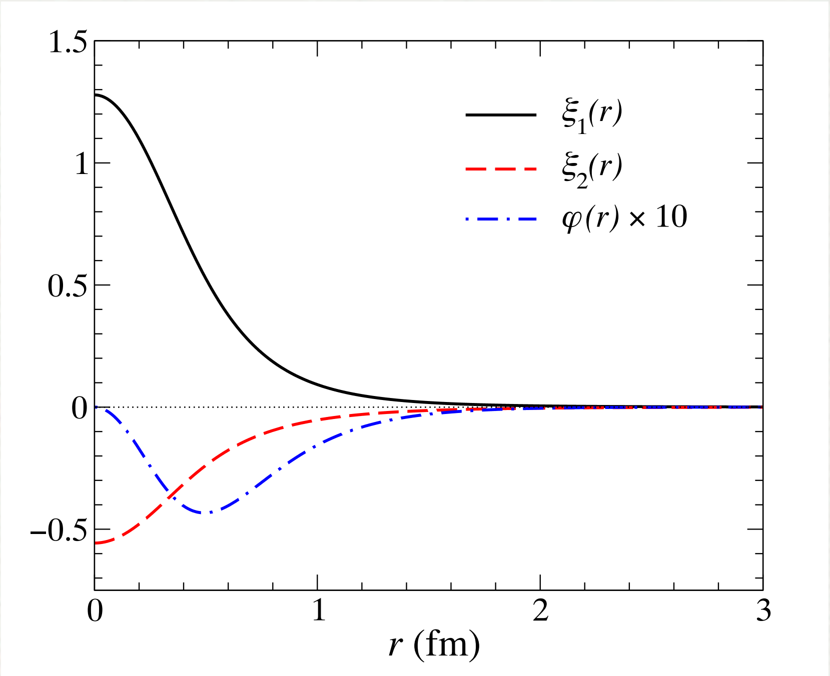
$$\mathcal{I}_{y_8}(r) = \frac{1}{3}\sin^2 F\left[\left(rgW - \frac{g\varphi}{2r}\right)^2 - 4\left(G+2\sin^2\frac{F}{2}\right)\left(\xi_1 - 2\sin^2\frac{F}{2}\right)\right],$$

$$\mathcal{I}_{y_9}(r) = \frac{r^2}{6}g^2W^2\sin^2 F + \frac{r^2}{6}F'^2\left(\xi_1 - 2\sin^2\frac{F}{2}\right)^2 - \frac{r^2}{12}\left(F'^2 - \frac{2}{r^2}\sin^2 F\right)(\xi_1+\xi_2)^2 + \frac{1}{24}g^2\varphi^2\left(F'^2 + \frac{2}{r^2}\sin^2 F\right),$$

Solutions

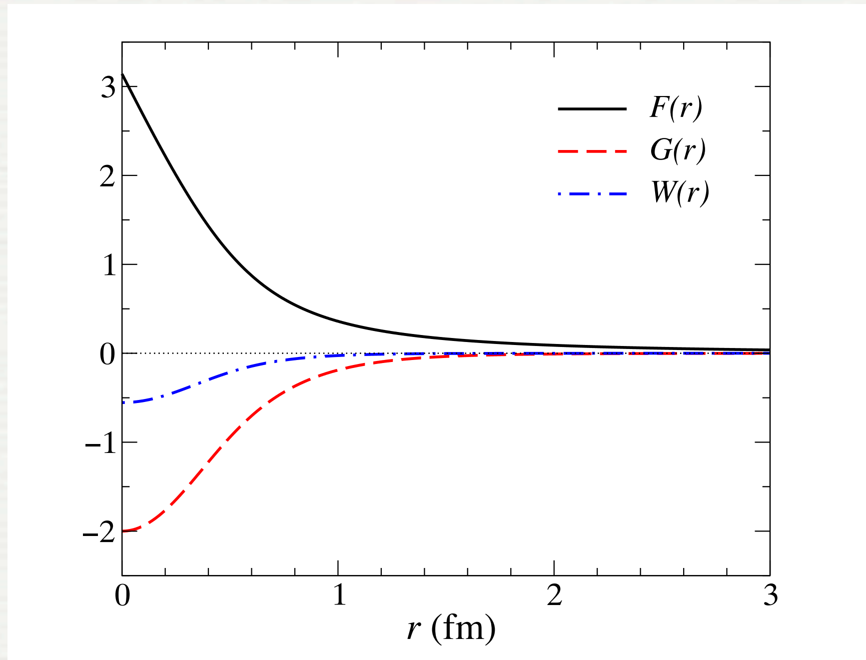
HLS(π, ρ, ω) model

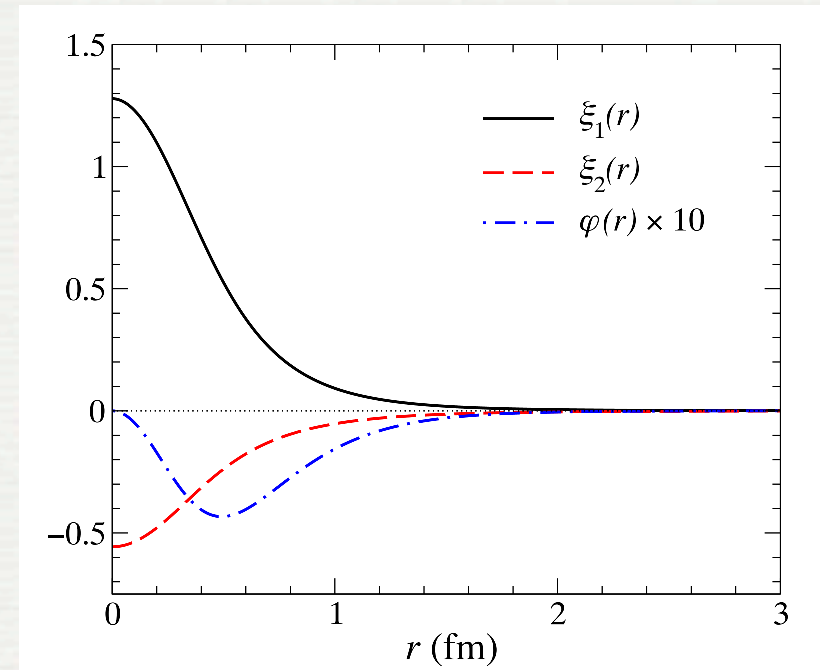




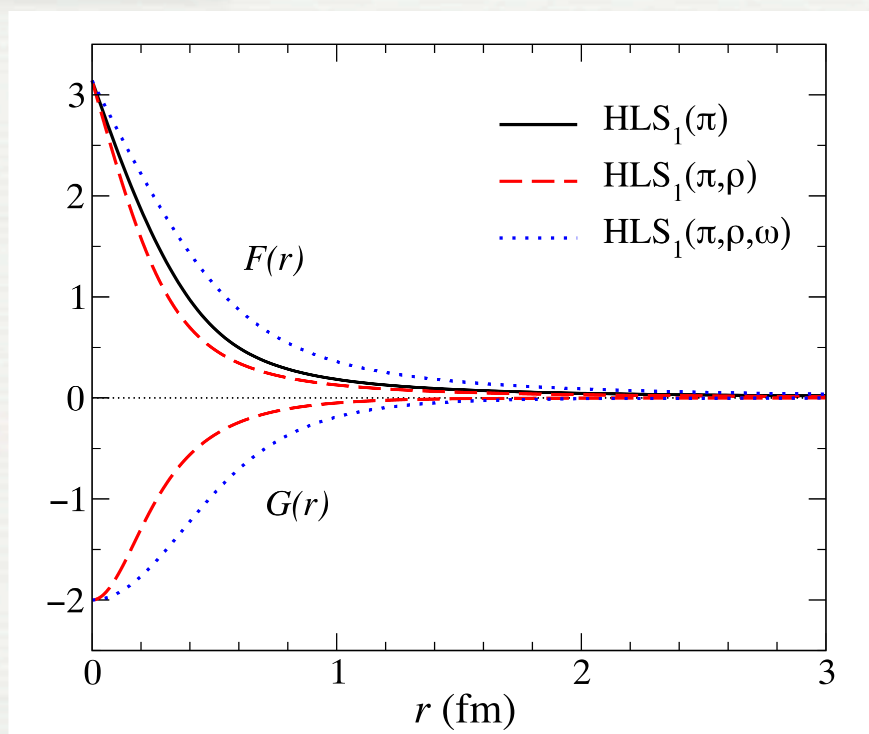
Solutions

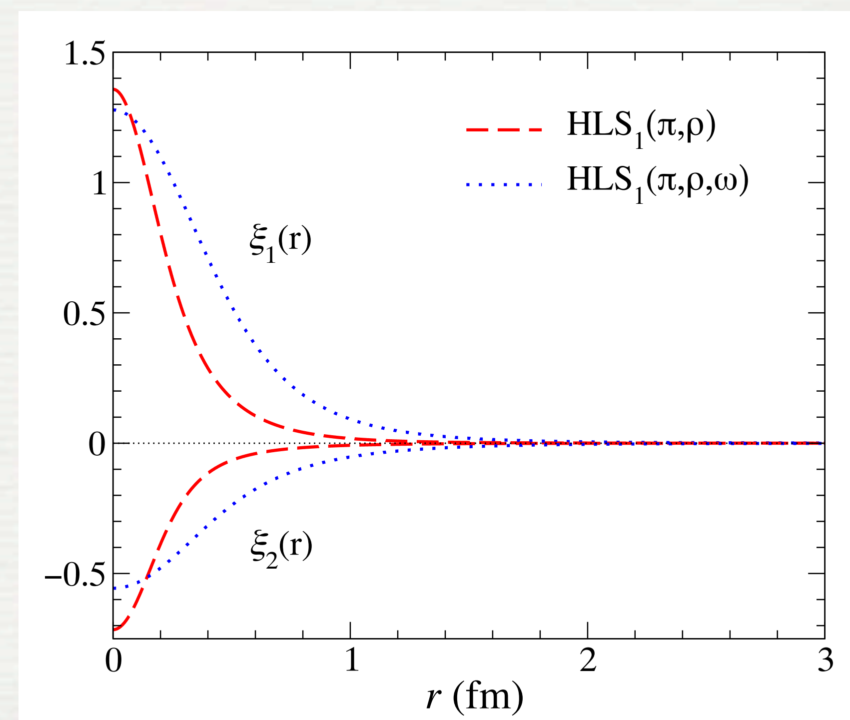
HLS(π, ρ, ω) model





Comparison of the three models





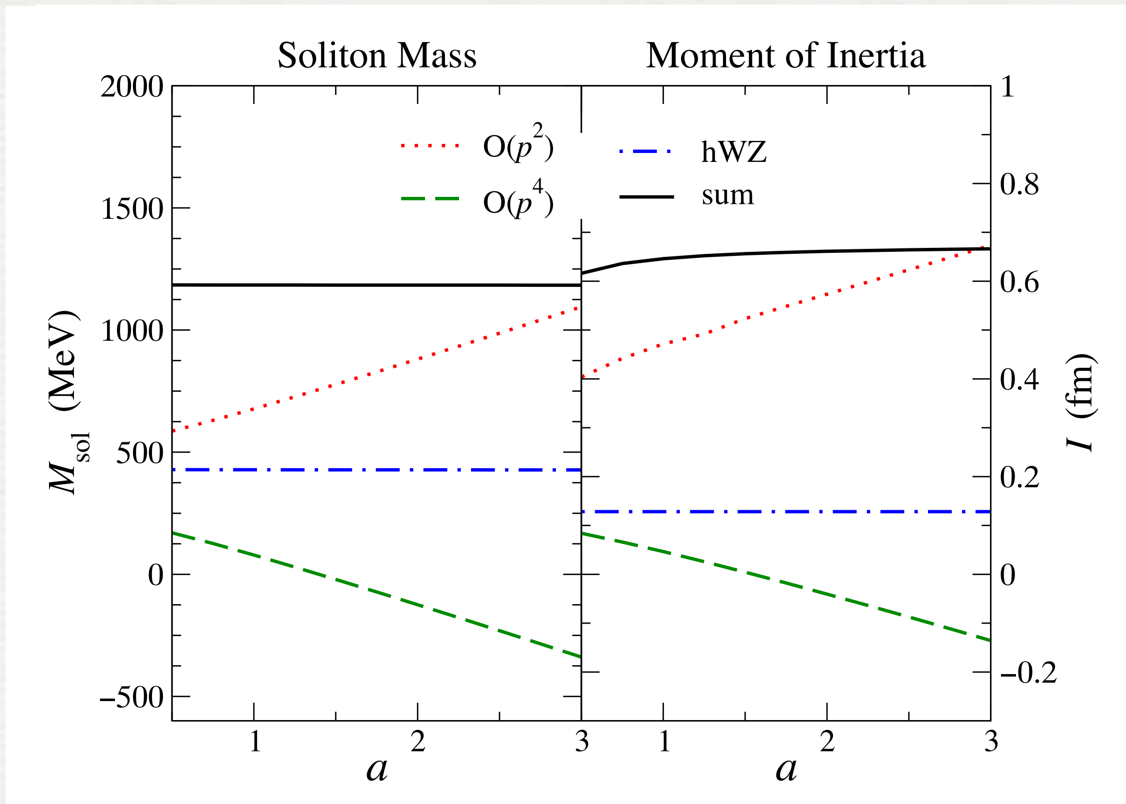
Results

TABLE II. Skyrmion mass and size calculated in the HLS with the SS and BPS models with $a = 2$. The soliton mass $M_{\rm sol}$ and the Δ-N mass difference Δ_M are in unit of MeV while $\sqrt{\langle r^2 \rangle_W}$ and $\sqrt{\langle r^2 \rangle_E}$ are in unit of fm. The column of $O(p^2) + \omega_\mu B^\mu$ is "the minimal model" of Ref. [20] and that of $O(p^2)$ corresponds to the model of Ref. [19]. See the text for more details.

	$\mathrm{HLS}_1(\pi,\rho,\omega)$	$\mathrm{HLS}_1(\pi,\rho)$	$\mathrm{HLS}_1(\pi)$	$\mathrm{BPS}(\pi,\rho,\omega)$	$\mathrm{BPS}(\pi,\rho)$	$\mathrm{BPS}(\pi)$	$O(p^2) + \omega_\mu B^\mu$ [20]	$O(p^2)$ [19]
$M_{\rm sol}$	1184	834	922	1162	577	672	1407	1026
Δ_M	448	1707	1014	456	4541	2613	259	1131
$\sqrt{\langle r^2 \rangle_W}$	0.433	0.247	0.309	0.415	0.164	0.225	0.540	0.278
$\sqrt{\langle r^2 \rangle_E}$	0.608	0.371	0.417	0.598	0.271	0.306	0.725	0.422

$$\Delta_M \equiv M_\Delta - M_N$$

a independence of the Skyrmion properties



13년 6월 26일 수요일

Discussions

1. The role of ρ meson

- reduction of the soliton mass: from 922 MeV to 834 MeV
- increase of the Δ-N mass difference: from 1014 MeV to 1707 MeV
- shrink the soliton profile: from 0.417 fm to 0.371 fm

2. The role of ω meson

- increase of the soliton mass: from 834 MeV to 1184 MeV
- decrease of the Δ-N mass difference: from 1707 MeV to 448 MeV
- expand the soliton profile: from 0.371 fm to 0.608 fm

3. Without ω meson

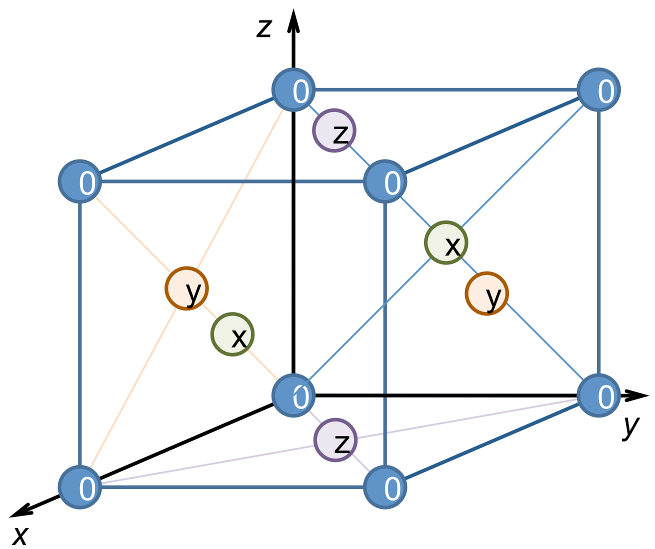
- the Δ-N mass difference of $O(1/N_c)$ > the soliton mass of $O(N_c)$

4. The independence of *a*

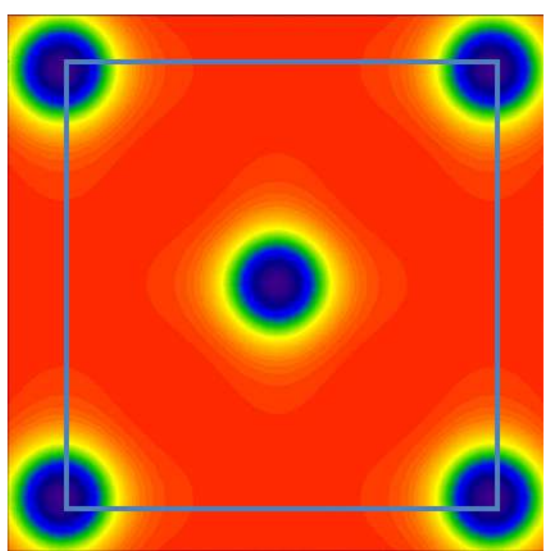
- Direct consequence from hQCD

Nuclear Matter: Skyrme Crystal

Skyrme Crystal (FCC)

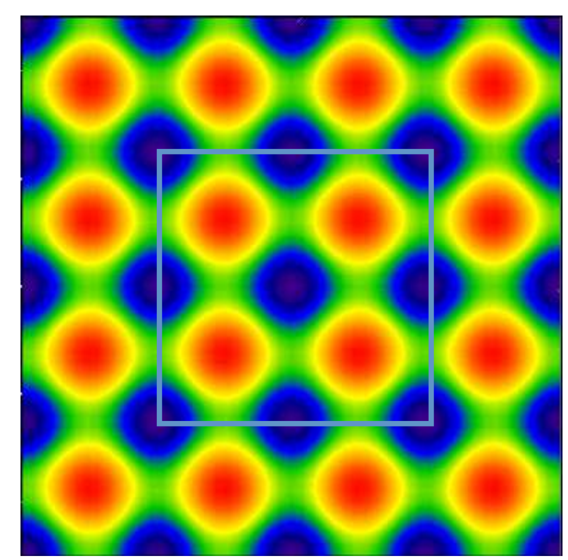


I. Klebanov, Nucl. Phys. B262, 133 (1985)
M. Kugler et al., Phys. Lett. B208, 491 (1988)
H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho, and V. Vento, Nucl. Phys. A723, 427 (2003)

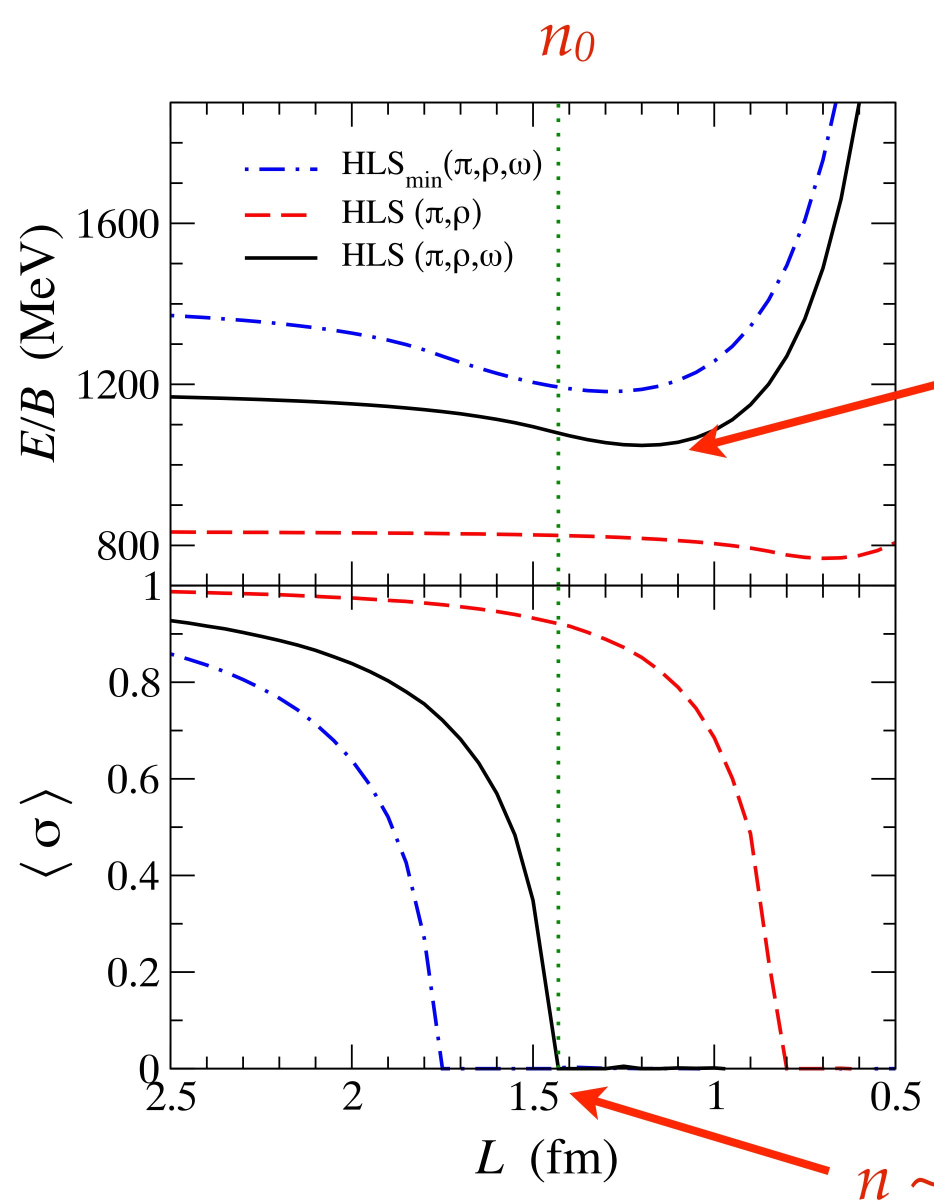


increasing density

Half-Skyrmion Phase



Skyrme Crystal



- Skyrmion number density $n = 1/(2L^3)$
- normal nuclear density n_0 : 0.17 fm^{-3}

corresponds to $L \sim 1.43$ fm

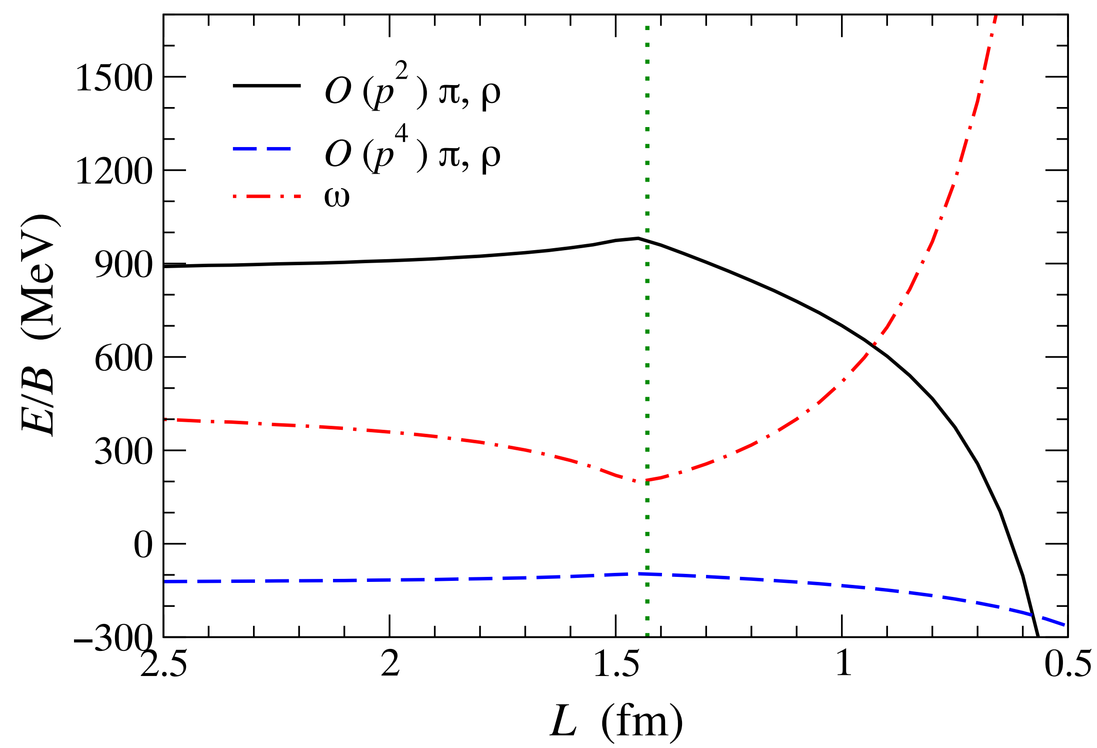
$n \sim 2n_0$

binding energy per baryon
~150 MeV, ~100 MeV, ~50 MeV (too big!)

transition to the half-Skyrmion phase
$<\sigma> = 0$

$n \sim n_0$

13년 6월 26일 수요일



O (p2) π, ρ
O (p4) π, ρ
ω
E/B (MeV)
L (fm)
1500
1200
900
600
300
0
−300
2.5
2
1.5
1
0.5

Change of Meson Properties

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2$$

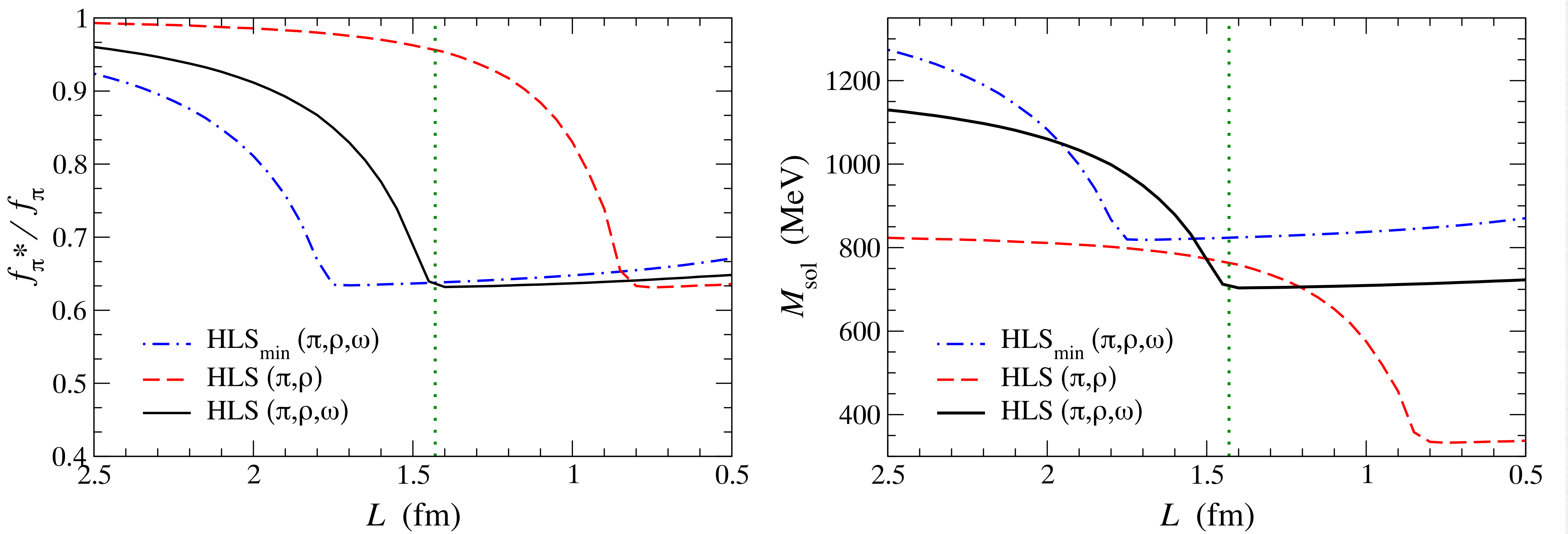
$$U_\pi = \exp(i\vec{\tau}\cdot\vec{\pi})$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \cdots$$

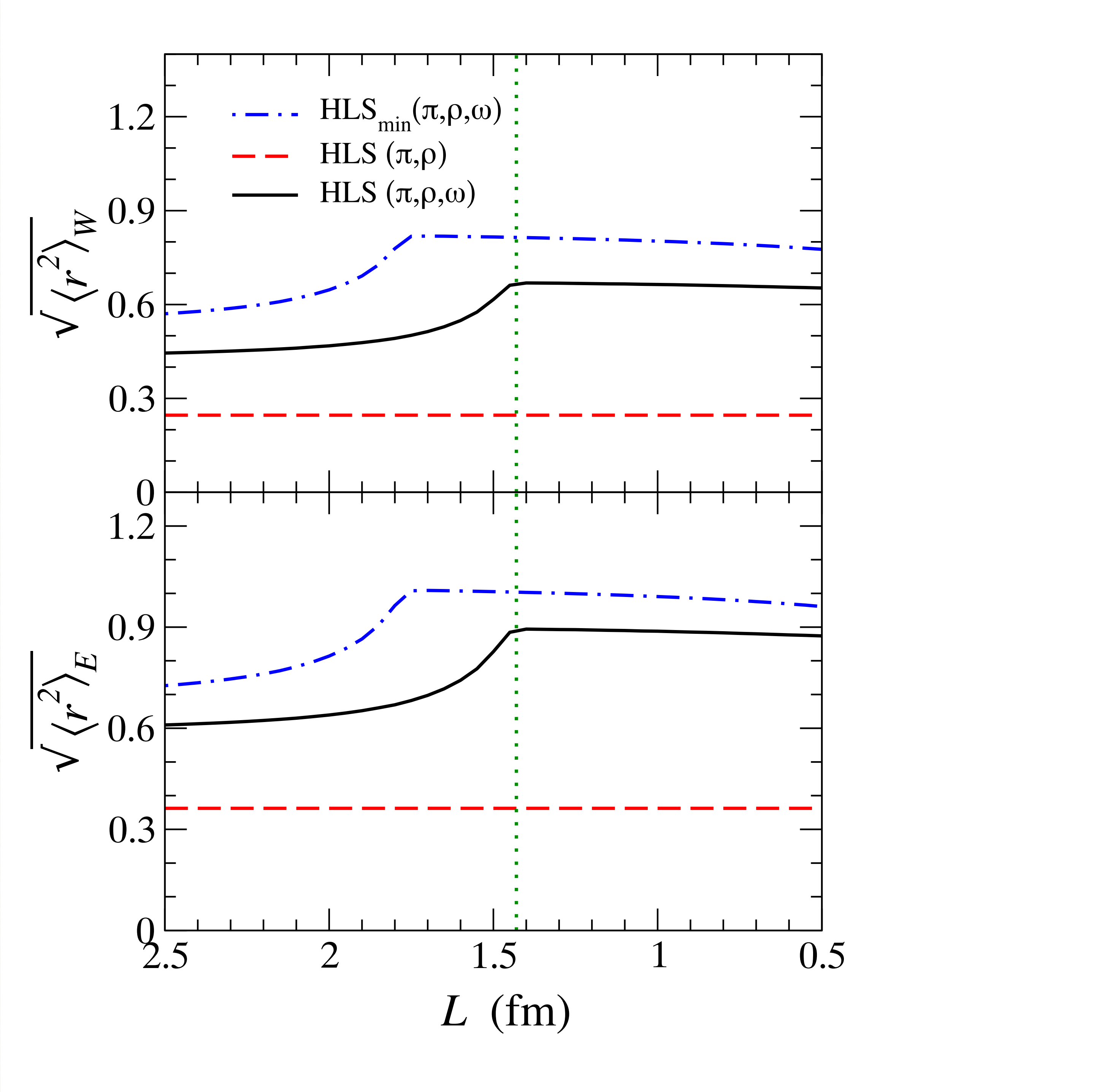
$$U = \sqrt{U_\pi}U_B\sqrt{U_\pi}$$

$$\mathcal{L} = \frac{1}{2}G_{ab}(U_B)\partial_\mu\pi^a\partial^\mu\pi^b + \cdots$$

$$\frac{f_\pi^*}{f_\pi} = \sqrt{\langle G_{aa}(U_B)\rangle}$$



f_π^* / f_π
HLS$_{min}$ (π,ρ,ω)
HLS (π,ρ)
HLS (π,ρ,ω)
L (fm)
M_{sol} (MeV)
HLS$_{min}$ (π,ρ,ω)
HLS (π,ρ)
HLS (π,ρ,ω)
L (fm)



HLS$_{min}$(π,ρ,ω)
HLS (π,ρ)
HLS (π,ρ,ω)
$\sqrt{\langle r^2\rangle_W}$
$\sqrt{\langle r^2\rangle_E}$
1.2
0.9
0.6
0.3
0
2.5
2
1.5
1
0.5
L (fm)

Summary

1. The nontrivial role of the ω meson
2. The presence of topological change from Skyrmions to half-Skyrmions
   at slightly above the normal nuclear density of higher density
3. Problems: minimal energy occurs at too high density with too high binding energy
4. The structure may be robust, but the numbers?
   1/Nc corrections: Casimir energy

F. Meier and H. Walliser, Phys. Rep. 289, 383 (1997)

Table 4.1
Tree and one-loop contribution to various quantities for parameter set A ($e = 4.25$, $g_\omega = 0$)

	Tree	One-loop	$\sum$	Exp.
M (MeV)	1629	−683	946	939
σ (MeV)	54	−22	32	45 ± 7
$\langle r^2 \rangle^S$ (fm^2)	1.0	+0.3	1.3	1.6 ± 0.3
g_A	0.91	−0.25	0.66	1.26
$\langle r^2 \rangle_A$ (fm^2)	0.45	−0.04	0.41	$0.42^{+0.18}_{-0.08}$
$\langle r^2 \rangle_E^S$ (fm^2)	0.62	−0.11	0.51	0.59
μ^V	1.62	+0.62	2.24	2.35
$\langle r^2 \rangle_M^V$ (fm)^2	0.77	−0.13	0.64	0.73
α (10^{-4} fm^3)	17.8	−8.0	9.8	9.5 ± 5

Outlook

1. The role of vector mesons

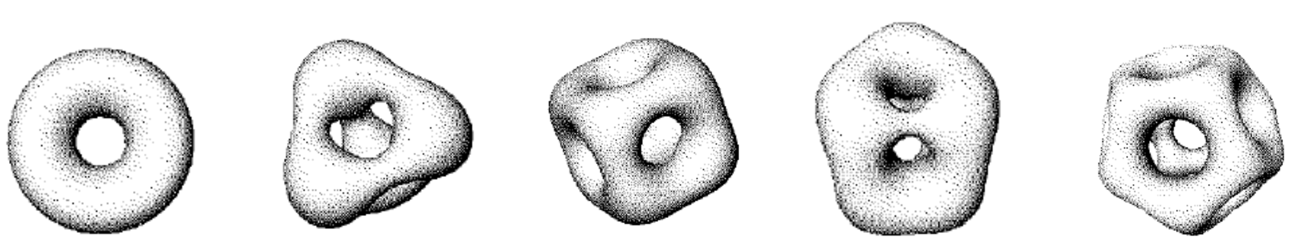
- previous works: more VMs lead to the Bogomolny bound
- the inclusion of the ρ meson confirms it
- but, the ω meson has the opposite role:
  important from both the theoretical and phenomenological views

2. Issues

- next order corrections: O(N_c^0) pion fluctuation
- next order terms in the HLS: in N_c and in p

3. Final goal

- few-nucleon systems ⇒ semi-empirical mass formula?
- nuclear matter, Skyrmion crystal
- equation of state, nuclear symmetric energy



Thank you very much

for your attention