Solitons, Large N and Holography

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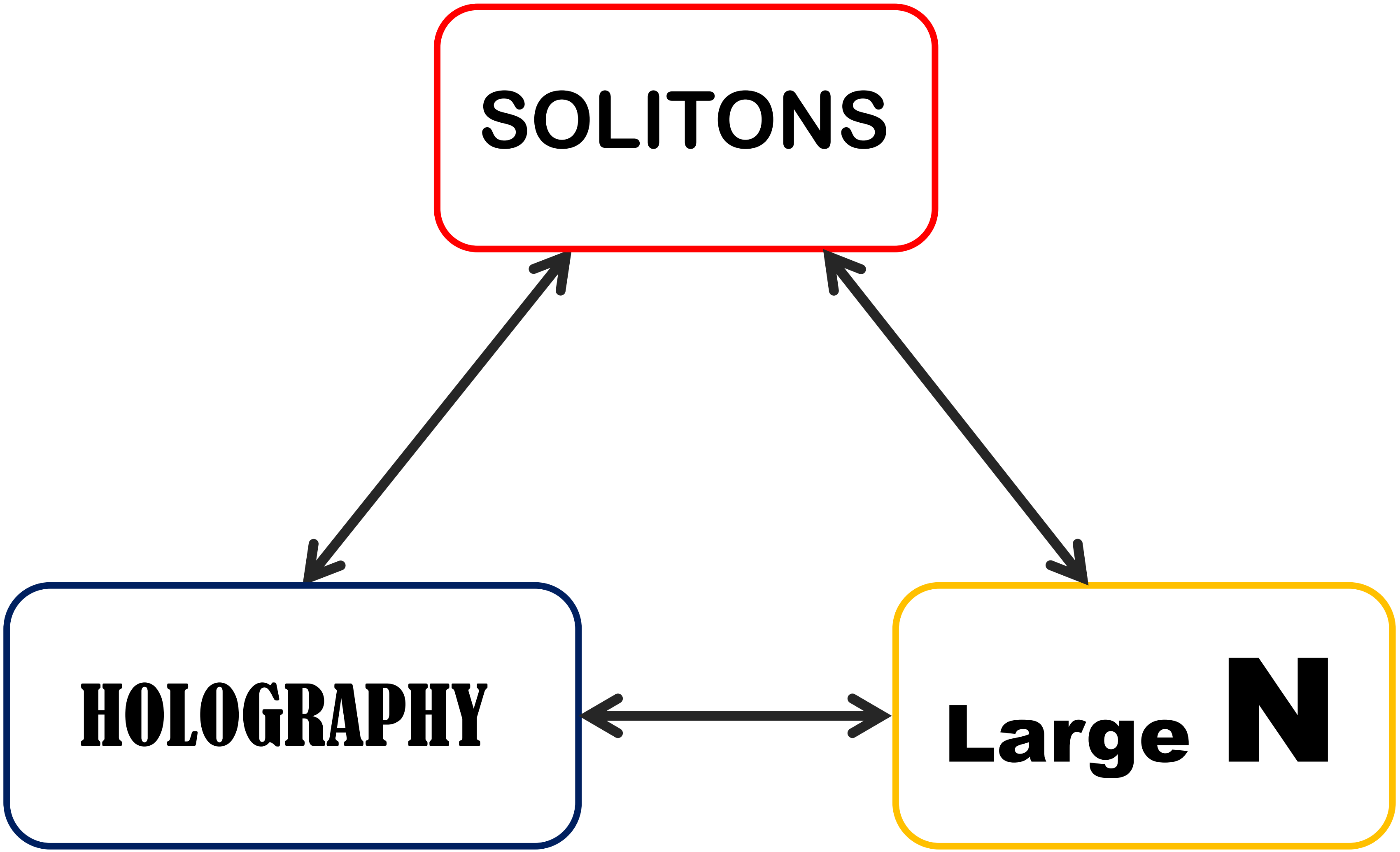
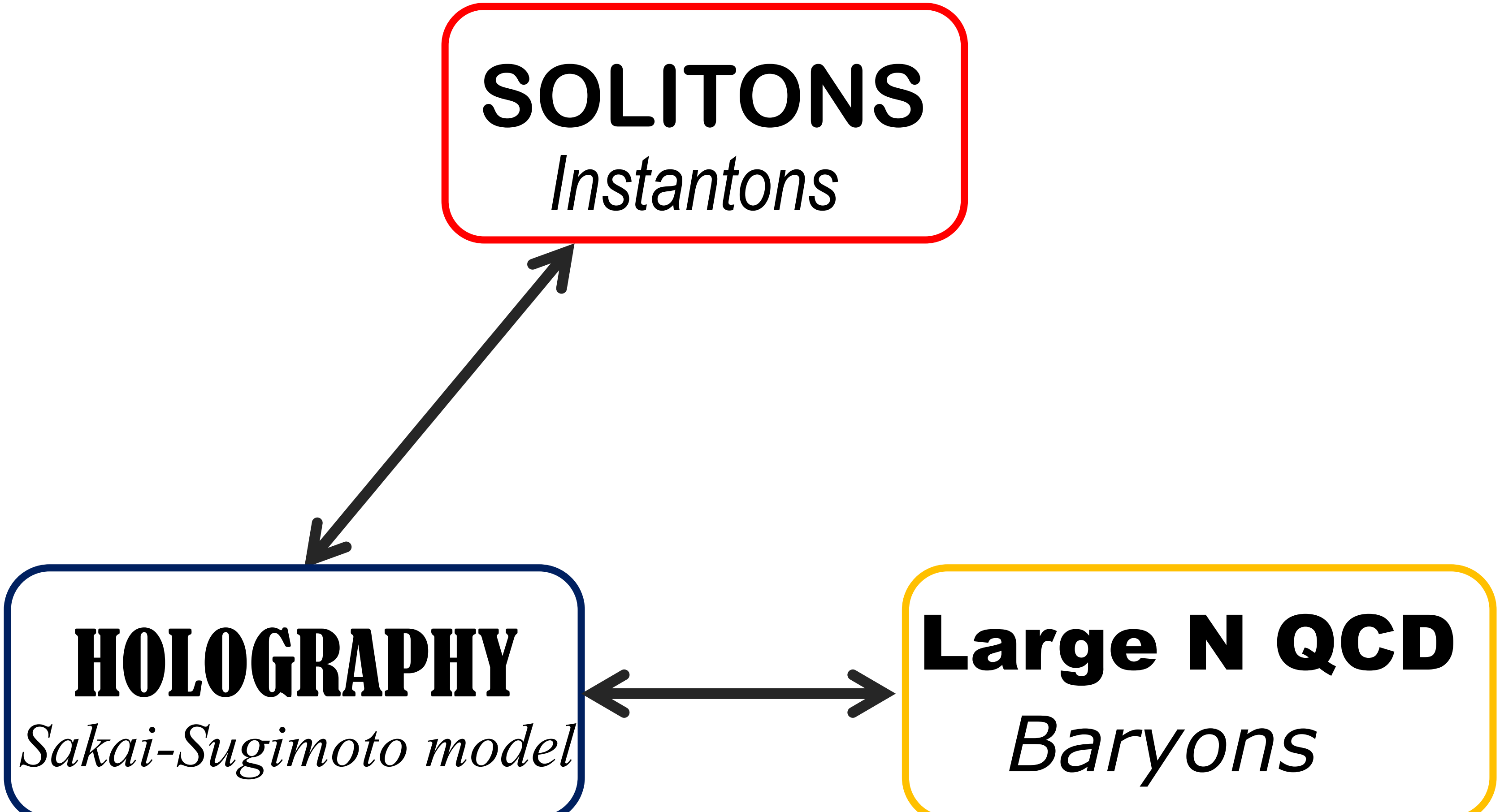
"eNlarge horizons" Madrid 3/6/15


SOLITONS
HOLOGRAPHY
Large N

Plan of the talk (part 1)

- Models of holographic baryons



Plan of the talk (part 1)

- Introduction to the models of holographic baryons
- Two main issues

  1) *Core properties*
  2) *Large distance properties*

based on work with **P. Sutcliffe** arXiv:1309.1396

Some geometry to begin with…

We consider the following 5D Lorentzian metric

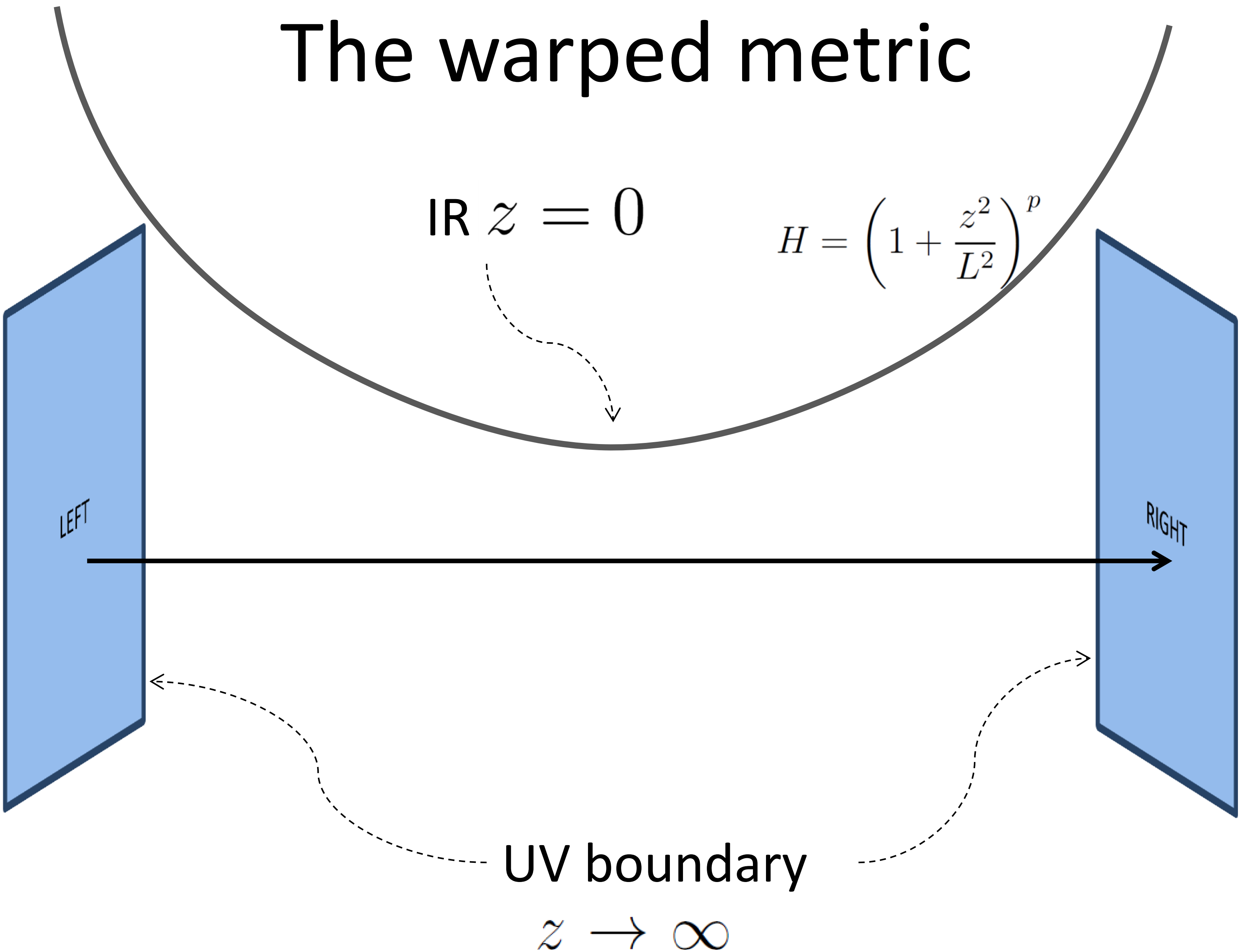
$$ds^2 = H(z)\, dx_\mu dx^\mu + \frac{1}{H(z)} dz^2$$

with $H = \left(1 + \frac{z^2}{L^2}\right)^p$ $\qquad p \in (\frac{1}{2}, 1]$

Conformal boundary

p=2/3 for the Sakai-Sugimoto model

The warped metric



Brief holographic dictionary

Bulk theory		"QCD" defined on the boundary	
String coupling **String length** **Curvature scale**	↔	N_c λ	Colors 't Hooft coupling
Gauge group	↔	N_f	Flavors

The Sakai-Sugimoto model

$$\mathcal{S} = -\frac{N_c \lambda}{216\pi^3} \int \sqrt{-g}\, \frac{1}{2} \mathrm{tr}\left(\mathcal{F}_{\Gamma\Delta}\mathcal{F}^{\Gamma\Delta}\right) d^4x\, dz + \frac{N_c}{24\pi^2} \int \omega_5(\mathcal{A})\, d^4x\, dz$$

Yang–Mills

Chern-Simons

U(2) = SU(2) x U(1)

$$\mathcal{A}_\Gamma = A_\Gamma + \frac{1}{2}\widehat{A}_\Gamma$$

The Sakai-Sugimoto model

$$\mathcal{S} = -\frac{N_c \lambda}{216\pi^3} \int \sqrt{-g}\, \frac{1}{2} \mathrm{tr}\left(\mathcal{F}_{\Gamma\Delta}\mathcal{F}^{\Gamma\Delta}\right)\, d^4x\, dz + \frac{N_c}{24\pi^2} \int\, \omega_5(\mathcal{A})\, d^4x\, dz$$

Colors

The Sakai-Sugimoto model

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't Hooft coupling

The Sakai-Sugimoto model

$$\mathcal{S} = -\frac{N_c \lambda}{216\pi^3} \int \sqrt{-g}\, \frac{1}{2} \mathrm{tr}\left(\mathcal{F}_{\Gamma\Delta} \mathcal{F}^{\Gamma\Delta}\right)\, d^4x\, dz + \frac{N_c}{24\pi^2} \int \omega_5(\mathcal{A})\, d^4x\, dz$$

Static configuration

$$A_0 = 0, \qquad A_I = A_I(x_J), \qquad \widehat{A}_0 = \widehat{A}_0(x_J), \qquad \widehat{A}_I = 0$$

The Sakai-Sugimoto model

$$\mathcal{S} = -\frac{N_c \lambda}{216\pi^3} \int \sqrt{-g}\, \frac{1}{2} \mathrm{tr}\left(\mathcal{F}_{\Gamma\Delta}\mathcal{F}^{\Gamma\Delta}\right) d^4x\, dz + \frac{N_c}{24\pi^2} \int \omega_5(\mathcal{A})\, d^4x\, dz$$

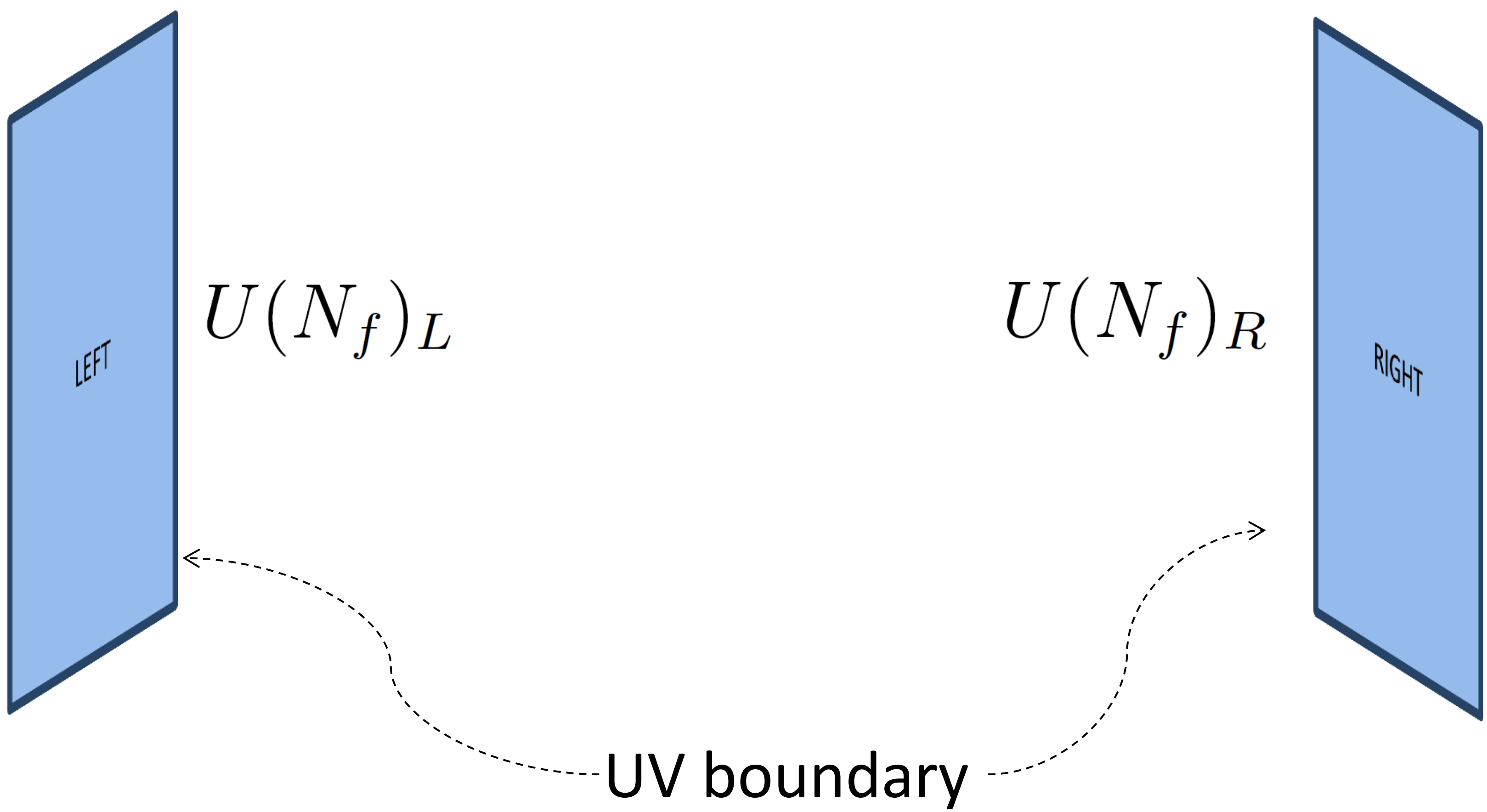
Static configuration

$$A_0 = 0, \qquad A_I = A_I(x_J), \qquad \widehat{A}_0 = \widehat{A}_0(x_J), \qquad \widehat{A}_I = 0$$

$$\int \widehat{A}_0\, \mathrm{tr}\left(F_{IJ}F_{KL}\right) \varepsilon_{IJKL}\, d^4x\, dz$$

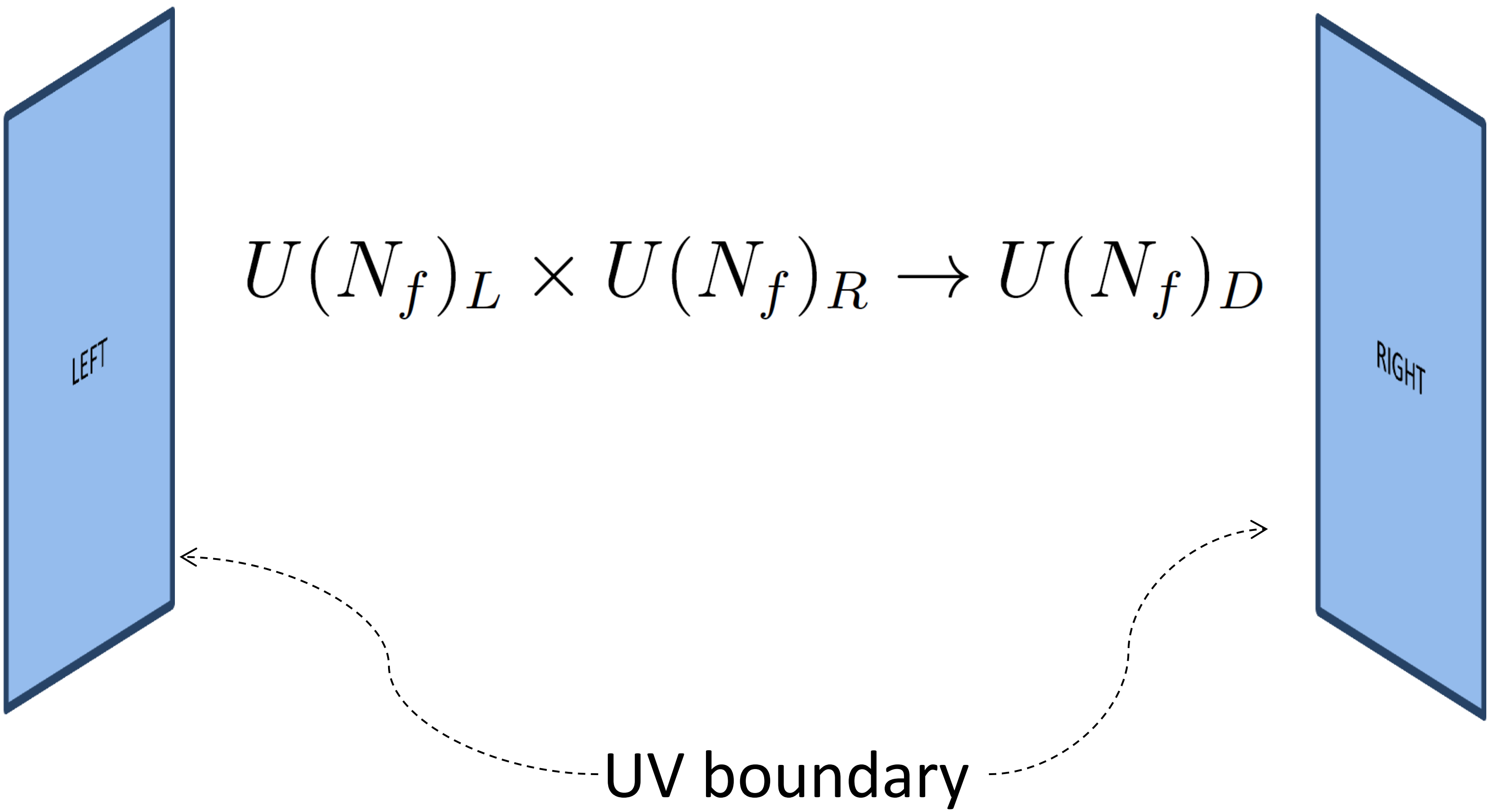
Instanton charge is an electric field source

Global symmetries

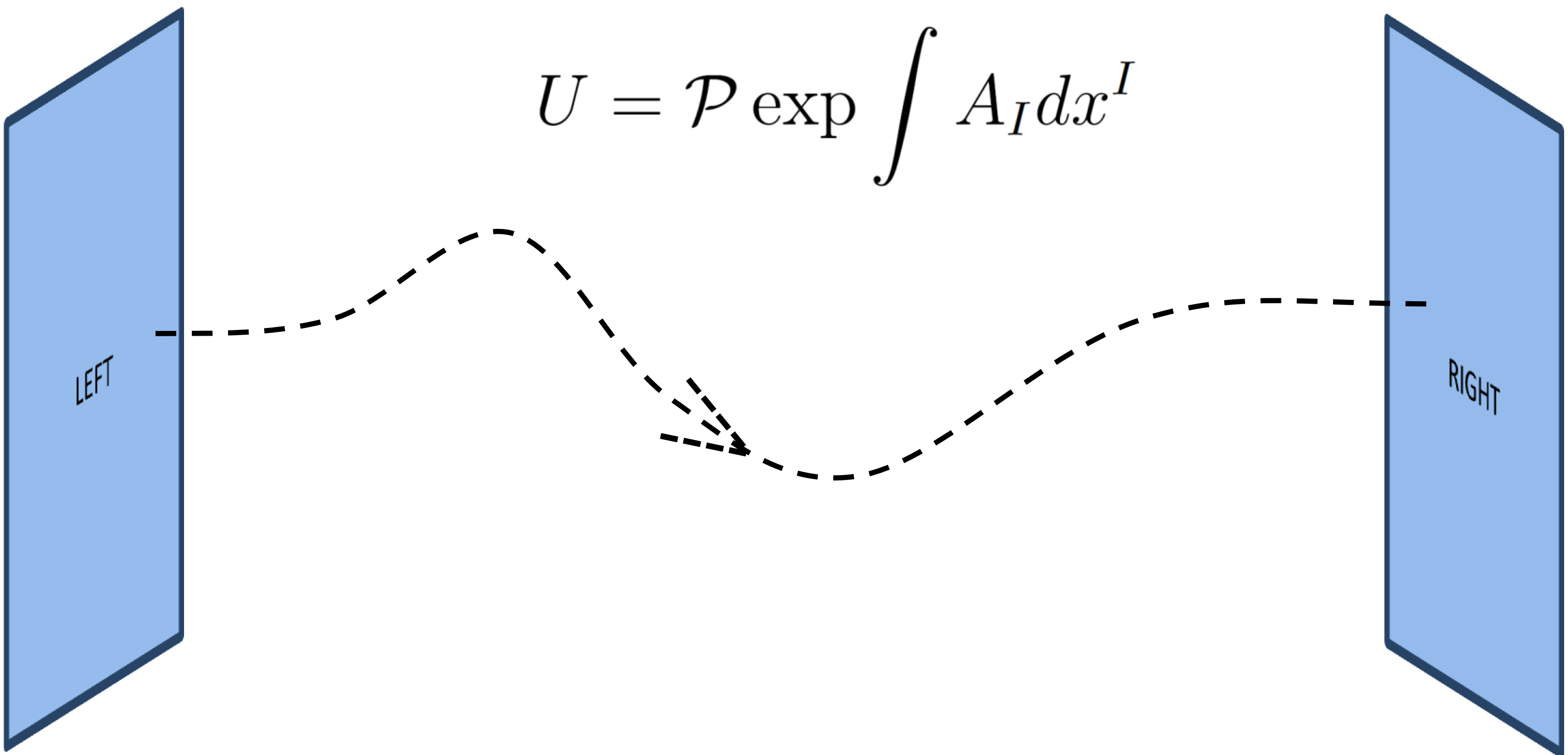


Chiral symmetry breaking

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_D$$

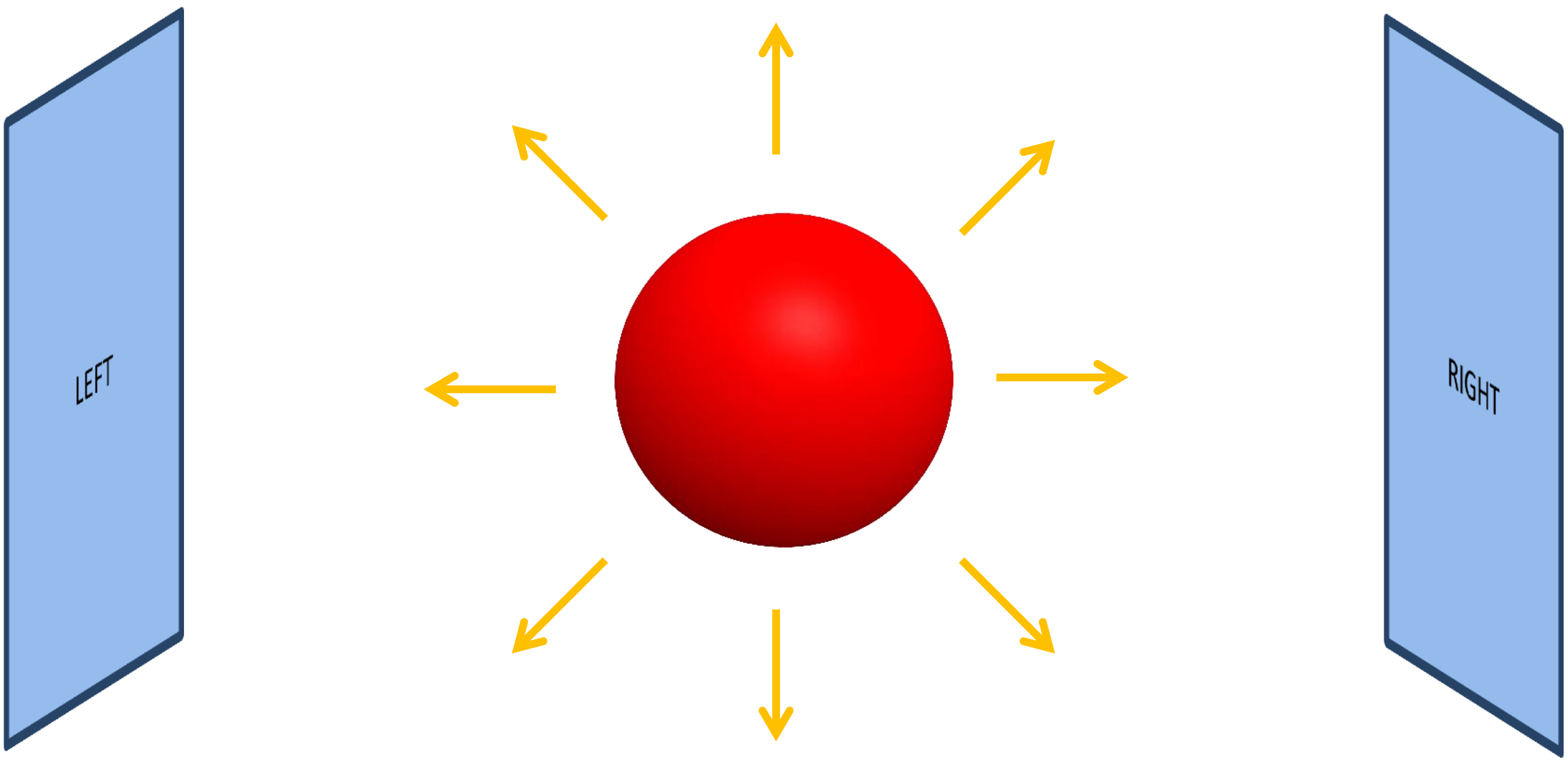


Instantons *are* holographic baryons



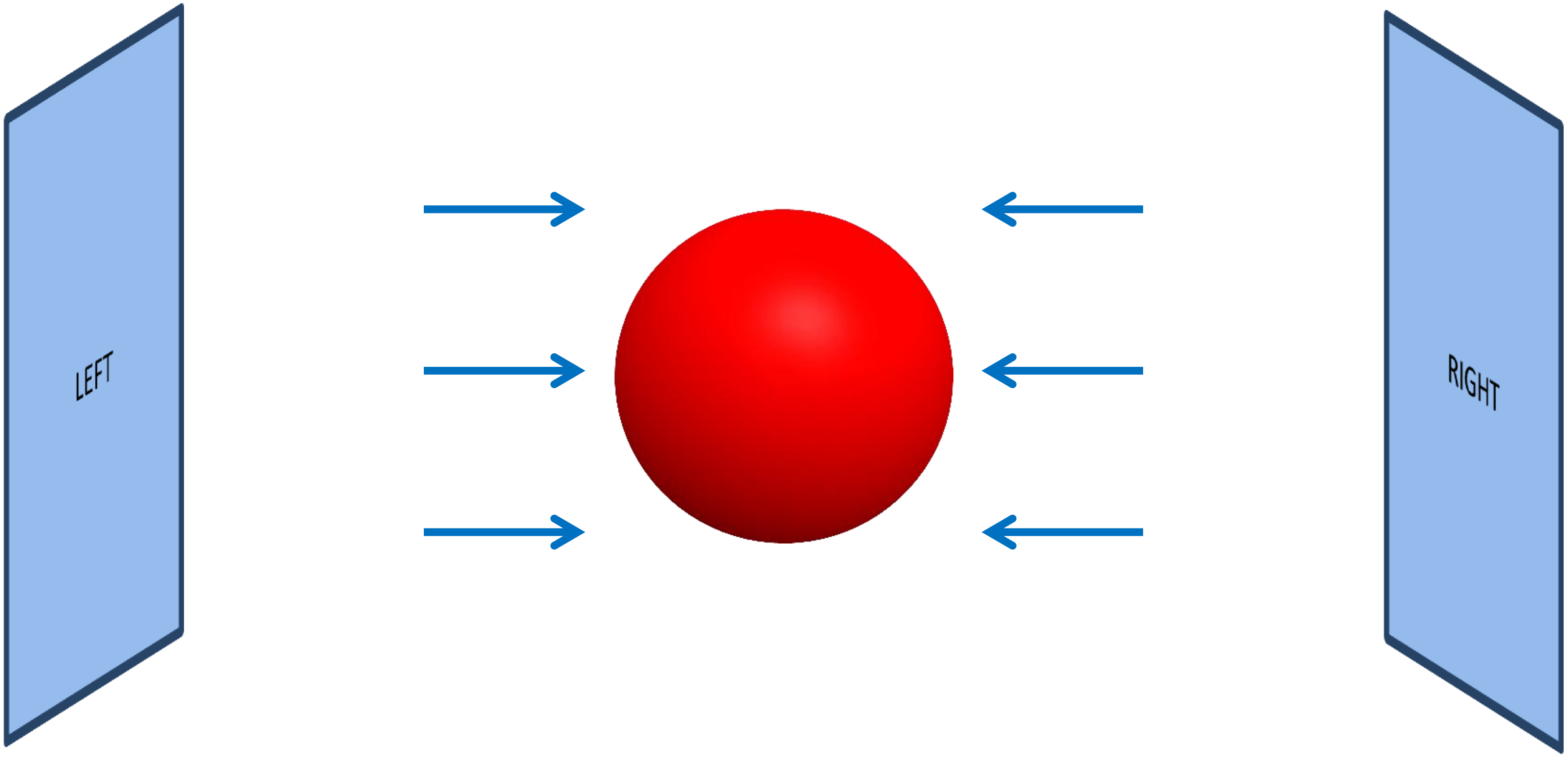
Atiyah-Manton correspondence

What stabilizes the instanton



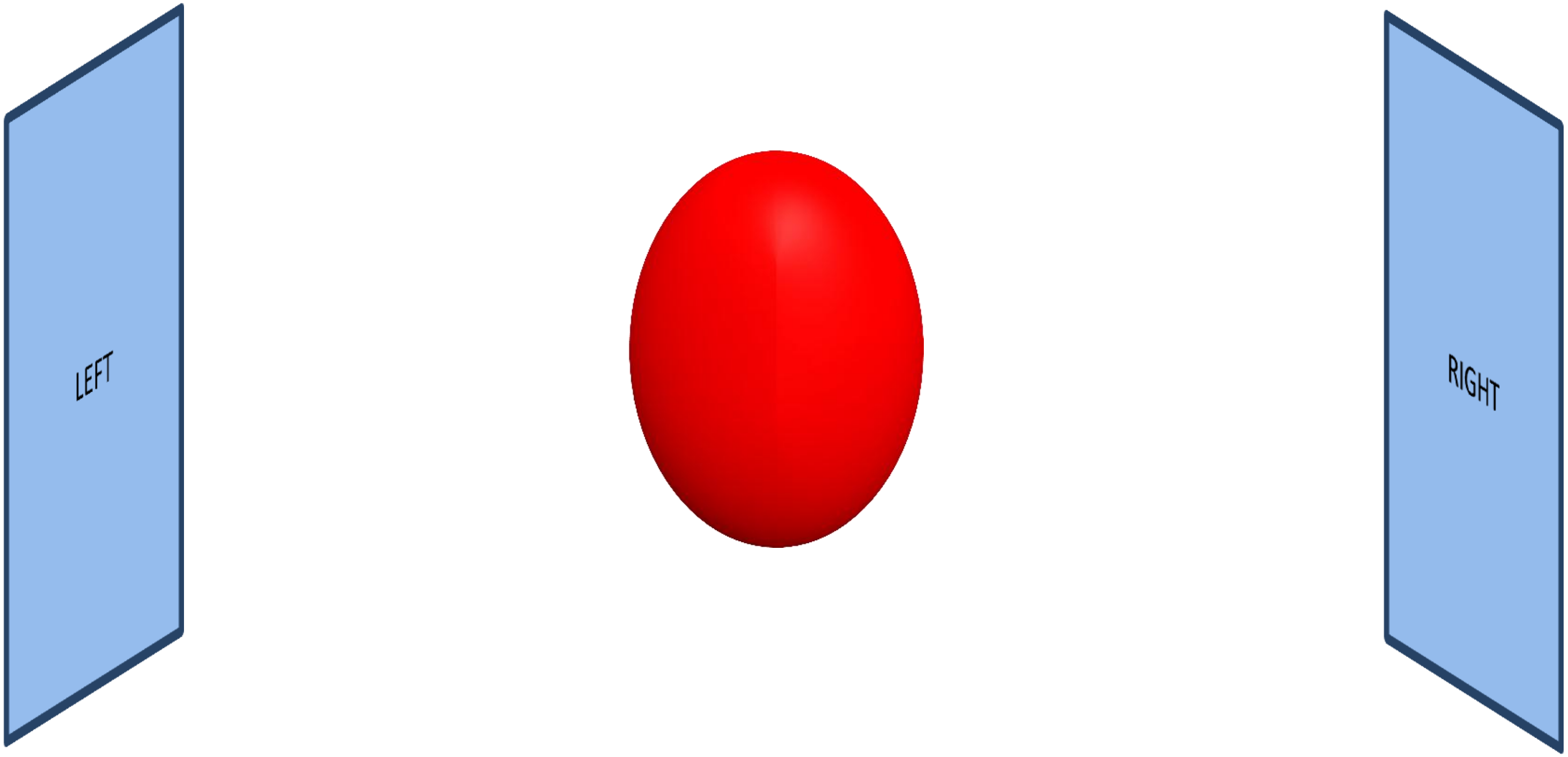
Electric field

What stabilizes the instanton



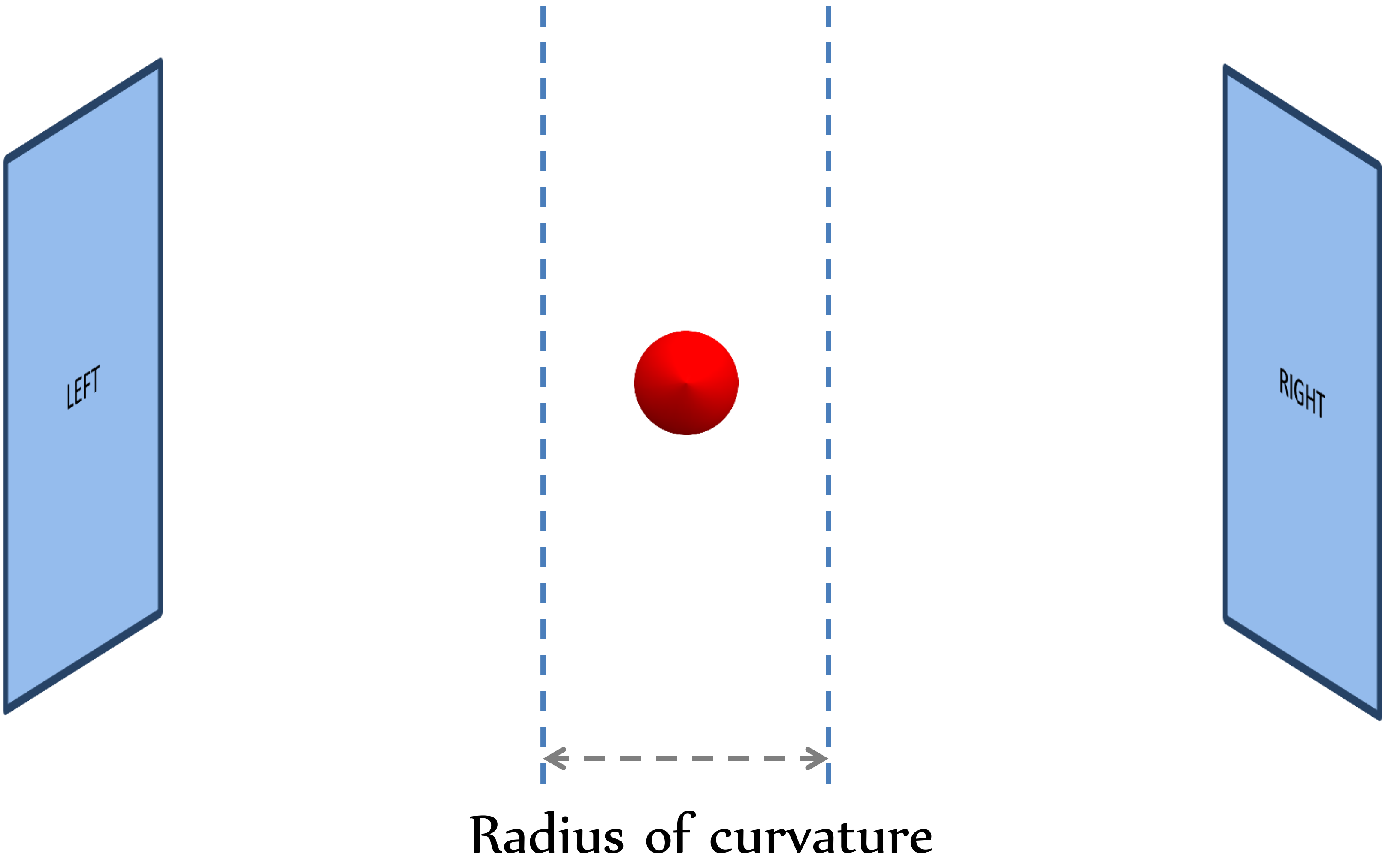
GRAVITY

What stabilizes the instanton



Squashed

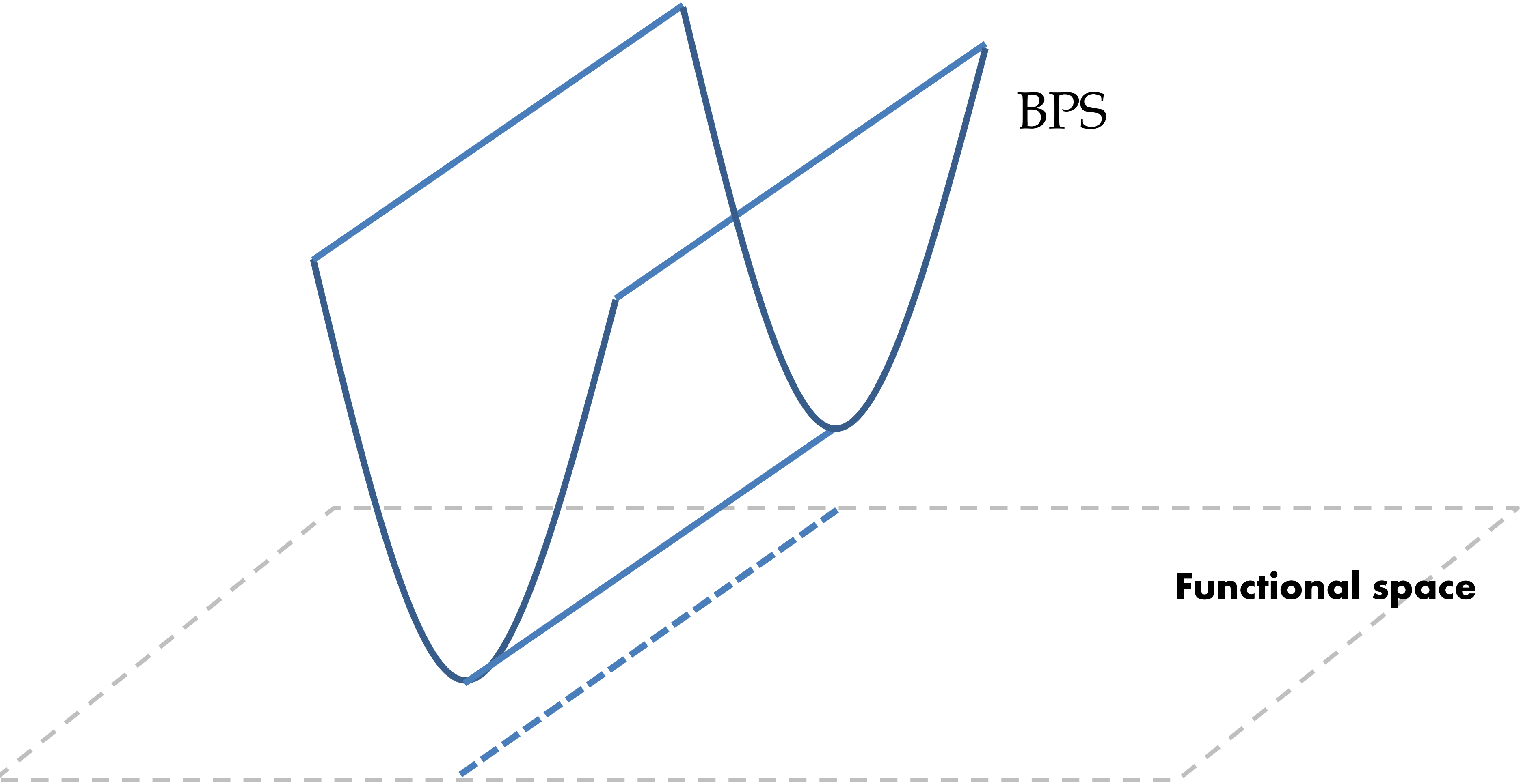
Self-Dual Approximation



Flow to the BPS solution

1311.2685 with P.Sutcliffe and 1407.3140 with W.Zakrzewski

Flow to the BPS solution



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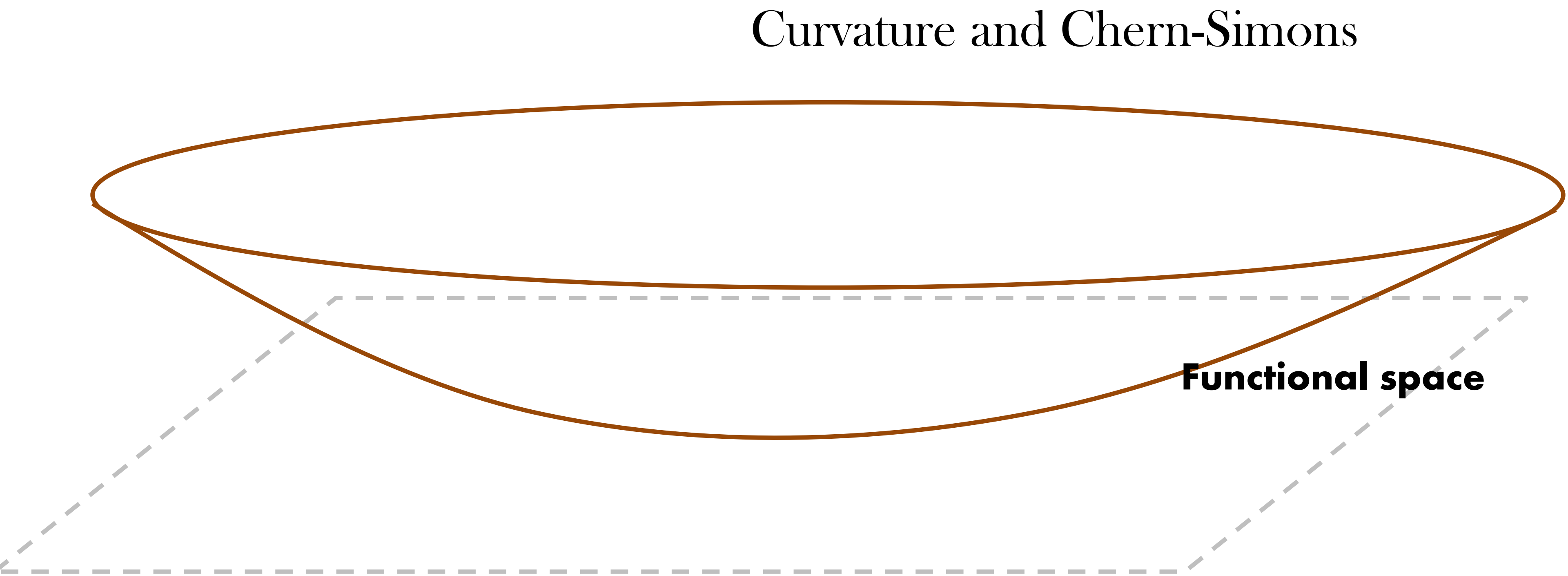

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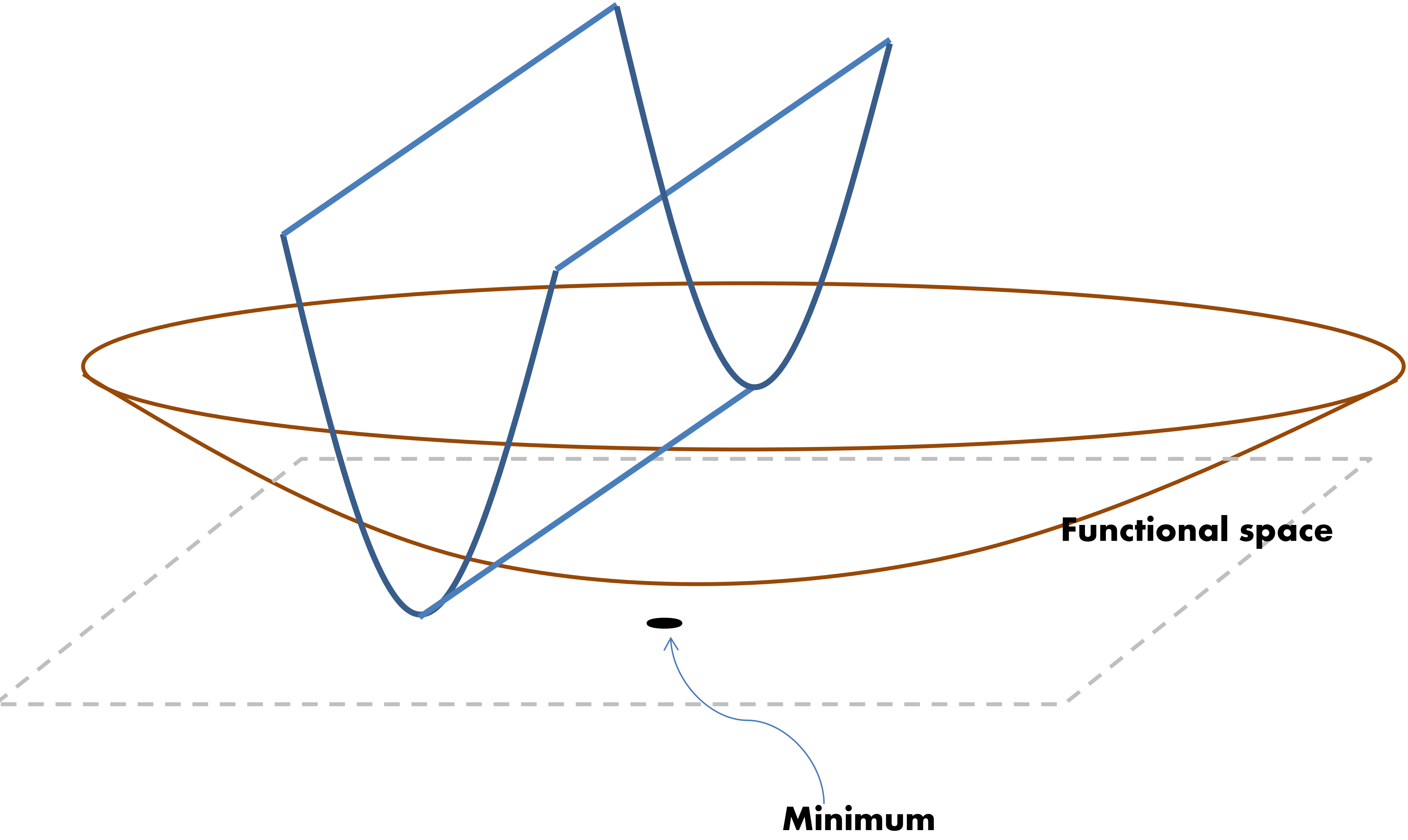
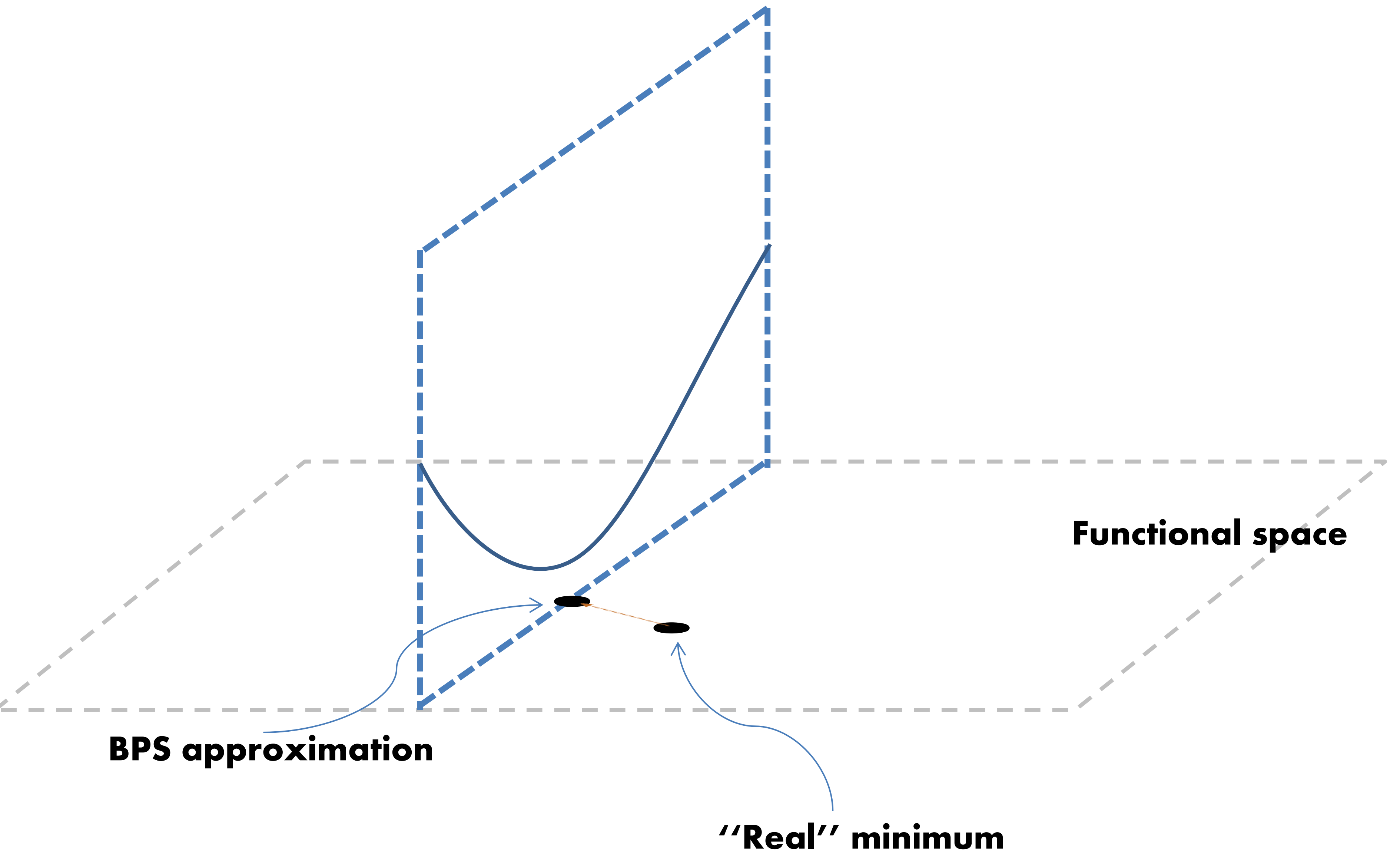
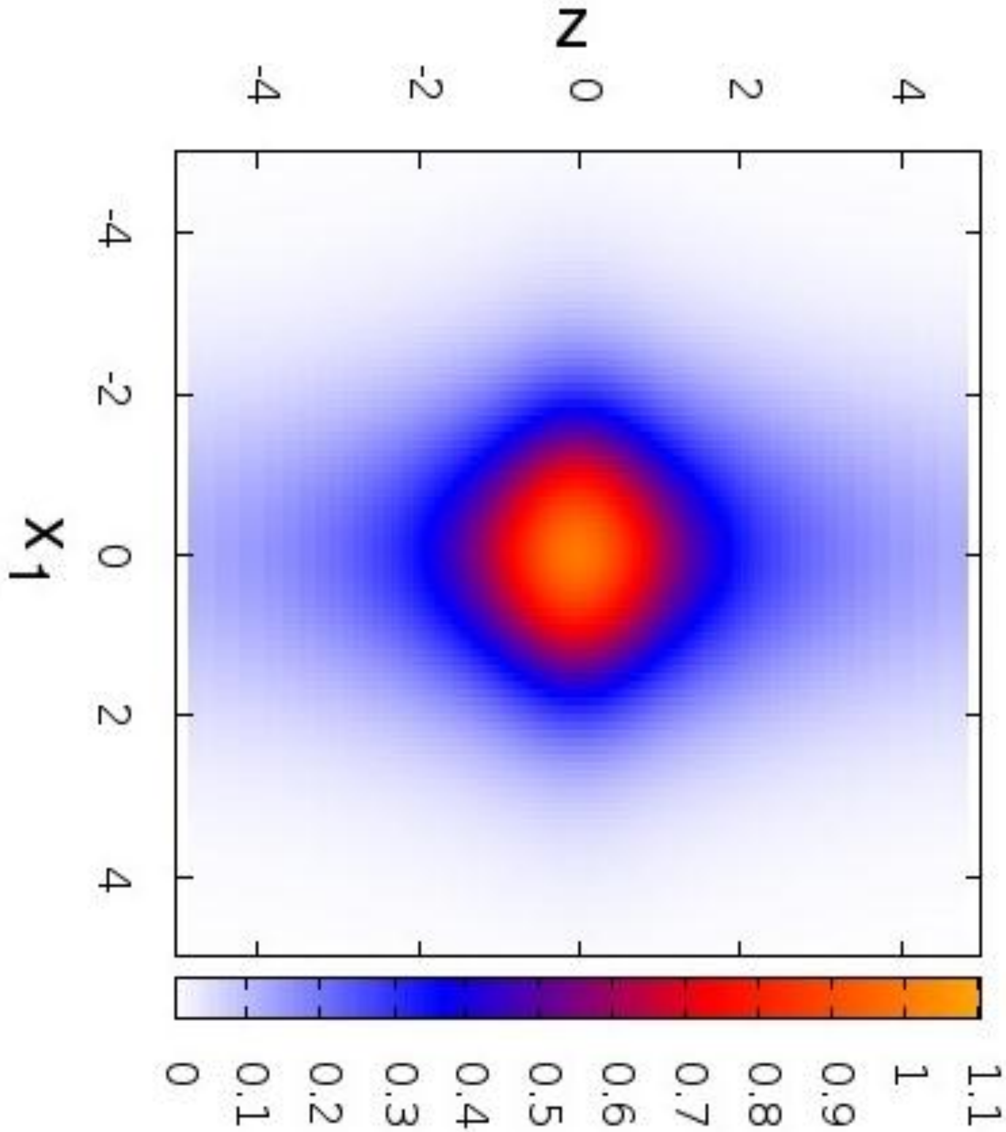


Flow to the BPS solution



Full numerical result



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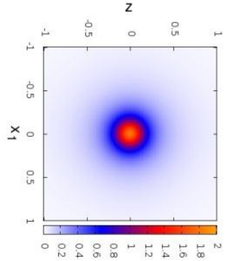

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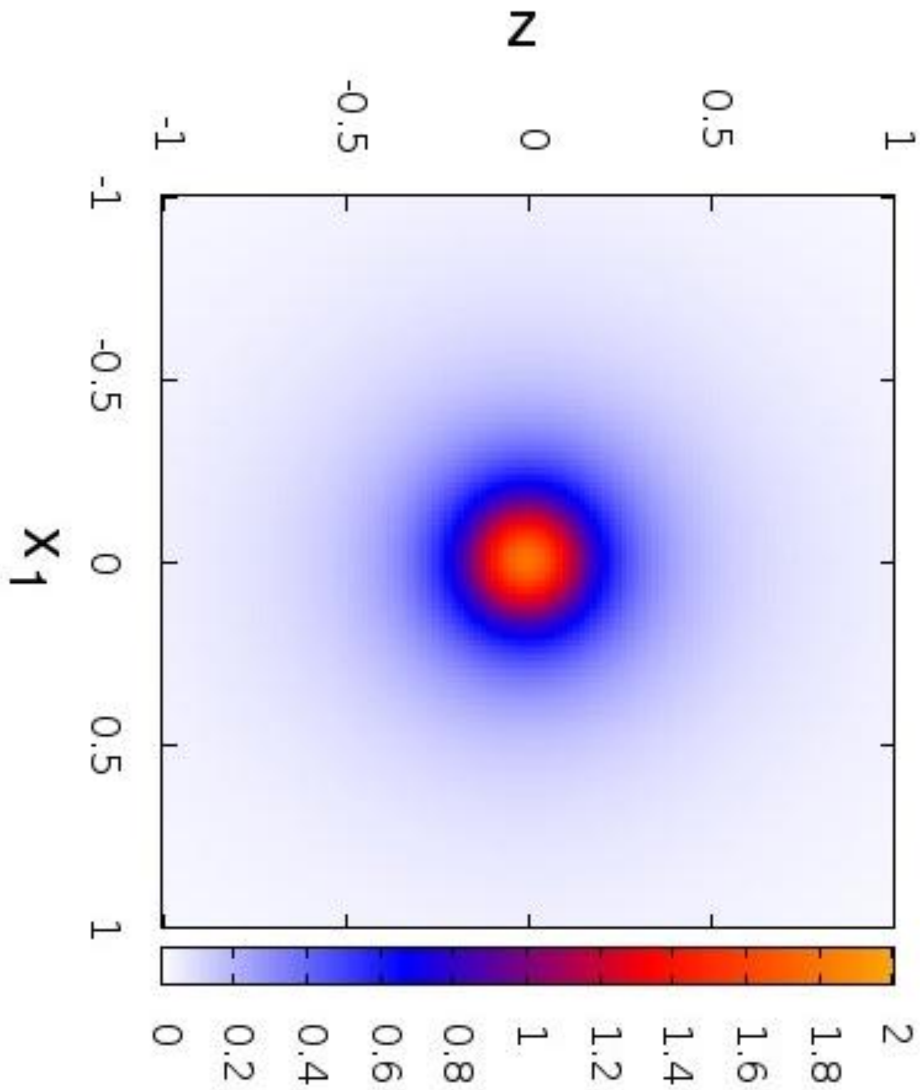
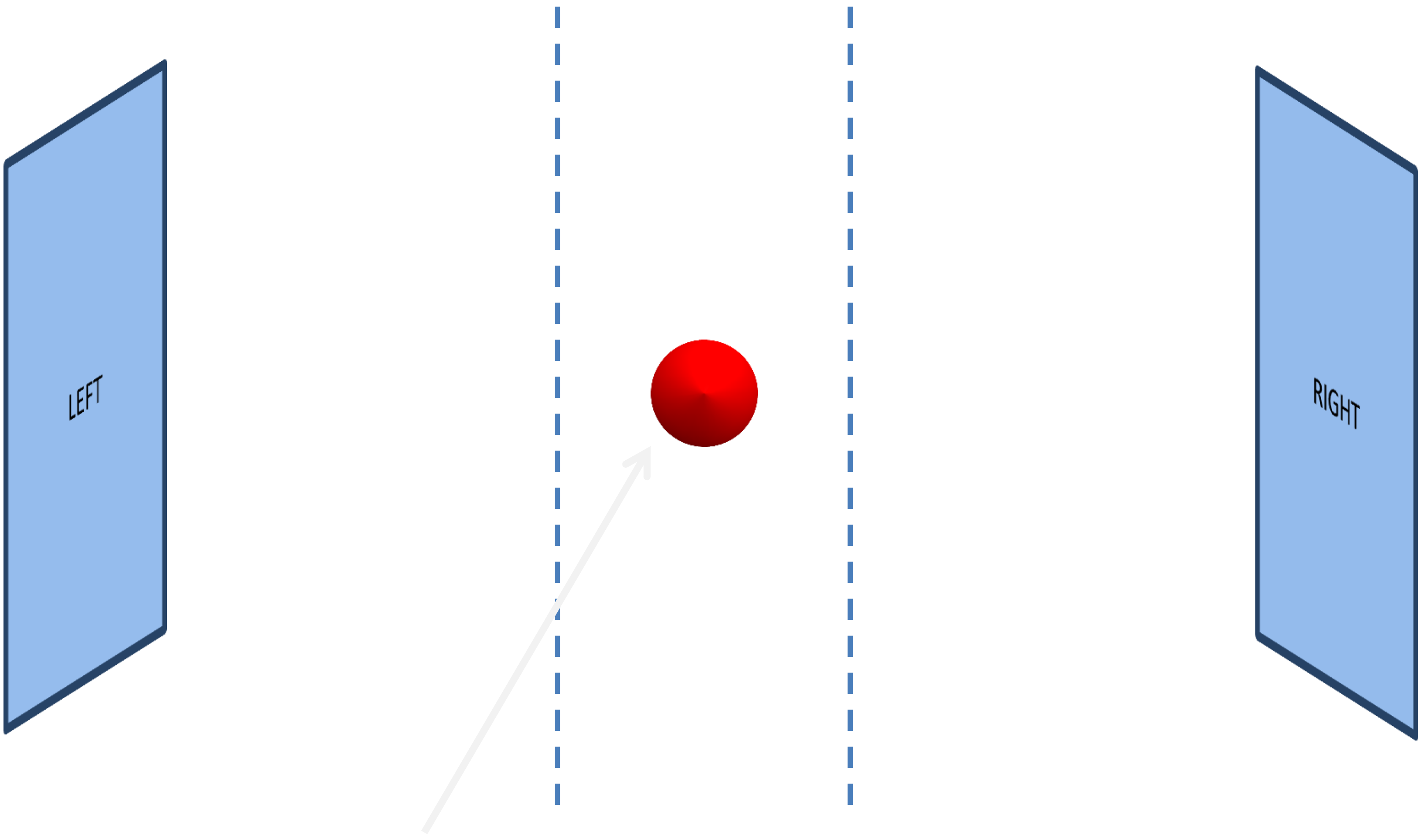
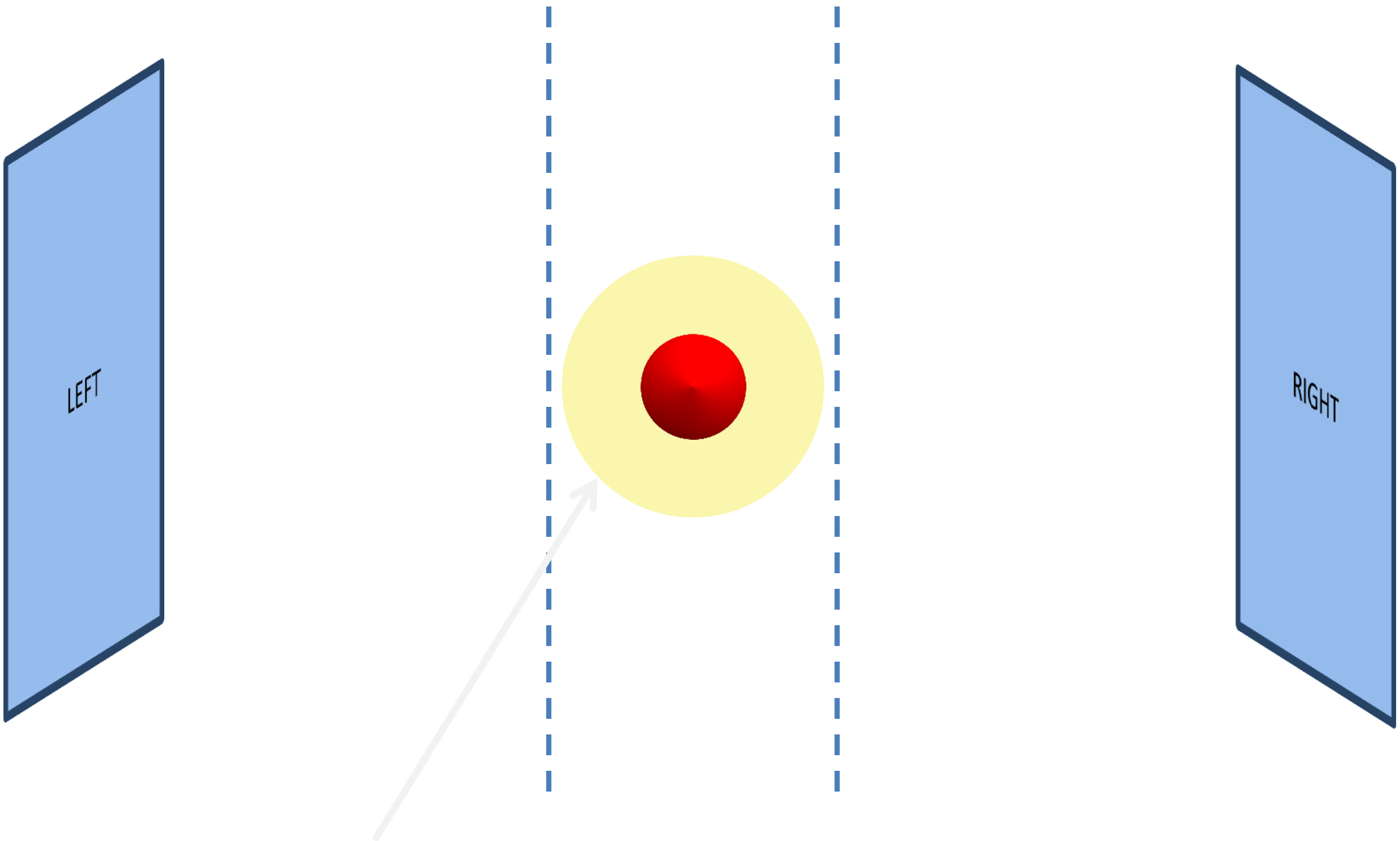


BPS core



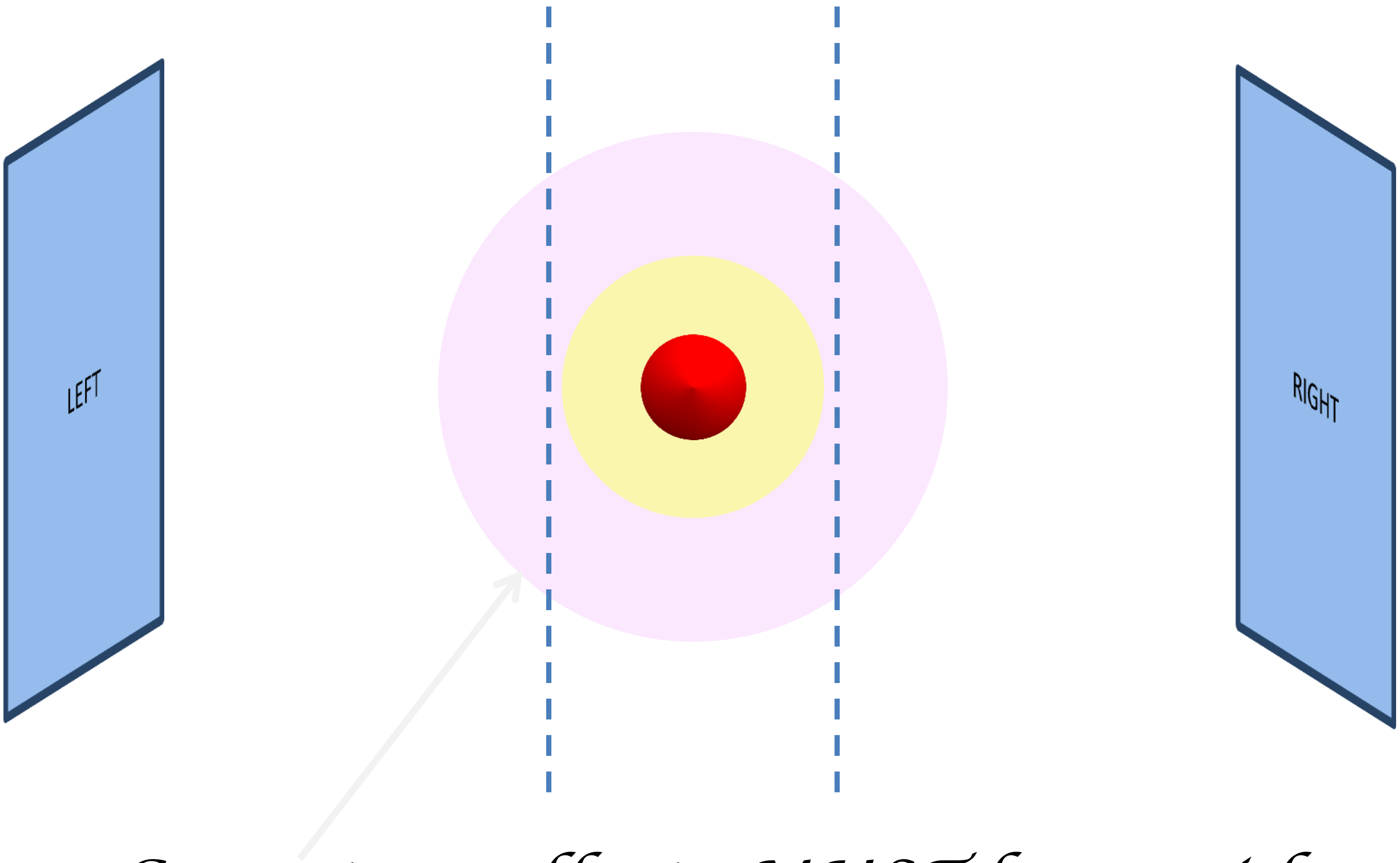
Full non-linear problem

Medium distance



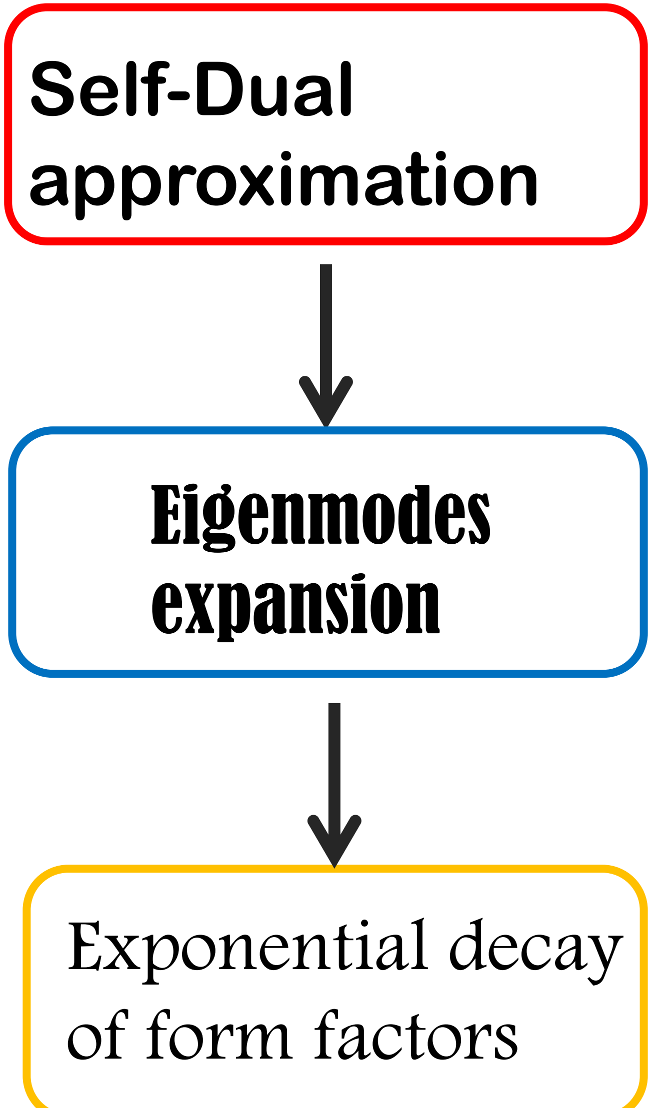
Linear and flat space-time

Large distance

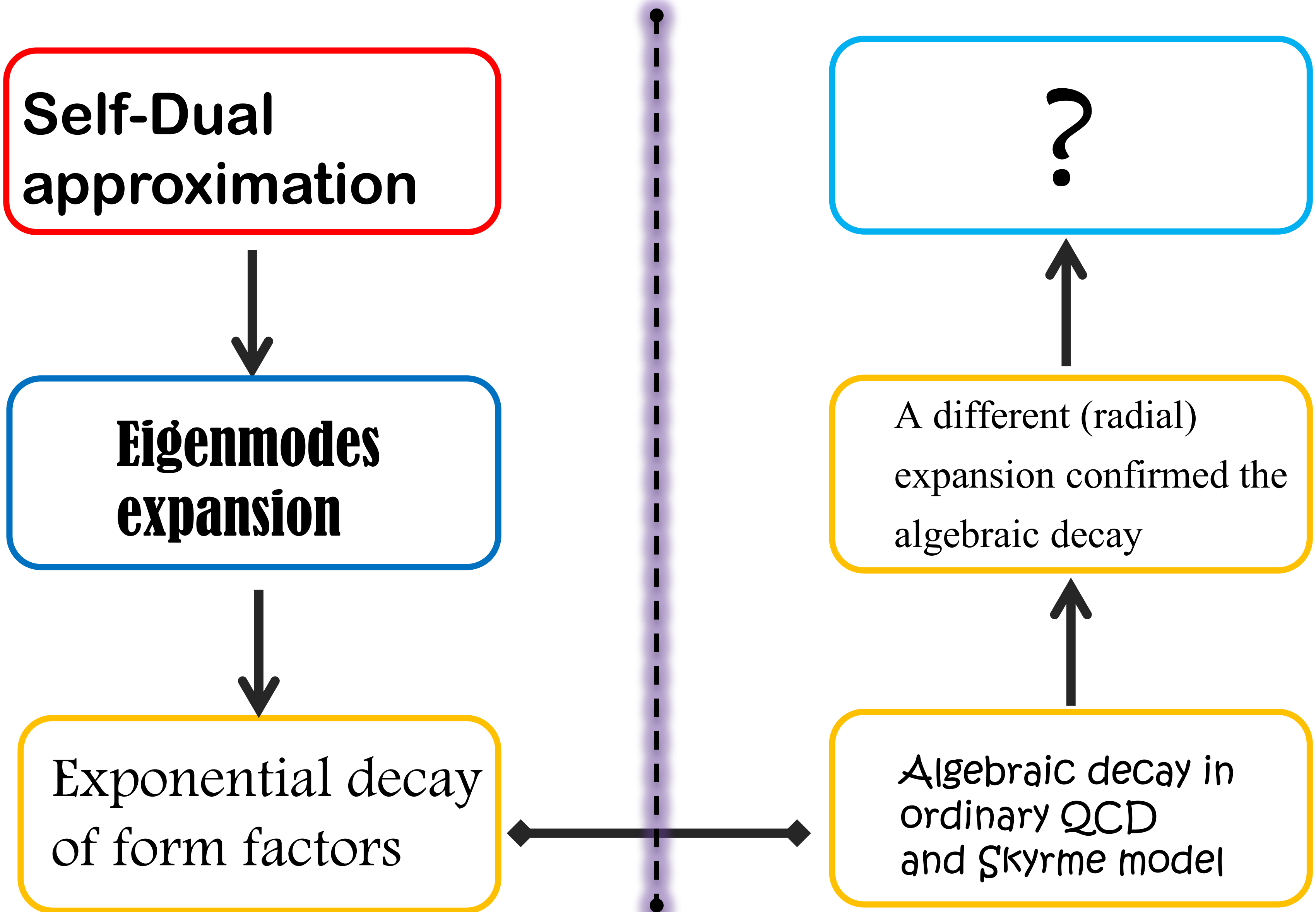


Curvature effects MUST be considered

Form factors puzzle



Form factors puzzle



Form factors puzzle

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[13] A. Cherman and T. Ishii, "Long-distance properties of baryons in the Sakai-Sugimoto model," Phys. Rev. D **86** (2012) 045011 [arXiv:1109.4665 [hep-th]].

[14] P. Colangelo, J. J. Sanz-Cillero and F. Zuo, "Large-distance properties of holographic baryons," arXiv:1306.6460 [hep-ph].

How to compute a soliton tail

BPST instanton in ‘t Hooft gauge

$$A_I = \frac{1}{2}\sigma_{IJ}\partial_J \log\left(1 + \frac{\mu^2}{\rho^2}\right)$$

How to compute a soliton tail

BPST instanton in ‘t Hooft gauge

$$A_I = \frac{1}{2}\sigma_{IJ}\partial_J \log\left(1+\frac{\mu^2}{\rho^2}\right)$$

At large distance form the core fields are small and equations "linearize"

$$A_I^{(1)} = -\sigma_{IJ}\frac{x_J\,\mu^2}{\rho^4} = \frac{\mu^2}{2}\sigma_{IJ}\partial_J\frac{1}{\rho^2}$$

Separation of variables

$$\frac{1}{\rho^2} = \frac{1}{r^2 + z^2} = \int_0^\infty \frac{e^{-kr}}{r} \cos(kz)\, dk$$

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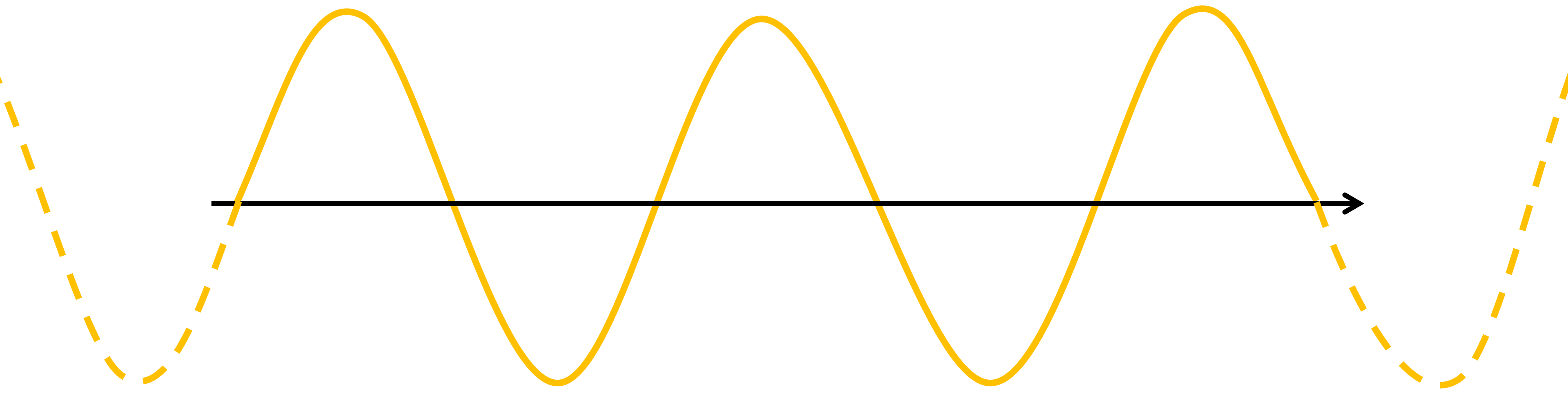
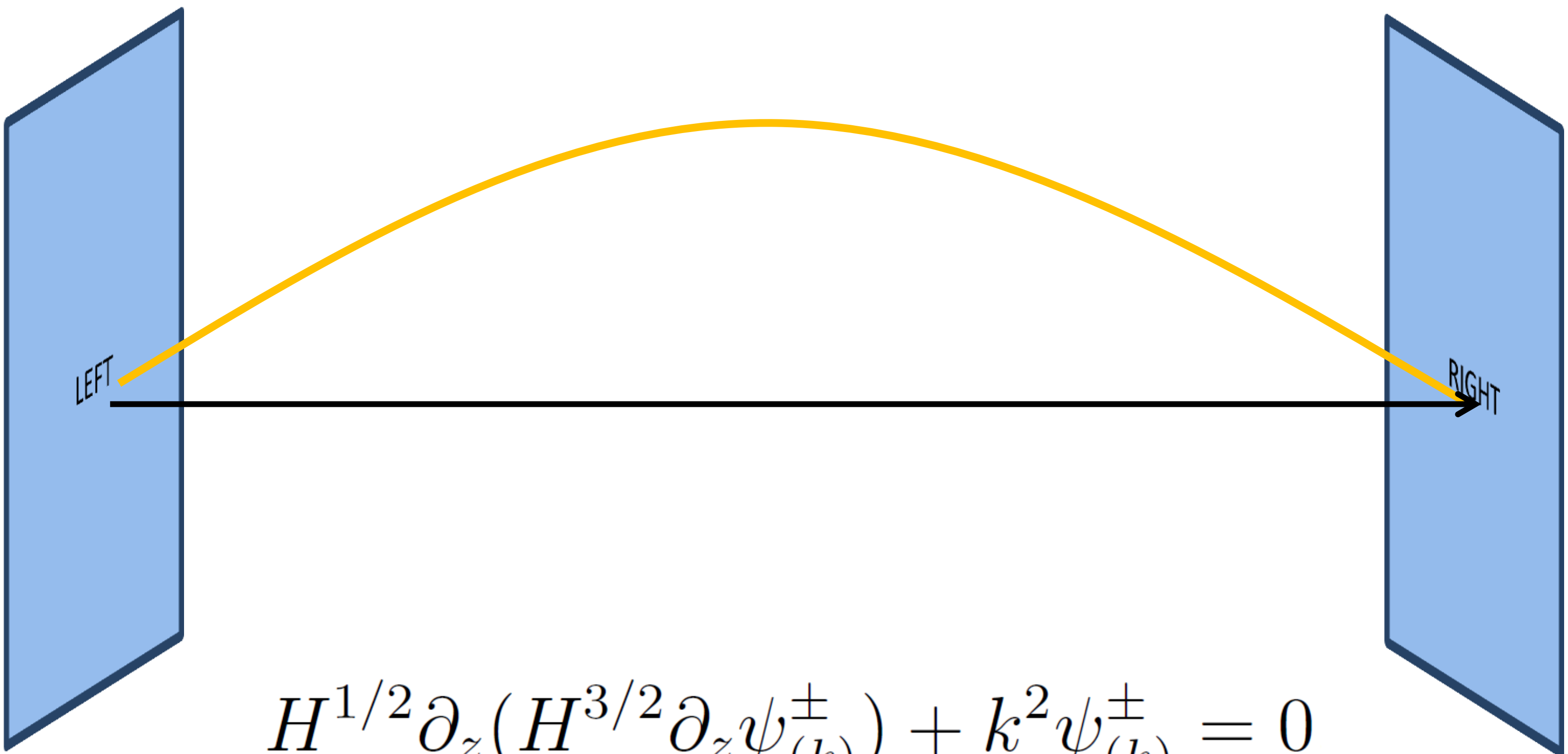
Laplace-Fourier expansion

Eigenmodes in flat space

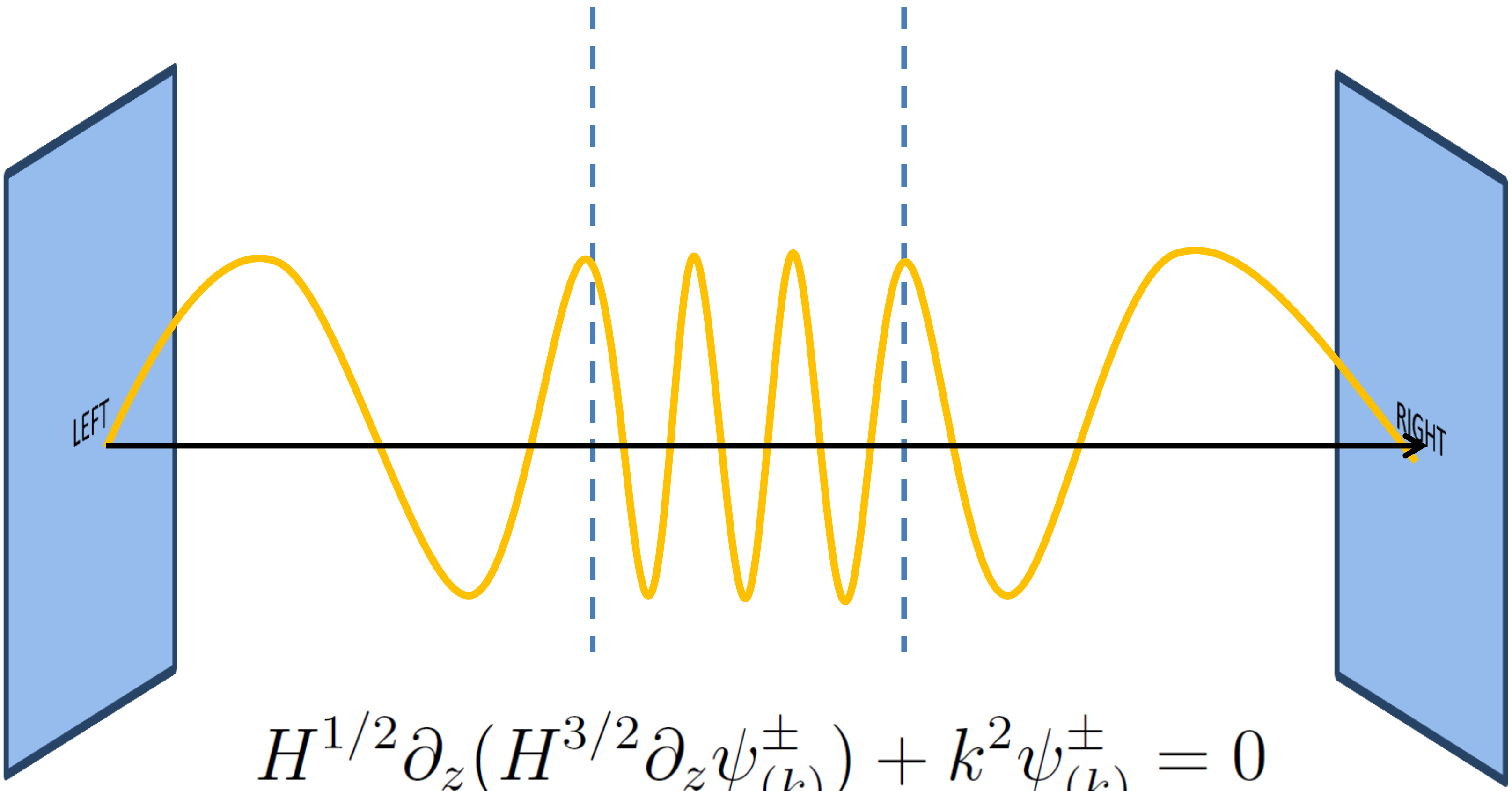


Eigenmodes in curved space



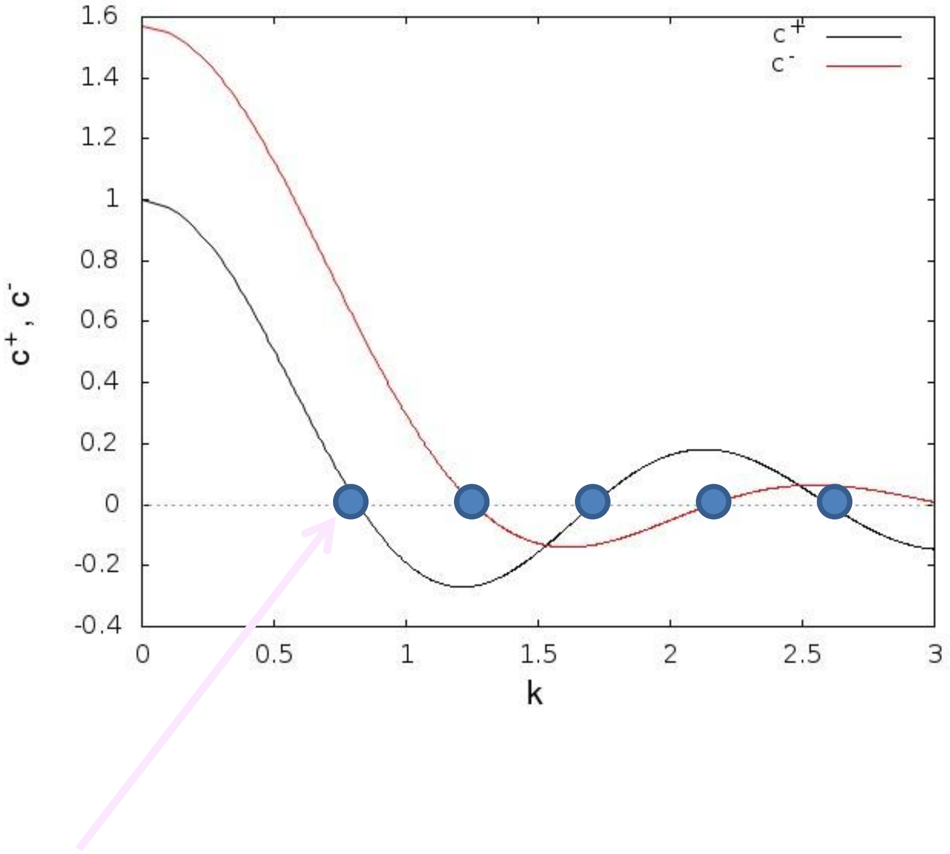
$$H^{1/2}\partial_z(H^{3/2}\partial_z\psi^{\pm}_{(k)}) + k^2\psi^{\pm}_{(k)} = 0$$

Eigenmodes expansion in curved space



$$H^{1/2}\partial_z(H^{3/2}\partial_z\psi^{\pm}_{(k)}) + k^2\psi^{\pm}_{(k)} = 0$$

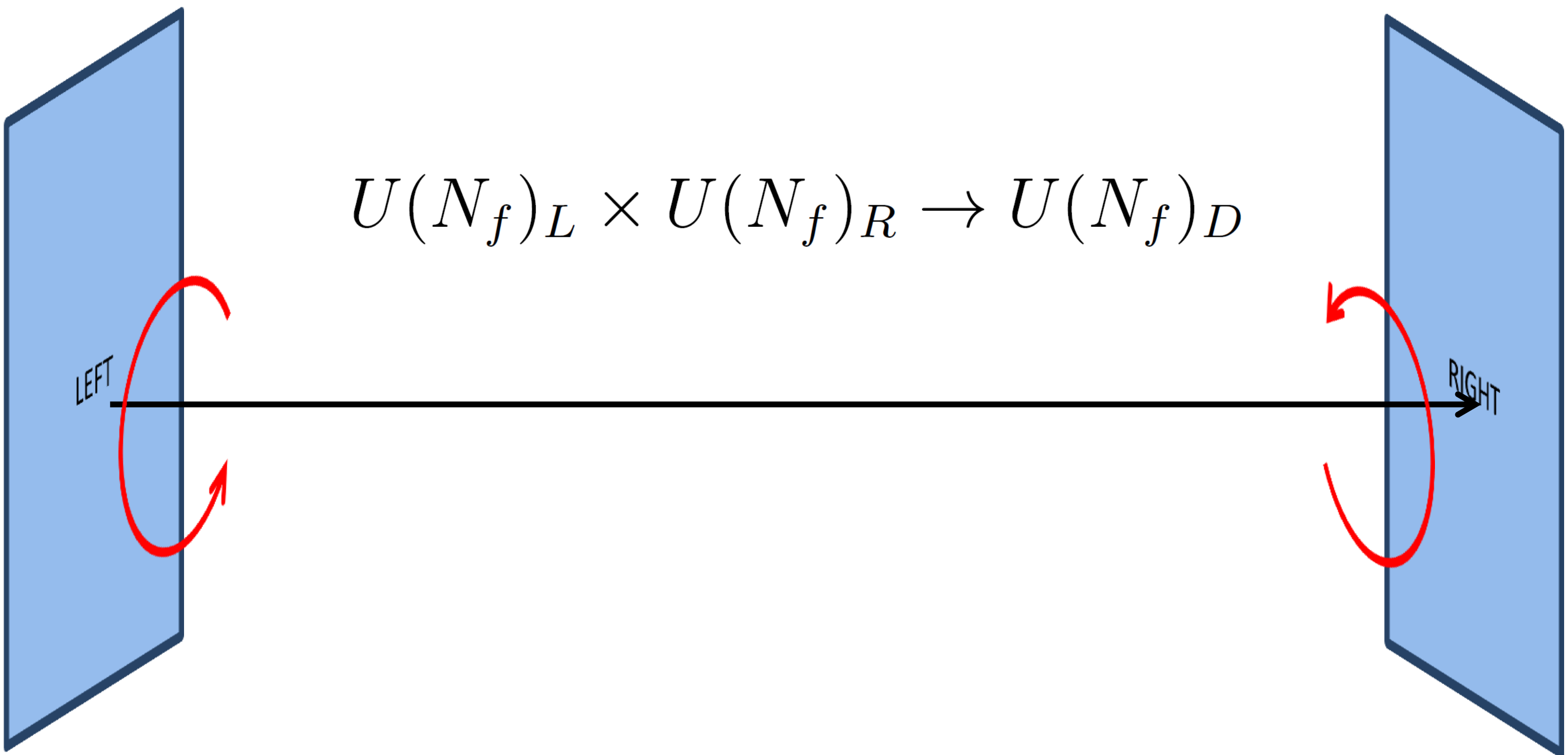
The effect of a conformal boundary



The conformal boundary, and the boundary condition, cause the discretization of the spectrum

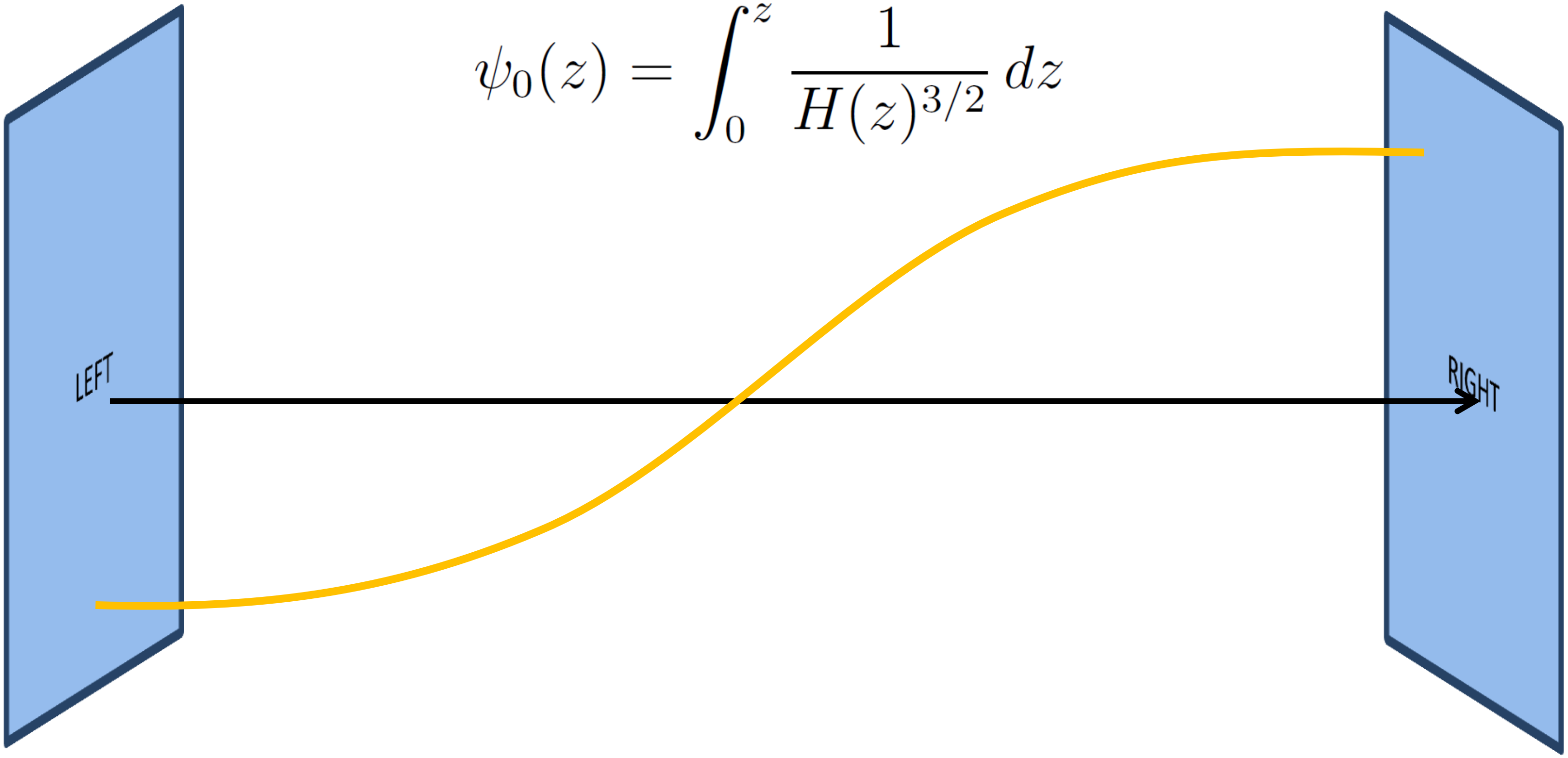
Goldstone boson

$$U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_D$$



The massless mode

$$\psi_0(z) = \int_0^z \frac{1}{H(z)^{3/2}}\, dz$$



The "remnants" at large distance (at linear level)

$$A_z^{(1)} \quad \propto \quad \frac{1}{\lambda}\,\frac{\sigma_i \hat{x}_i}{r^2}\,\psi_0'(z) + \mathcal{O}\left(\frac{e^{-k_2 r}}{r}\right)$$

$$A_i^{(1)} = \mathcal{O}\left(\frac{e^{-k_1 r}}{r}\right)$$

$$\widehat{A}_0^{(1)} = \mathcal{O}\left(\frac{e^{-k_1 r}}{r}\right)$$

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Pion contribution

Non-linear *contamination*

$$A_z^{(1)} \propto \frac{1}{\lambda} \frac{\sigma_i \hat{x}_i}{r^2} \psi_0'(z)$$

Pion tail

Non-linear *contamination*

$$A_z^{(1)} \propto \frac{1}{\lambda}\,\frac{\sigma_i \hat{x}_i}{r^2}\,\psi_0'(z)$$

Pion tail

$$A_i^{(2)} \propto \frac{1}{\lambda^2}\epsilon_{ijk}\frac{\sigma_j \hat{x}_k}{r^5}\,\eta(z)$$

Instanton charge

Non-linear *contamination*

$$A_z^{(1)} \propto \frac{1}{\lambda} \frac{\sigma_i \hat{x}_i}{r^2} \psi_0'(z)$$

Pion tail

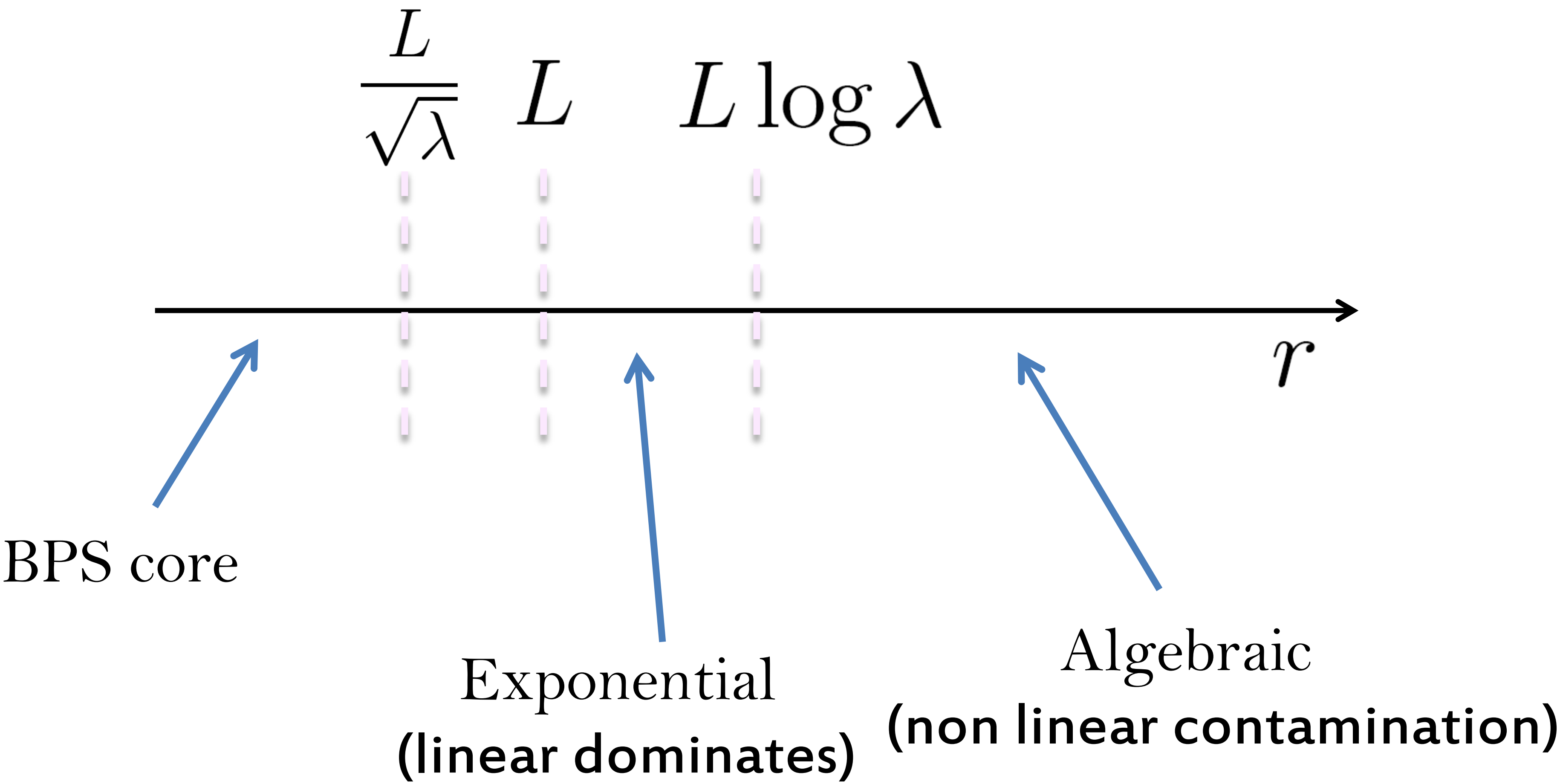
$$A_i^{(2)} \propto \frac{1}{\lambda^2} \epsilon_{ijk} \frac{\sigma_j \hat{x}_k}{r^5} \eta(z)$$

Instanton charge

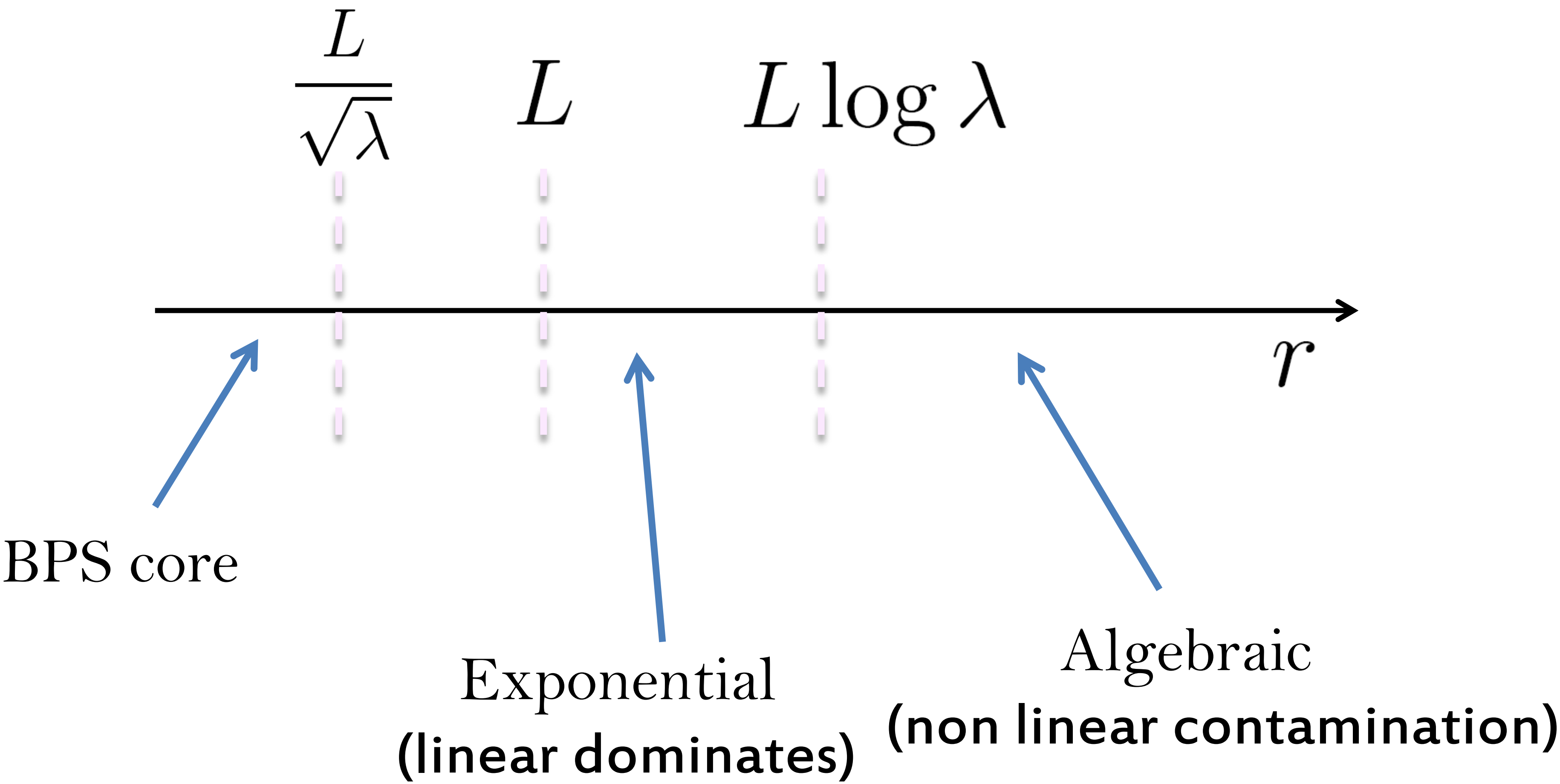
Baryon charge

$$\hat{A}_0^{(4)} \propto \frac{1}{\lambda^4} \frac{1}{r^9} \varrho(z)$$

The emergence of a new *BIG* scale



Noncommutativity of large ‘t Hooft coupling and large distance limits!



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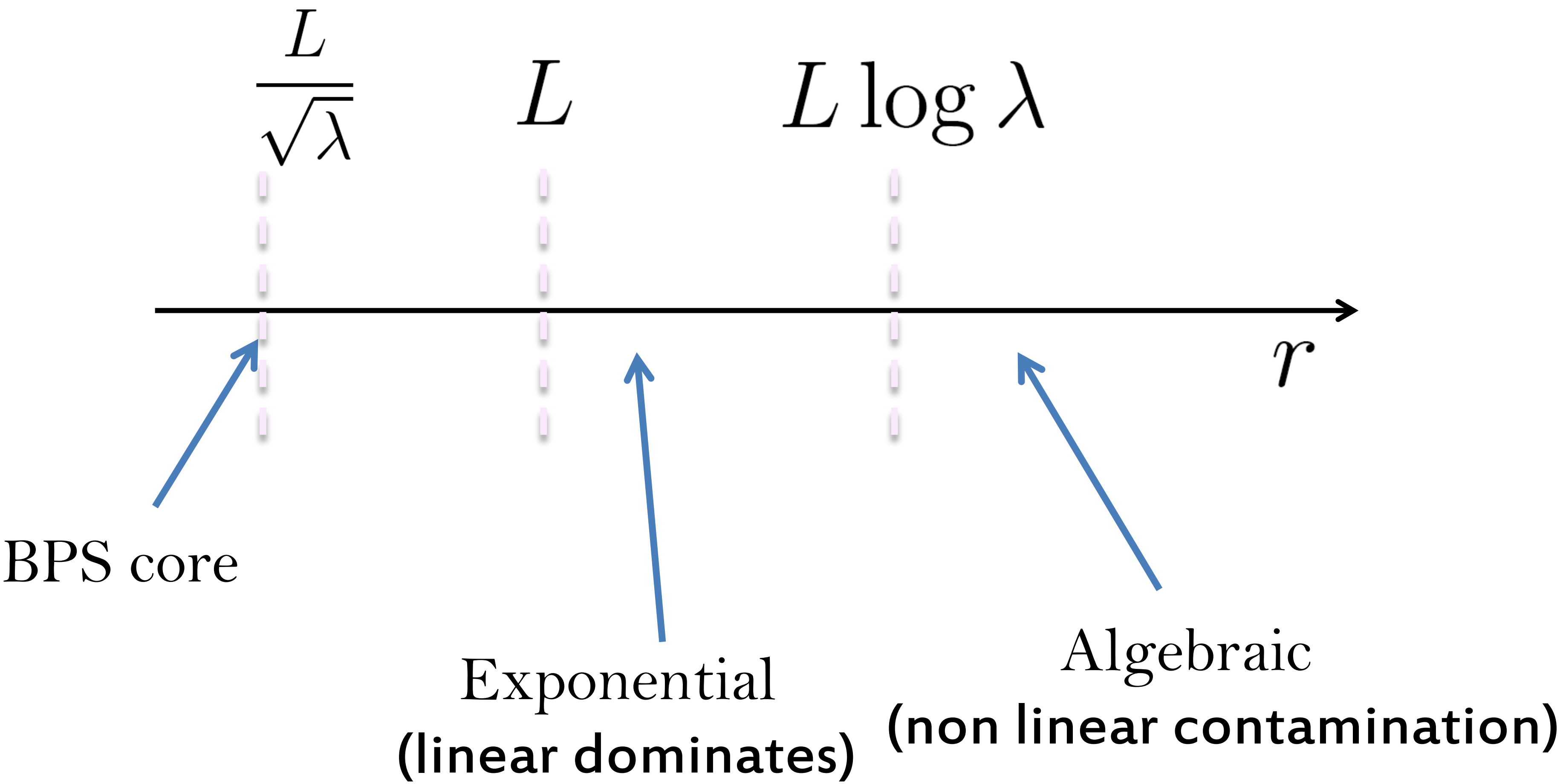
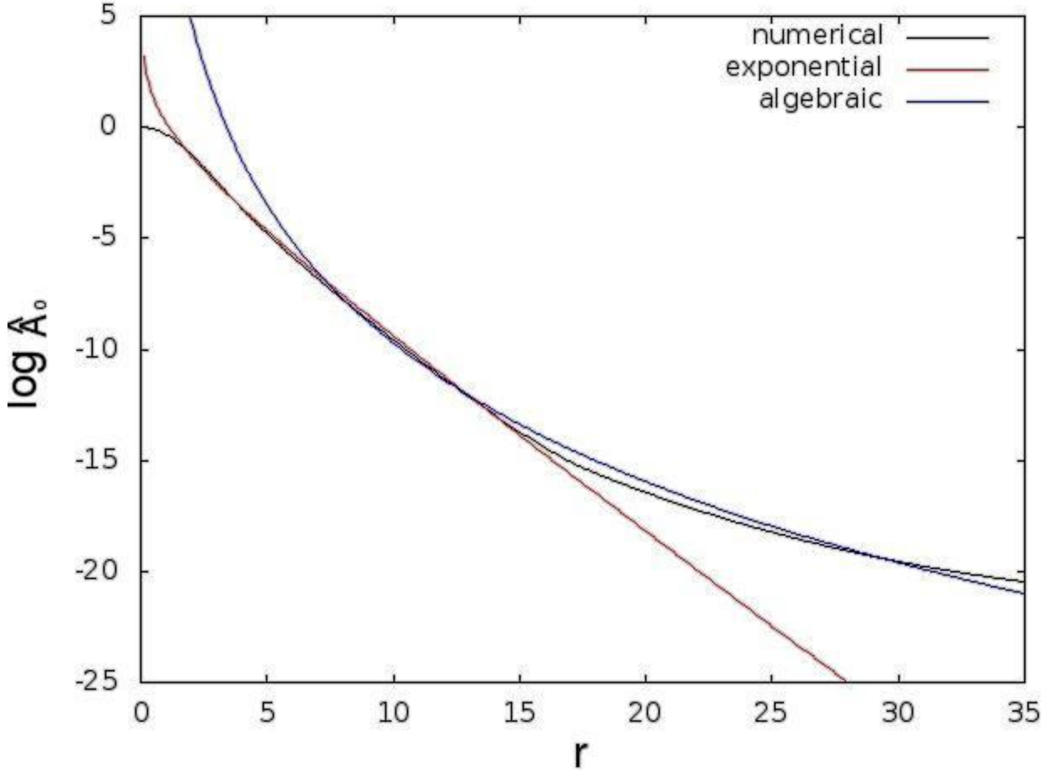


Numerical result for baryon charge



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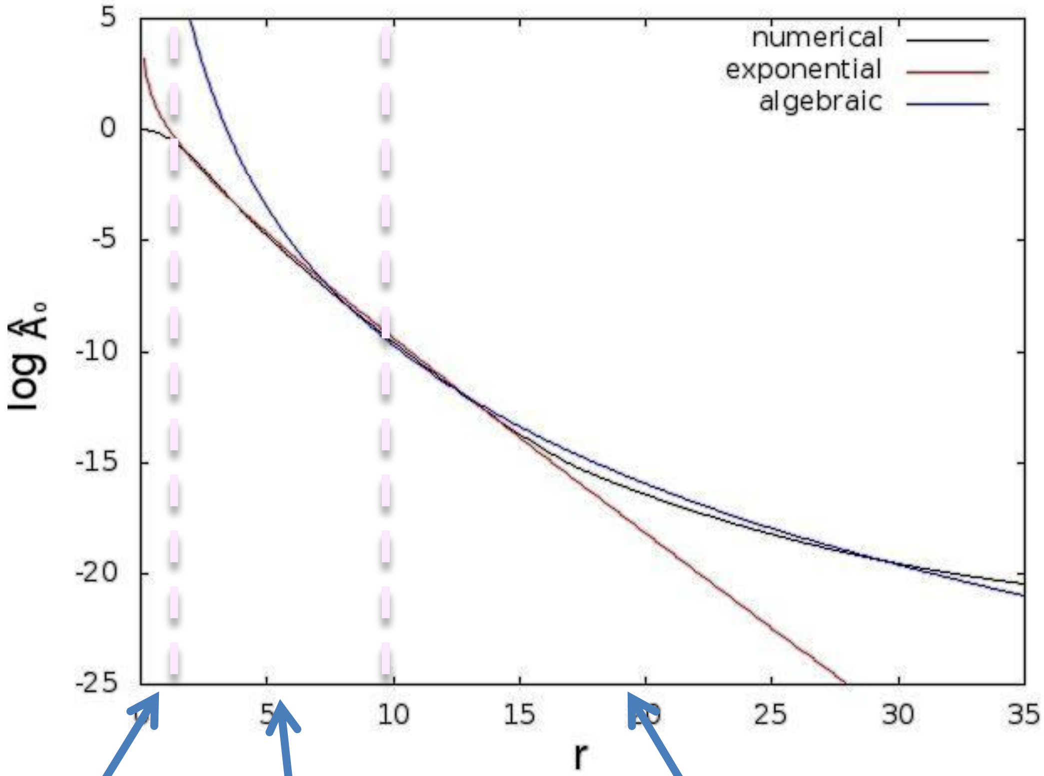

BPS core

Exponential
(linear)

Algebraic
(non linear)

Conclusion (part 1)

- We provided analytical and numerical understanding for various approximations used in holographic QCD
- All of them are correct, but only is applied to their region of validity

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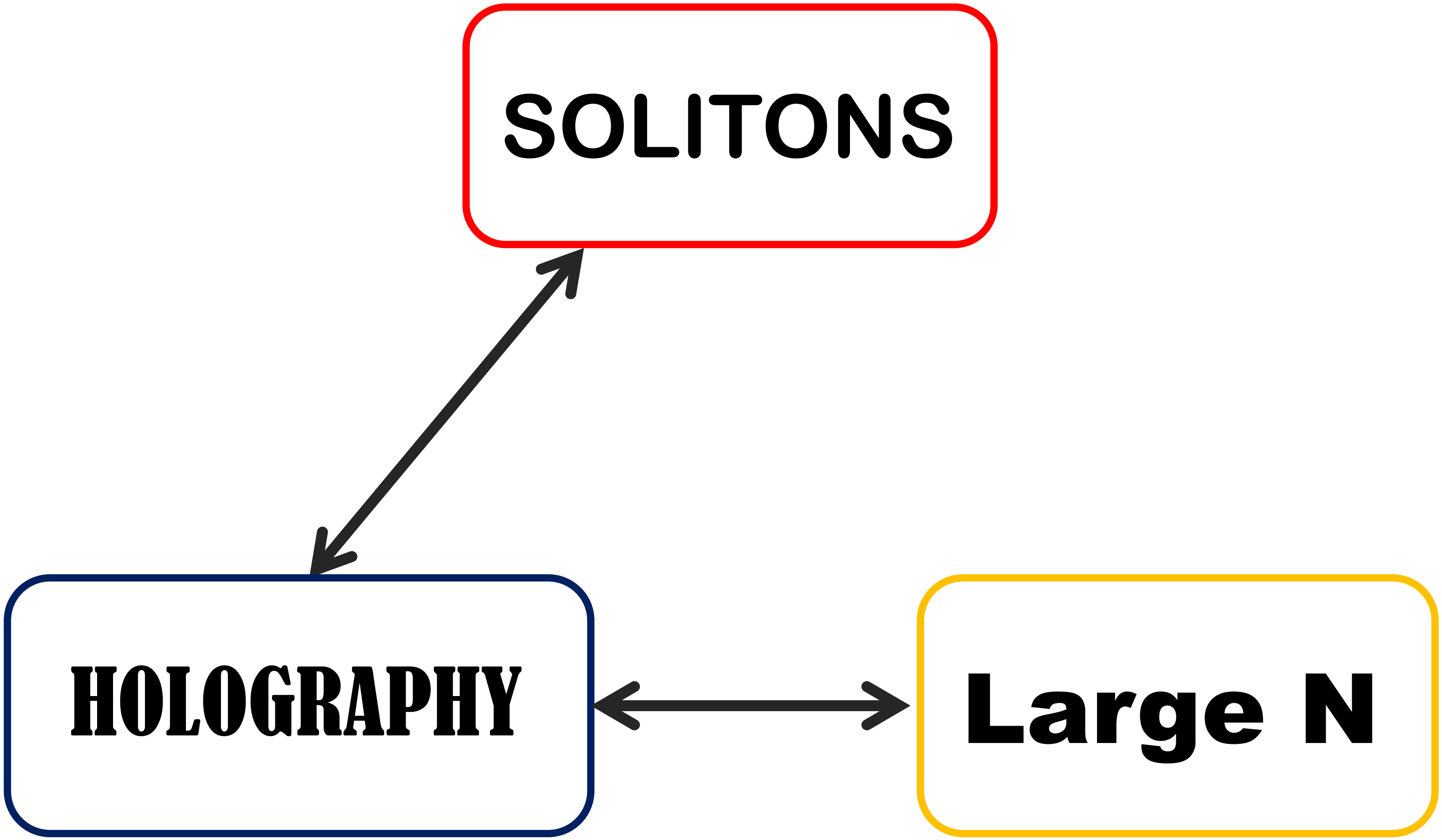
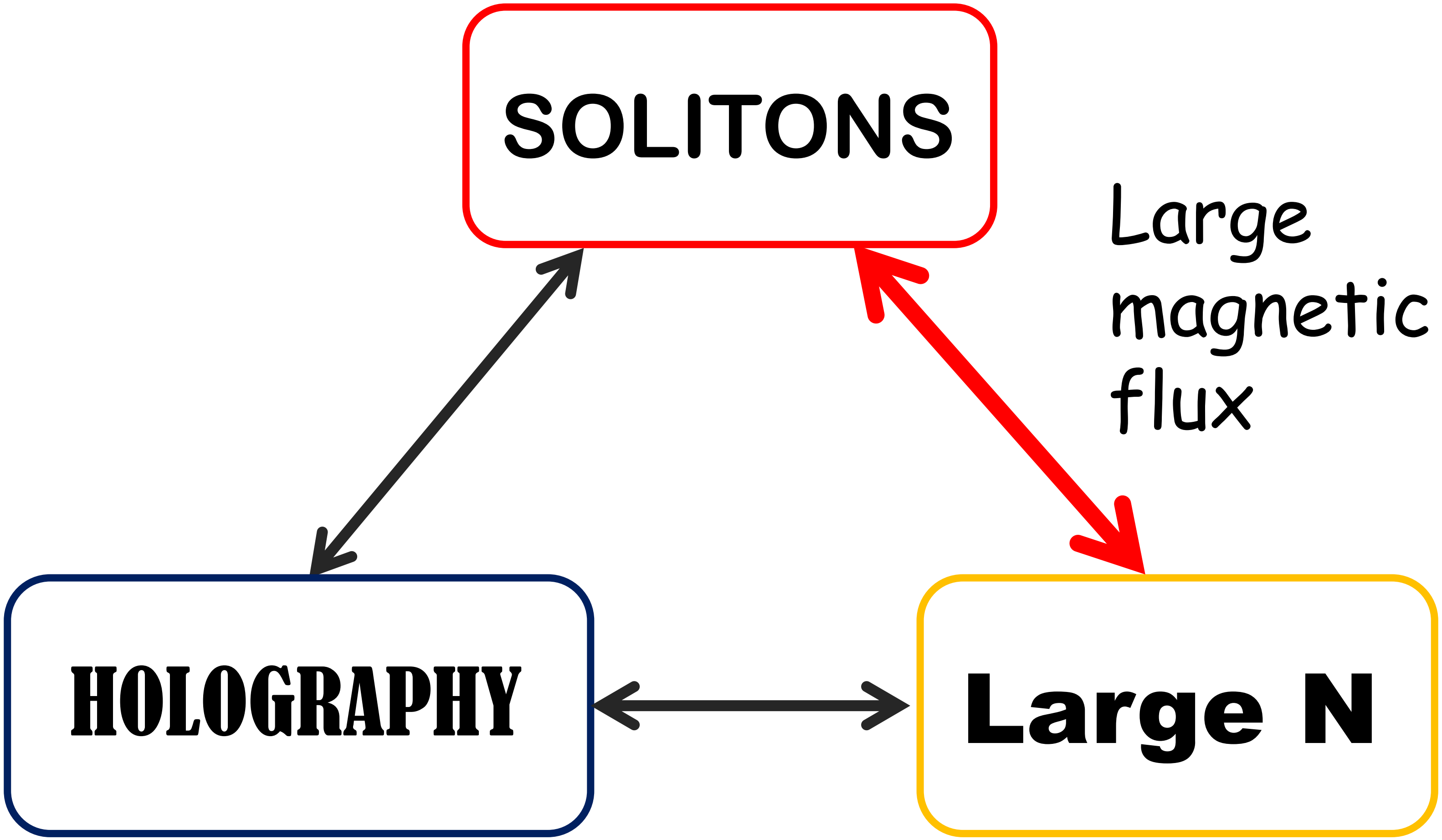
- We provided analytical and numerical understanding for various approximations used in holographic QCD
- All of them are correct, but only is applied to their region of validity
- Exponential vs. algebraic decay puzzle is solved by the existence of a new large scale. This explains non-commutativity of large ‘t Hooft and large radius limits.

Plan of the talk (part 2)



Plan of the talk (part 2)



Plan of the talk (part 2)

- Introduction to magnetic bags

- Instanton bags

- High density holographic QCD

based on arXiv:1406.0205

BPS Monopoles

$$E = \int_{\mathcal{M}} d^3x \sqrt{g}\ \mathrm{tr}\left(\frac{1}{2} F_{ij} F^{ij} + D_i \Phi D^i \Phi\right)$$

BPS Monopoles

$$E = \int_{\mathcal{M}} d^3x \sqrt{g}\ \mathrm{tr}\left(\frac{1}{2} F_{ij} F^{ij} + D_i \Phi D^i \Phi\right)$$

$$E = \int_{\mathcal{M}} d^3x \sqrt{g}\ \frac{1}{2}\mathrm{tr}\big(F_{ij} - \sqrt{g}\epsilon_{ijk} D^k \Phi\big)^2 + \int_{\partial\mathcal{M}} \mathrm{tr}(F_{ij}\Phi)$$

=0

Bogomolny Equation

Boundary term

BPS Monopoles

Boundary conditions:

Scalar field $$|\Phi| = \sqrt{2\,\mathrm{tr}\,\Phi^2} = v \qquad \text{on} \qquad \partial\mathcal{M}$$

Gauge field $$n = \frac{1}{4\pi v}\int_{\partial\mathcal{M}} \mathrm{tr}(F_{ij}\Phi) \qquad n \in \mathbb{Z}$$

BPS Monopoles

Boundary conditions:

Higgs vev

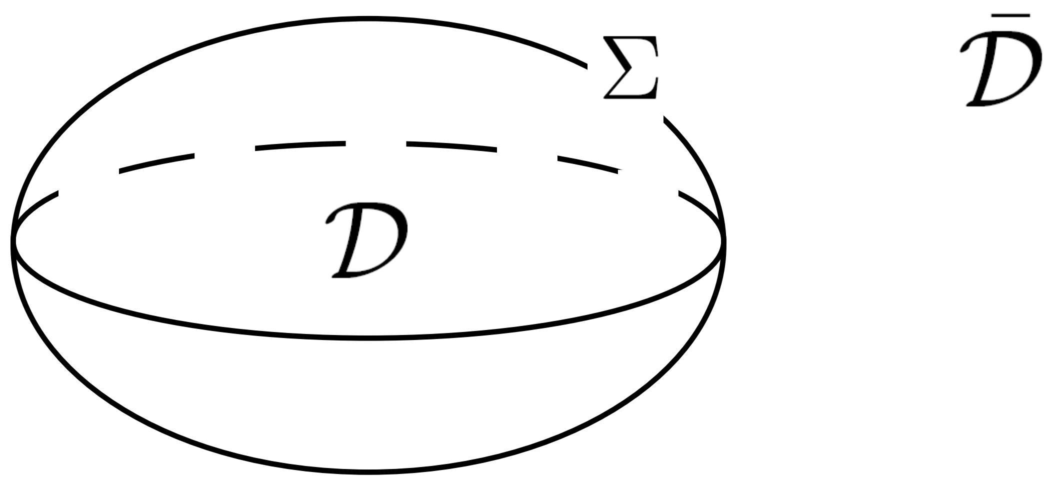
Scalar field $$|\Phi| = \sqrt{2\,\mathrm{tr}\,\Phi^2} = v \quad \text{on} \quad \partial\mathcal{M}$$

Gauge field $$n = \frac{1}{4\pi v}\int_{\partial\mathcal{M}} \mathrm{tr}(F_{ij}\Phi) \qquad n \in \mathbb{Z}$$

Magnetic charge

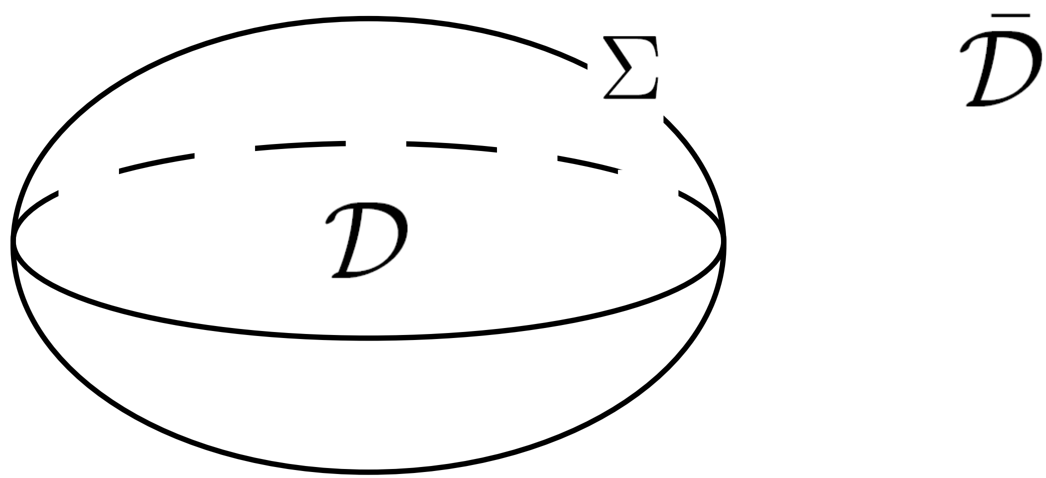
Definition of a magnetic bag

$\Sigma = \partial \mathcal{D}$ $\qquad$ $\mathcal{M} = \mathcal{D} \cup \bar{\mathcal{D}}$



Definition of a magnetic bag

$$E = \int_{\bar{\mathcal{D}}} d^3x \sqrt{g} \left(\frac{1}{4} f_{ij} f^{ij} + \frac{1}{2} \partial_i \phi \partial^i \phi \right)$$



Definition of a magnetic bag

$$E = \int_{\bar{D}} d^3x \sqrt{g} \left(\frac{1}{4} f_{ij} f^{ij} + \frac{1}{2} \partial_i \phi \partial^i \phi \right)$$

$$\phi \simeq |\Phi| \qquad \text{and} \qquad f_{ij} \simeq \frac{\mathrm{tr}(F_{ij}\Phi)}{2|\Phi|}$$

Definition of a magnetic bag

Boundary conditions:

$$\phi|_{\Sigma} = 0 \qquad \phi|_{\partial\mathcal{M}} = v$$

$$\frac{1}{2\pi}\int_{\partial\Sigma} f_{ij} = n$$

Definition of a magnetic bag

Boundary conditions:

$$\phi|_{\Sigma} = 0 \qquad \phi|_{\partial\mathcal{M}} = v$$

$$\frac{1}{2\pi}\int_{\partial\Sigma} f_{ij} = n$$

$$E = \int_{\mathcal{M}} d^3x\sqrt{g}\,\frac{1}{4}\left(f_{ij} \mp \sqrt{g}\epsilon_{ijk}\partial^k\phi\right)^2 \pm \int_{\partial\mathcal{M}} f_{ij}\phi$$

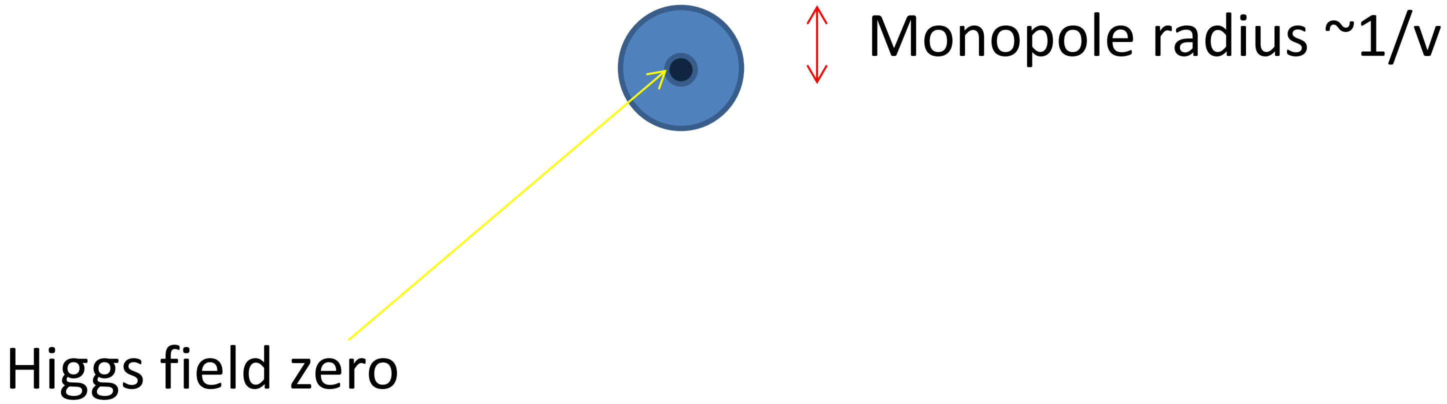
For any shape, the size of the bag is fixed

The magnetic bag limit

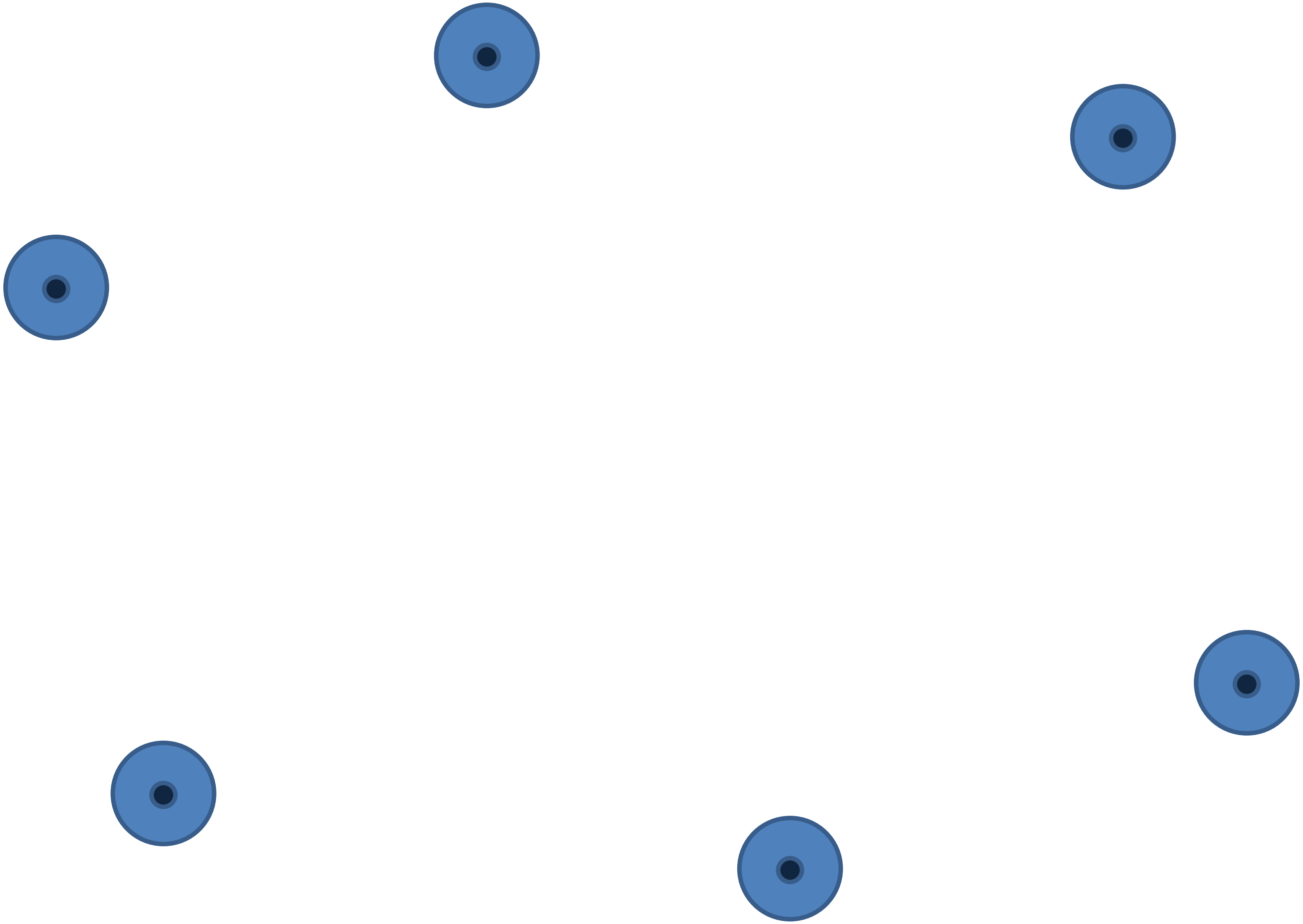
For any magnetic bag, there are multi monopole solutions which converge to it in the limit

$$n, v \to \infty \qquad \text{keeping} \qquad \frac{n}{v} = const$$

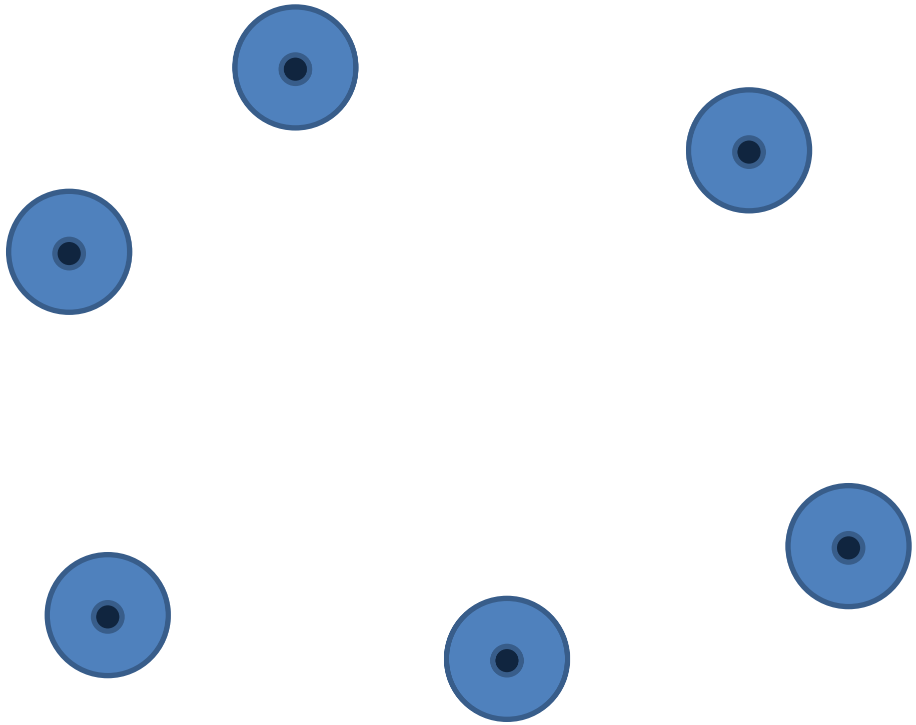
Heuristic explanation



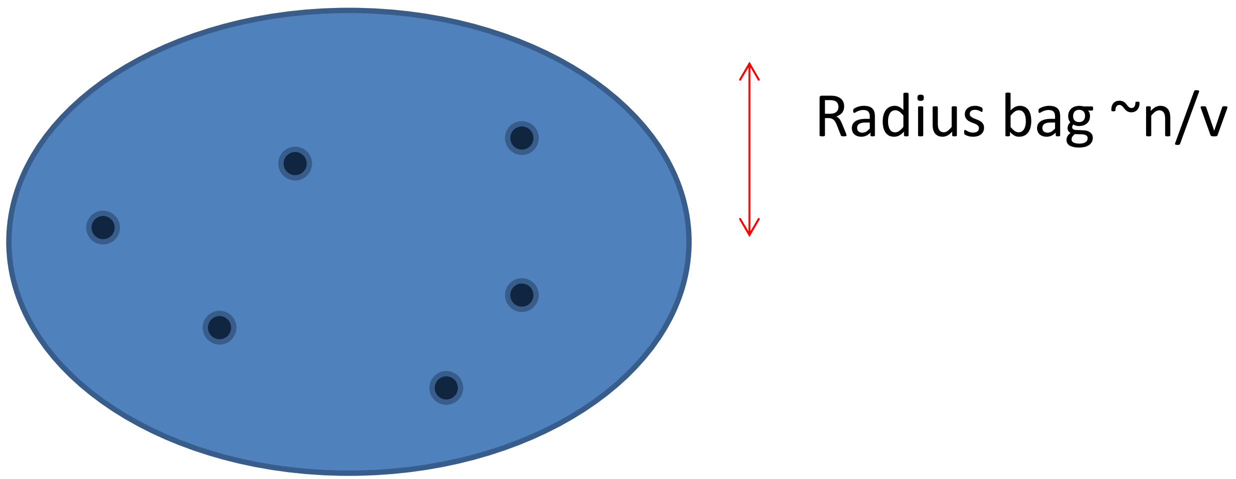
Heuristic explanation



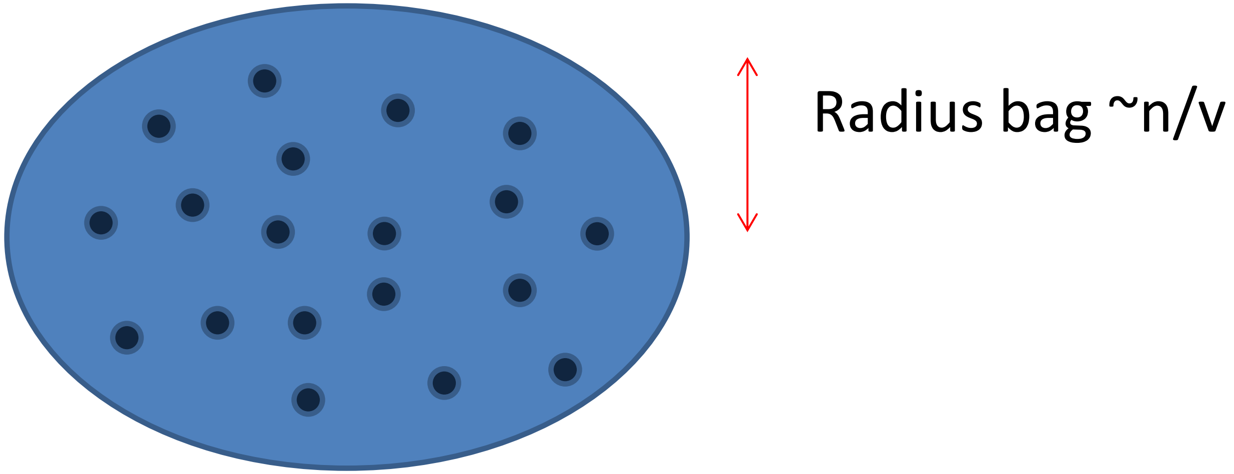
Heuristic explanation



Heuristic explanation



Heuristic explanation



Nahm Equation for monopoles

$$\frac{dT^i}{d\sigma} = -\frac{i}{2}\varepsilon_{ijk}[T^j, T^k]$$

Triplet of n x n matrices on a line

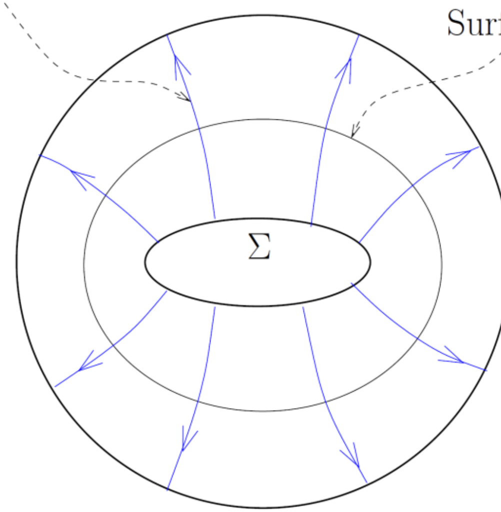
One-to-one correspondence with n monopole solution of Bogomoln'y equation

u(∞)-Nahm and magnetic bags

Nahm equations are "fuzzy sphere'' versions of the following commutative limit:

$$t^i : S^2 \times I \longrightarrow \mathbb{R}^3$$

$$\frac{dt^i}{ds} = \frac{1}{N}\epsilon_{ijk}\{t^j, t^k\}$$

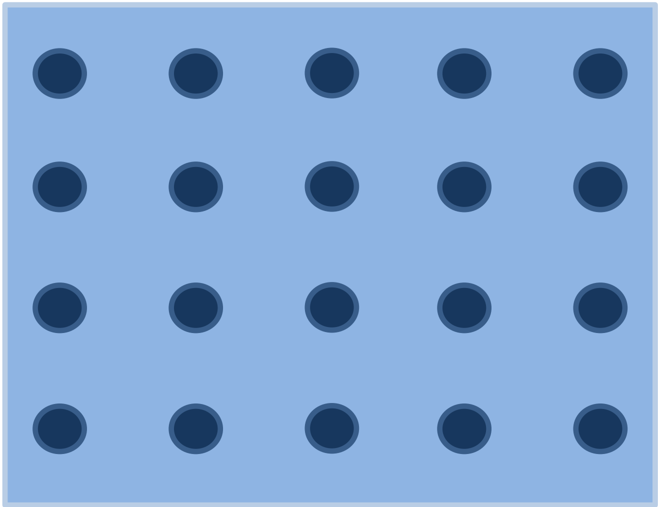


D. Harland

Monopole Wall

R. Ward

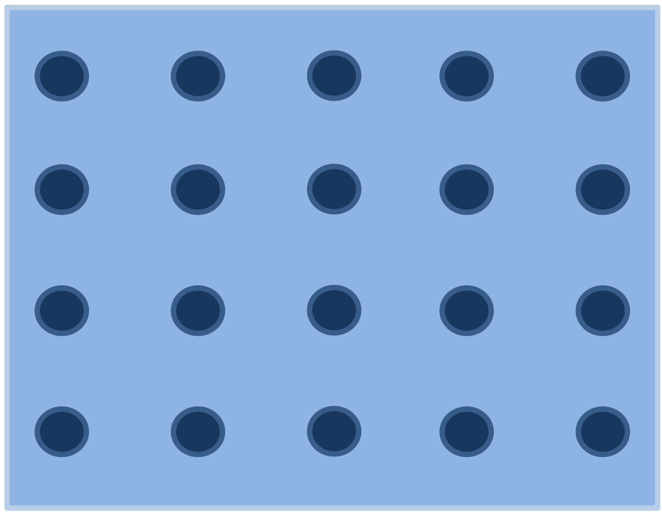
Lattice of monopoles periodic in two direction

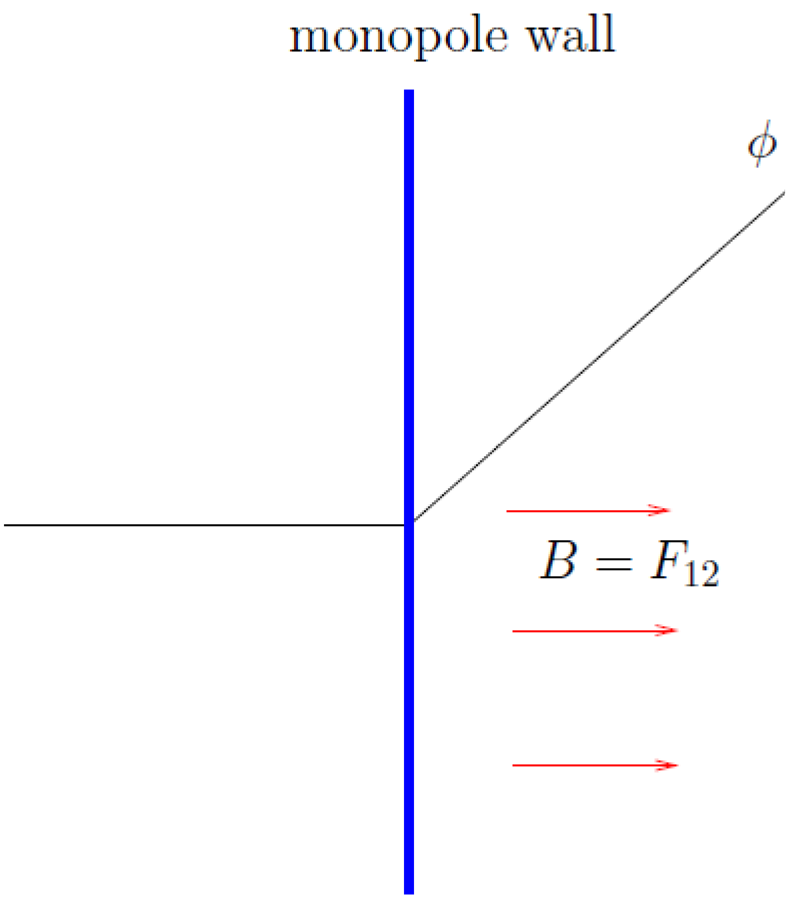


Monopole Wall

R. Ward

Lattice of monopoles periodic in two direction

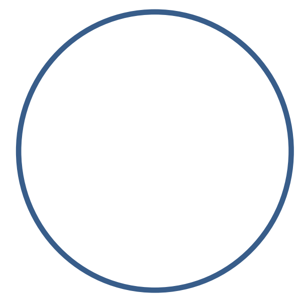




From instantons to monopoles

$$S_{YM5} = -\int dt d^4x \frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a}$$

Kaluza-Klein compactification



R_3 radius

$$S_{YMH4} = -\int dt dx_1 dx_2 dx_4 \frac{\pi R_3}{2g^2} \left(F^a_{\mu\nu} F^{\mu\nu a} + D\phi^a D\phi^a \right)$$

“Large” gauge transformation

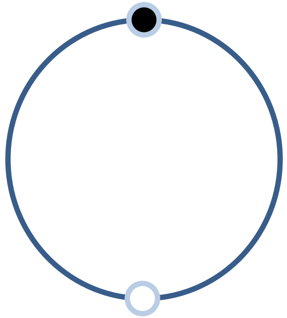
$$e^{-ix_3 t_{\mathrm{su}(2)}/R_3}$$

$$x_3 \simeq x_3 + 2\pi R_3$$

“Large’’ gauge transformation

$e^{-ix_3 t_{\mathrm{su}(2)}/R_3}$ $x_3 \simeq x_3 + 2\pi R_3$

$$A_3 \longrightarrow A_3 - \frac{t_{\mathrm{su}(2)}}{R_3}$$

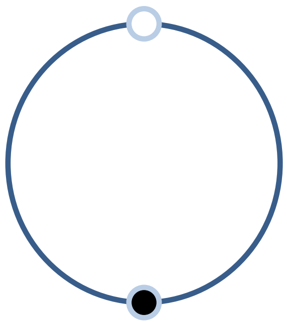


A_3 lives in the T-dual circle

"Large'' gauge transformation

$$e^{-ix_3 t_{\mathrm{su}(2)}/R_3} \qquad\qquad x_3 \simeq x_3 + 2\pi R_3$$

$$A_3 \quad \longrightarrow \quad A_3 - \frac{t_{\mathrm{su}(2)}}{R_3}$$



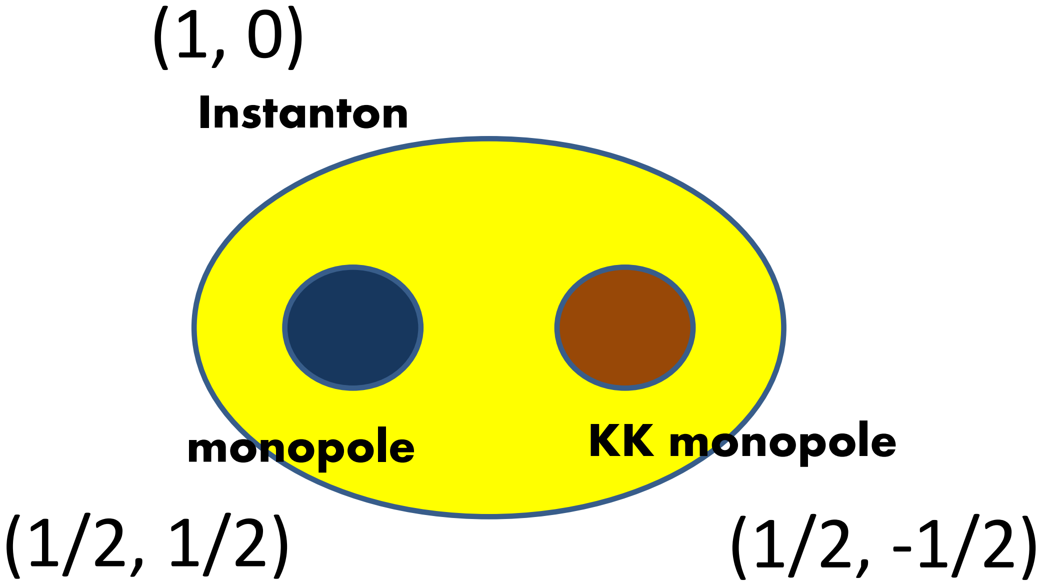
A_3 lives in the T-dual circle

Monopole and KK monopole

	Large gauge transformation		Global transformation	
(1/2, 1/2)	→	(-1/2, -1/2)	→	(-1/2, 1/2)
Monopole				**KK monopole**

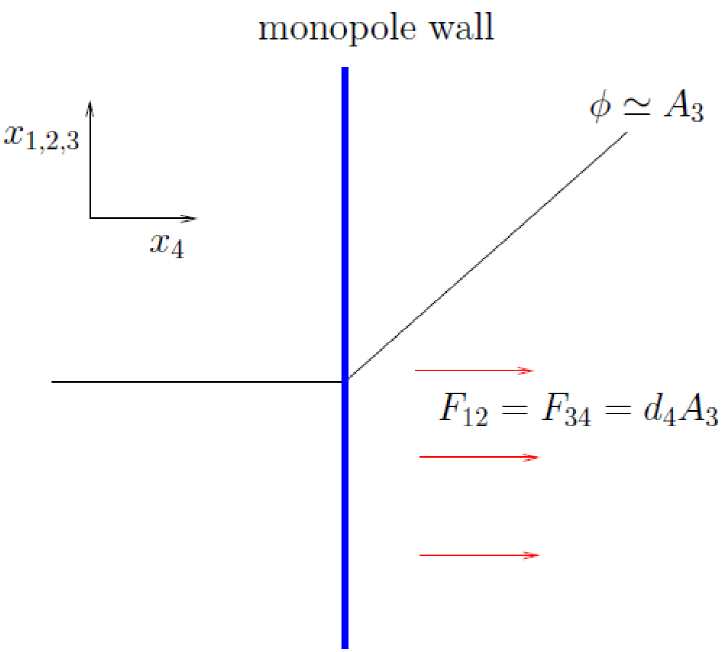
(Inst. Charge, Mag. Charge)

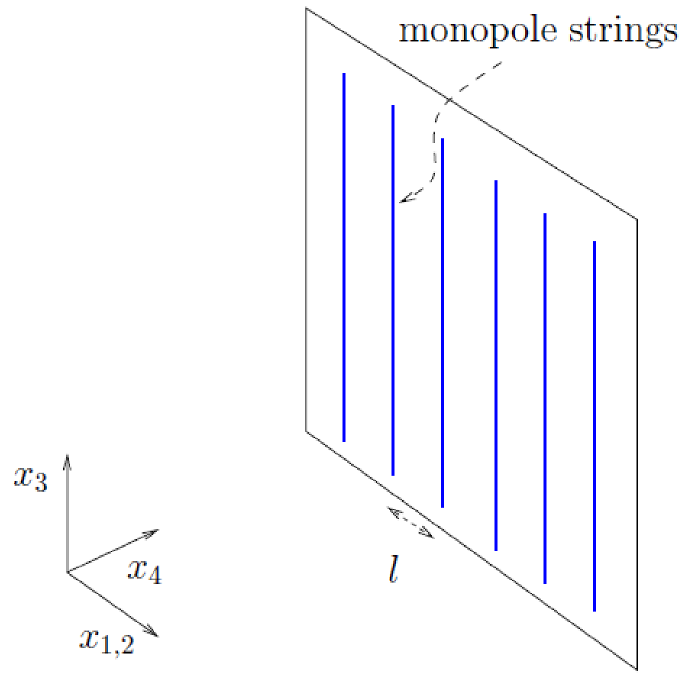
Instanton constituents



Lifting the Monopole Wall to 4+1

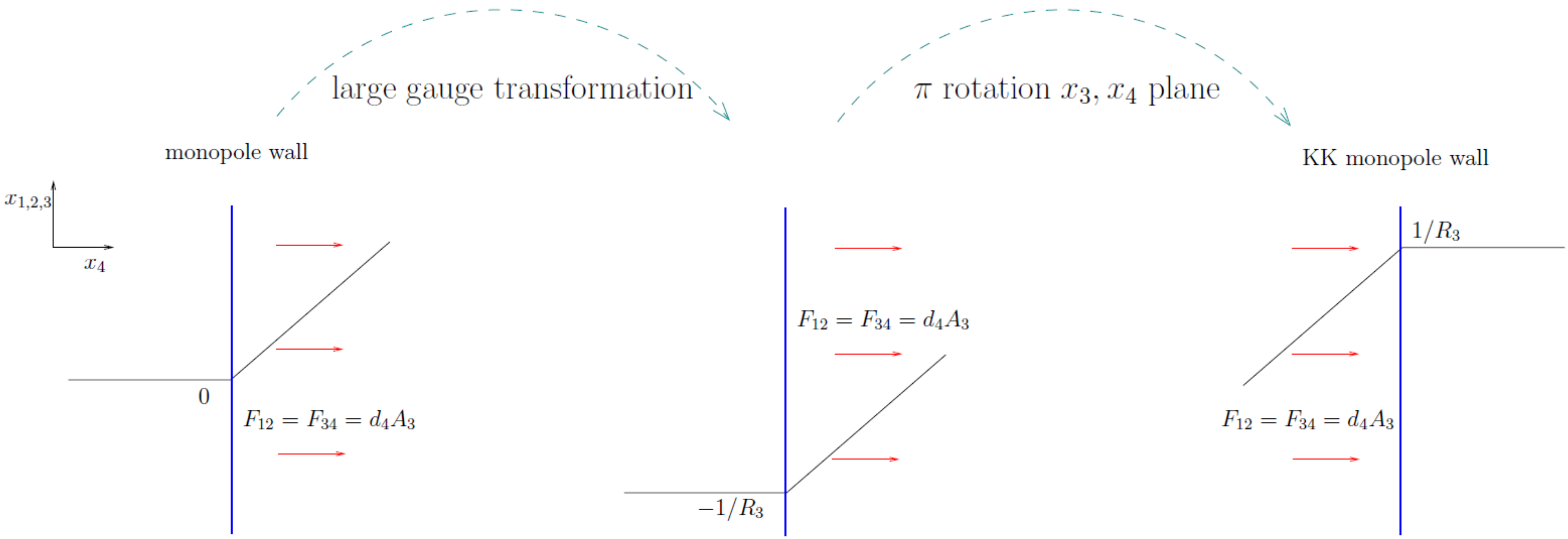
Lifting the monopole wall is easy, we just keep everything constant in the extra direction





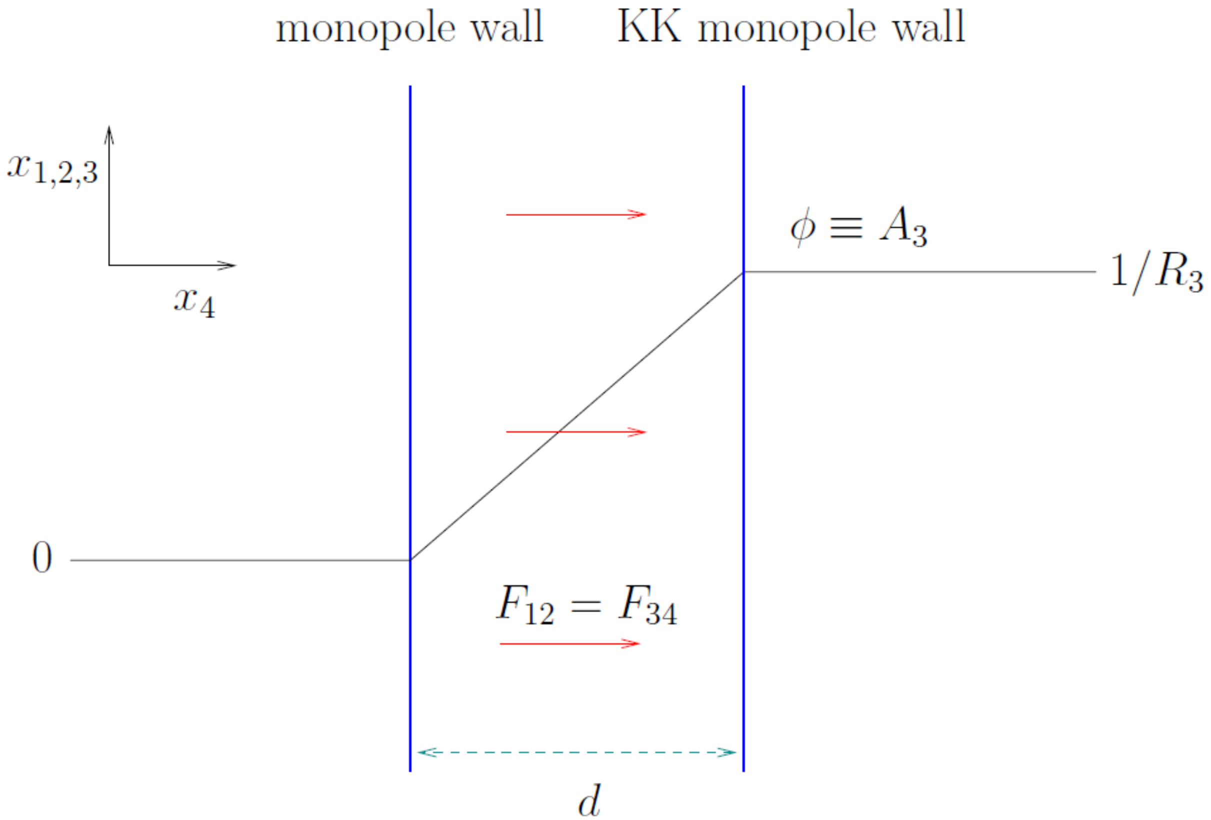
KK monopole wall

A KK monopole wall can be constructed starting from the monopole wall with two transformations



Monopole wall and KK monopole wall

Joining them together we obtain an instanton capacitor, or *instanton bag*

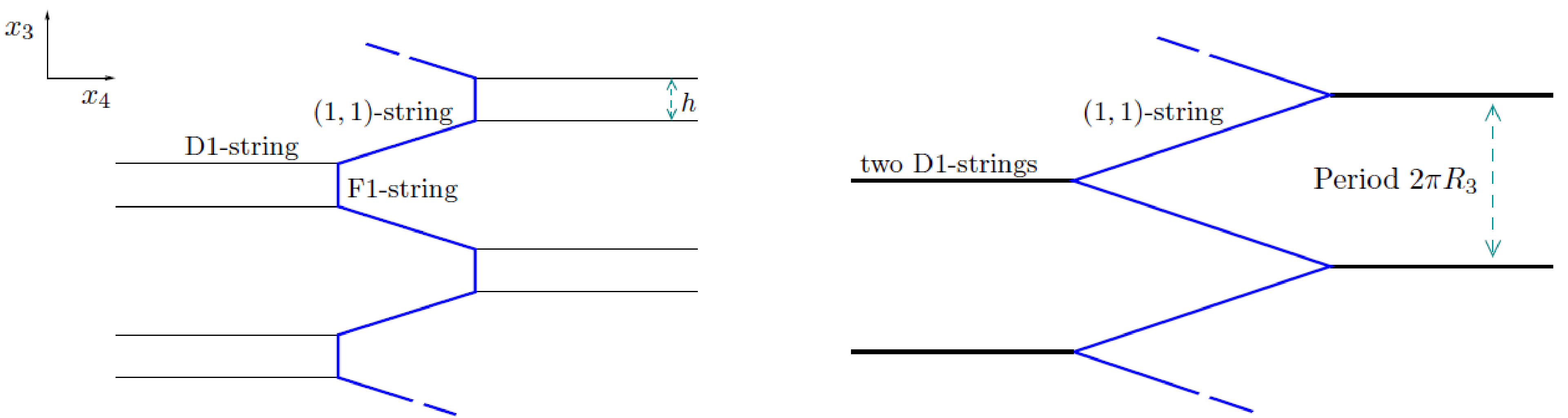


$$Q = \frac{B}{4\pi^2 R_3} \qquad B = \frac{1}{R_3 d}$$

The distance d is a modulus

String webs

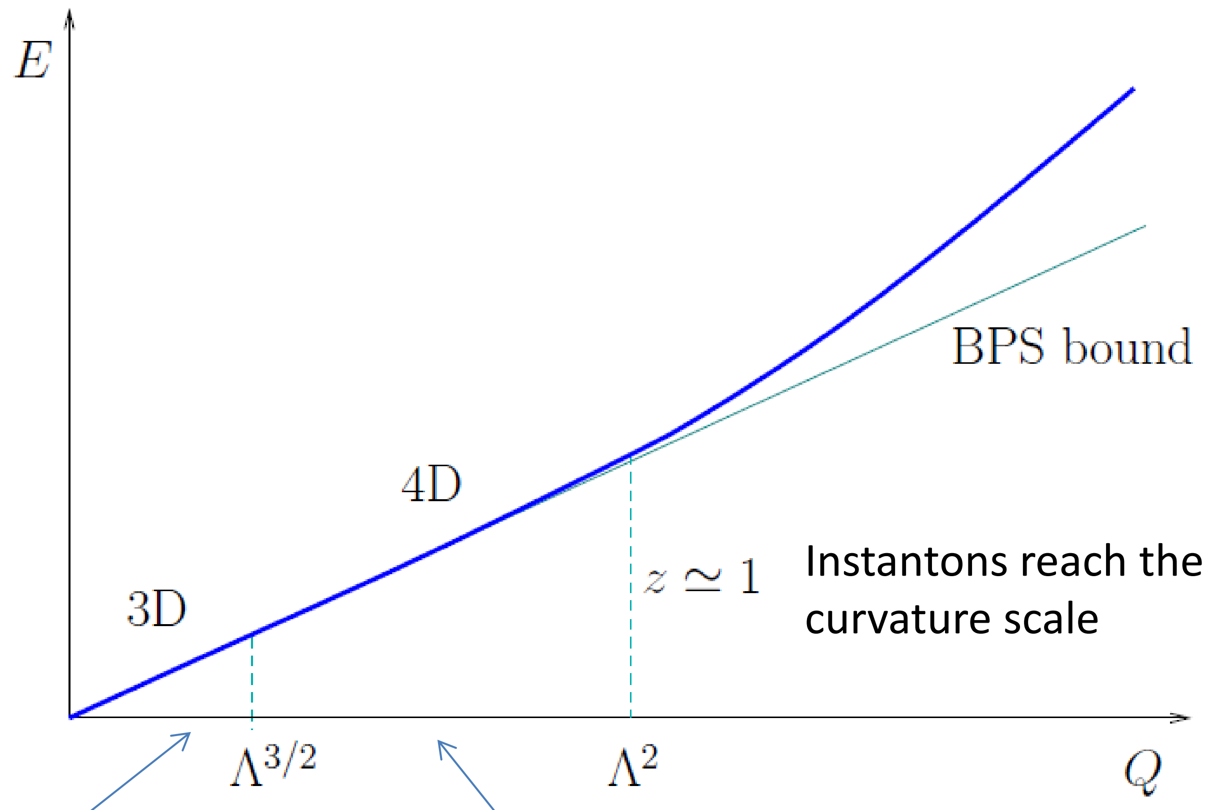
After a series of T and S dualities…



What we know so far…

- We were able to solve numerically the 4D problem only for one instanton
- When solitons start to populate the holographic direction they are no longer diluted!!!
- Instanton bag is good for these situations

Generic expectation

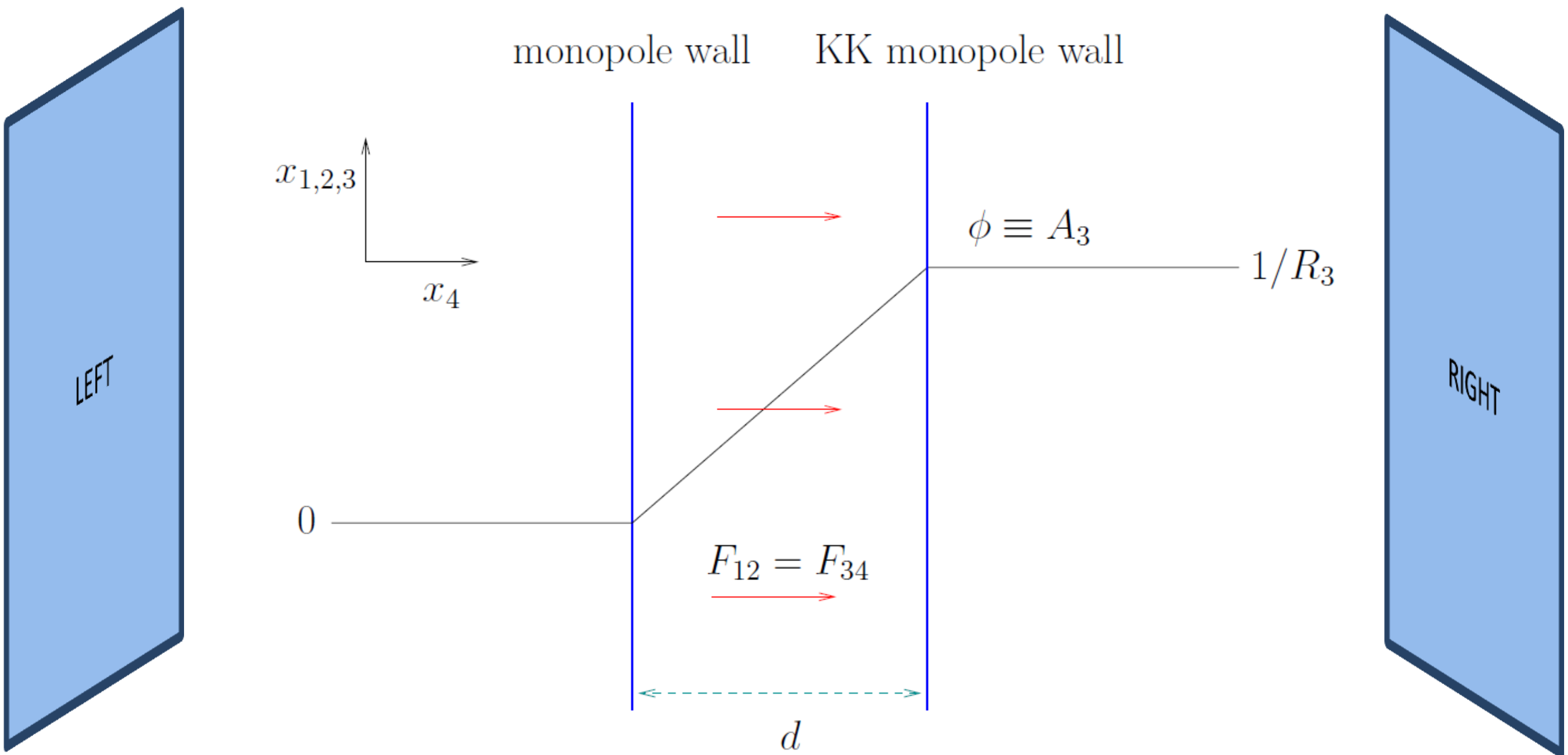


Instantons reach the curvature scale

3D lattice of instantons

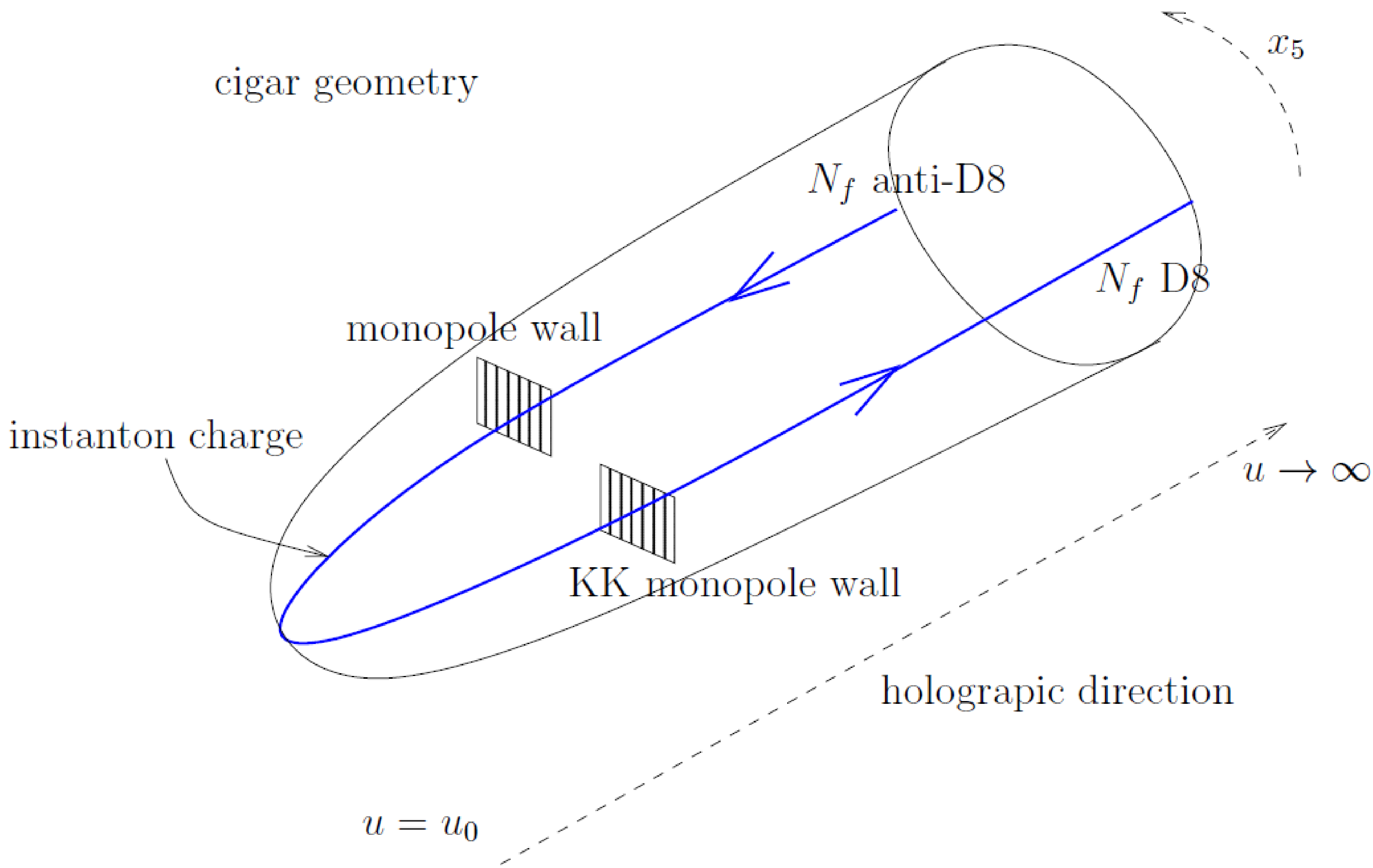
Instantons start to populate the holographic direction

Instanton Bag embedded in SS

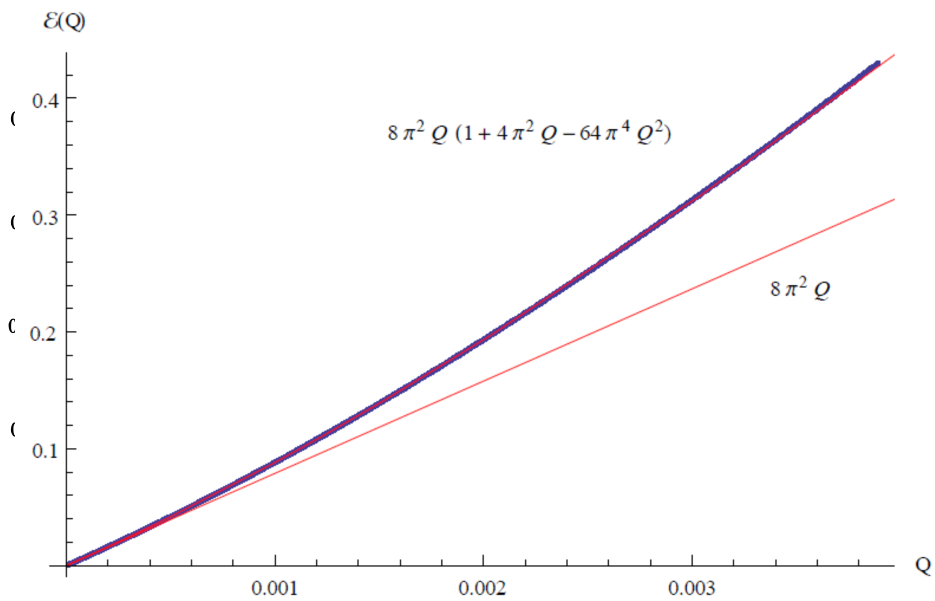


d is determined by energy minimization

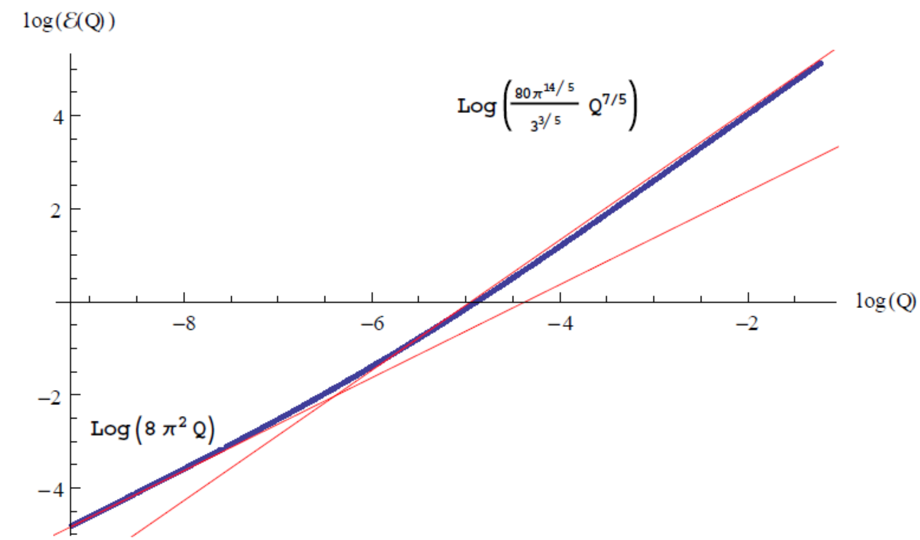
String Embedding



Results from moduli stabilization

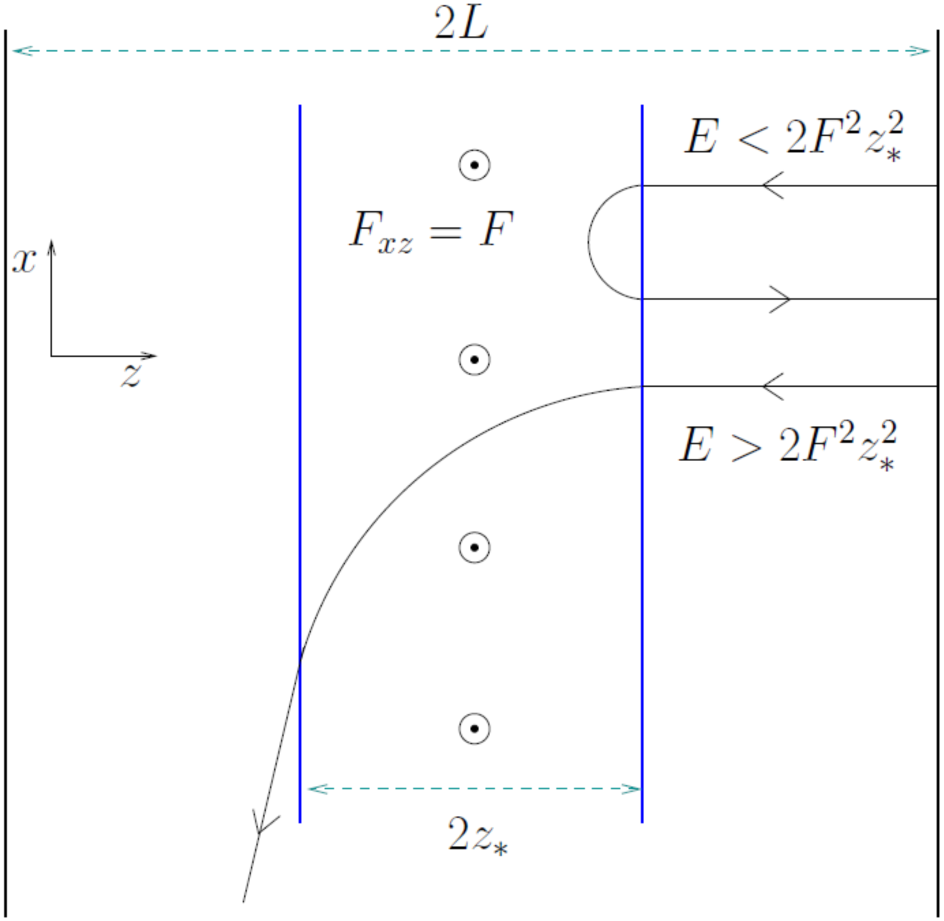


Small Q regime, almost BPS



Small-to-large Q regime,
in log plot

Chiral Symmetry Restoration



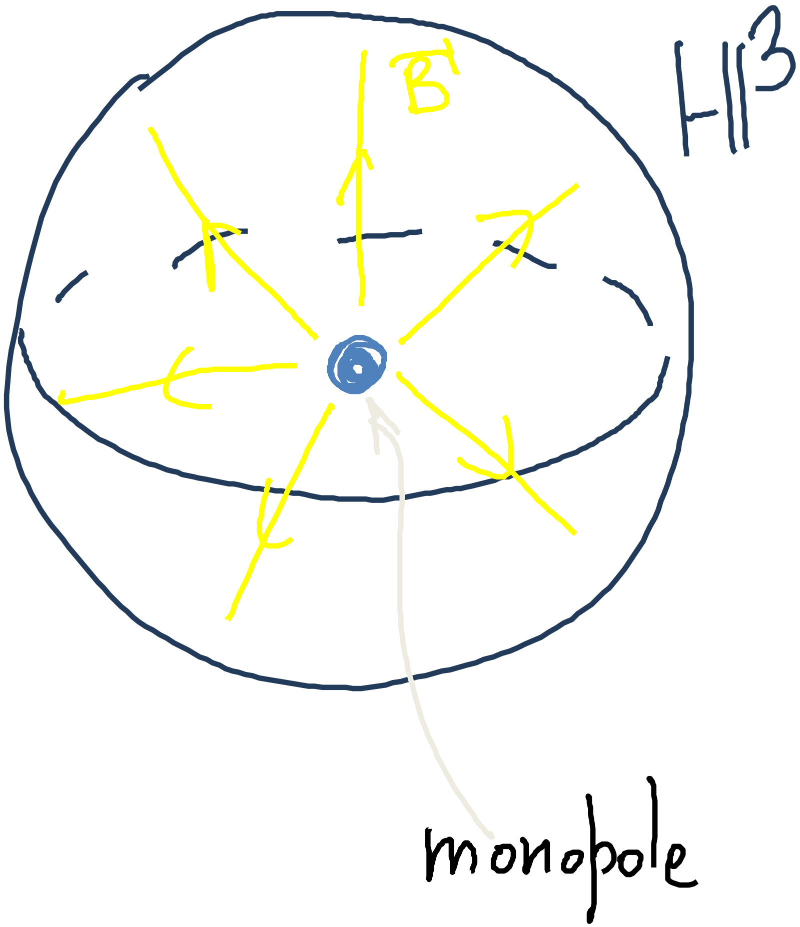
Effective separation between the two sides due to a magnetic trapping effect

Conclusion (part 2)

- First prototype of instanton bag constructed;

- This provides candidate for the state of HQCD at high density and explains chiral symmetry restoration.

Hyperbolic monopoles



Bogomolny equation

$$D\Phi = *F$$

Hyperbolic space

$$ds^2(\mathbb{H}^3) = \frac{4(dX_1^2 + dX_2^2 + dX_3^2)}{(1-R^2)^2}$$

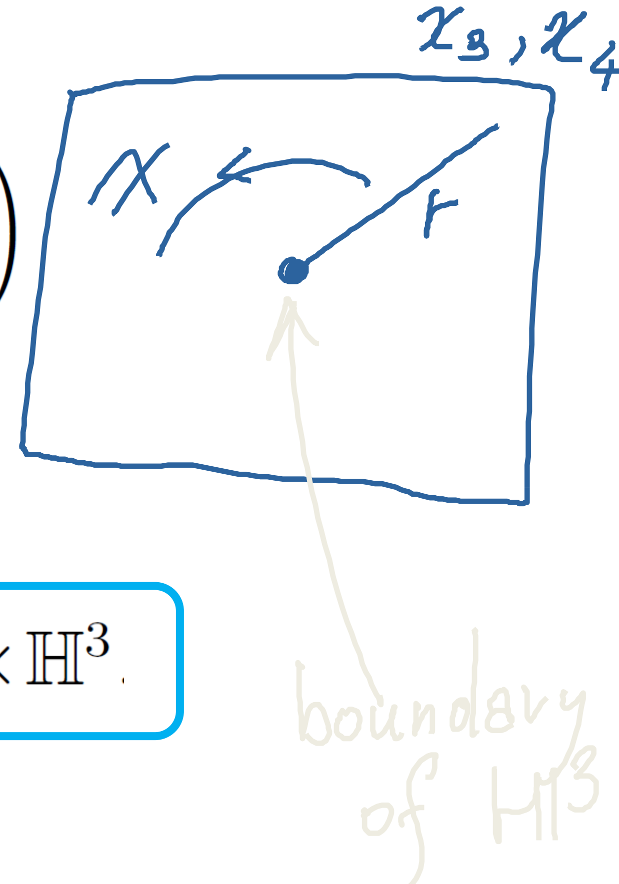
Plan of the talk (part III)

- Relation between instantons and Hyperbolic monopoles (Atiyah)
- Magnetic bags in hyperbolic space and Nahm equation from ADHM
- Examples of multi-monopole solutions from JNR

Based on arXiv:1404.1846 with A. Cockburn and P. Sutcliffe
on arXiv:1504.01477 with D. Harland and P. S.

Conformalities and invariant instantons

$$
\begin{aligned}
ds^2 &= dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 \\
&= r^2\left(d\chi^2 + \frac{1}{r^2}\left(dx_1^2 + dx_2^2 + dr^2\right)\right)
\end{aligned}
$$

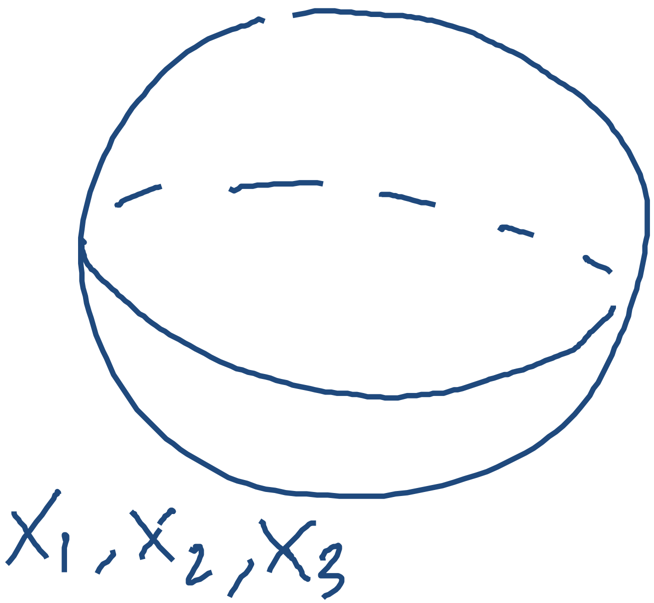


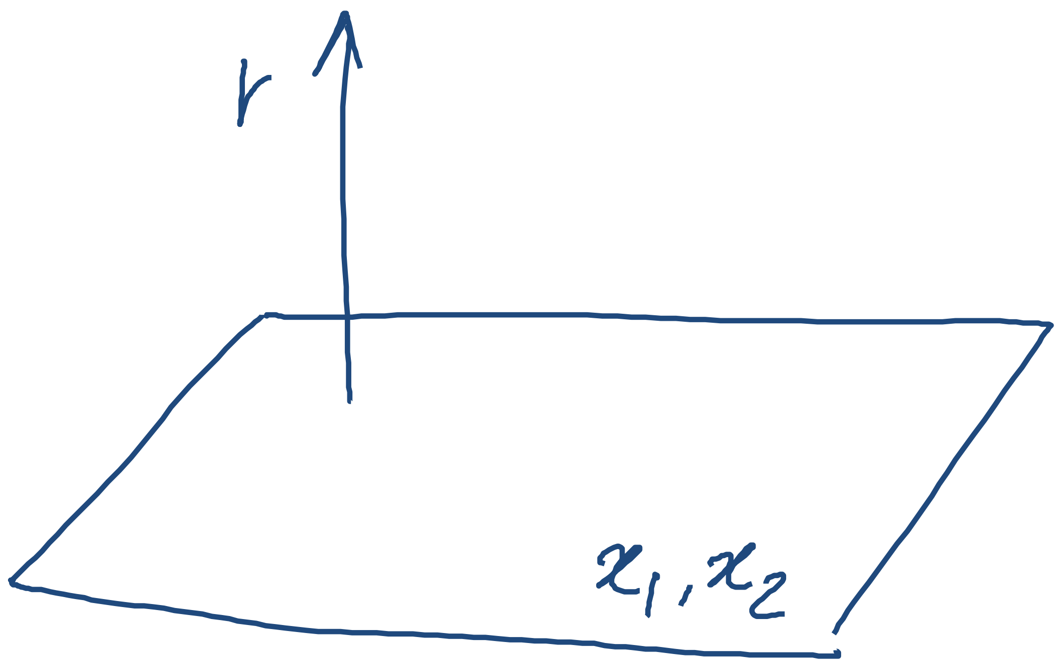
$\mathbb{R}^4 - \mathbb{R}^2$ is conformal equivalent to $S^1 \times \mathbb{H}^3$.

Ball and Poincare

$$ds^2(\mathbb{H}^3) = \frac{4(dX_1^2 + dX_2^2 + dX_3^2)}{(1-R^2)^2}$$

$$\frac{1}{r^2}\left(dx_1^2 + dx_2^2 + dr^2\right)$$





Circle invariant instantons

$$I = 2vN$$

Instanton number

Higgs vev (half integer)

Monopole charge

From ADHM to u(∞)-Nahm

$$\begin{array}{ccc} \text{Hyperbolic monopoles} & \longleftrightarrow & \text{Braam-Austin equation} \\ \downarrow & & \downarrow \\ \text{Magnetic bags in } \mathbb{H}^3 & \longleftrightarrow & u(\infty)\text{-Nahm in } \mathbb{H}^3 \end{array}$$

JNR ansatz provides a large class of accessible solutions

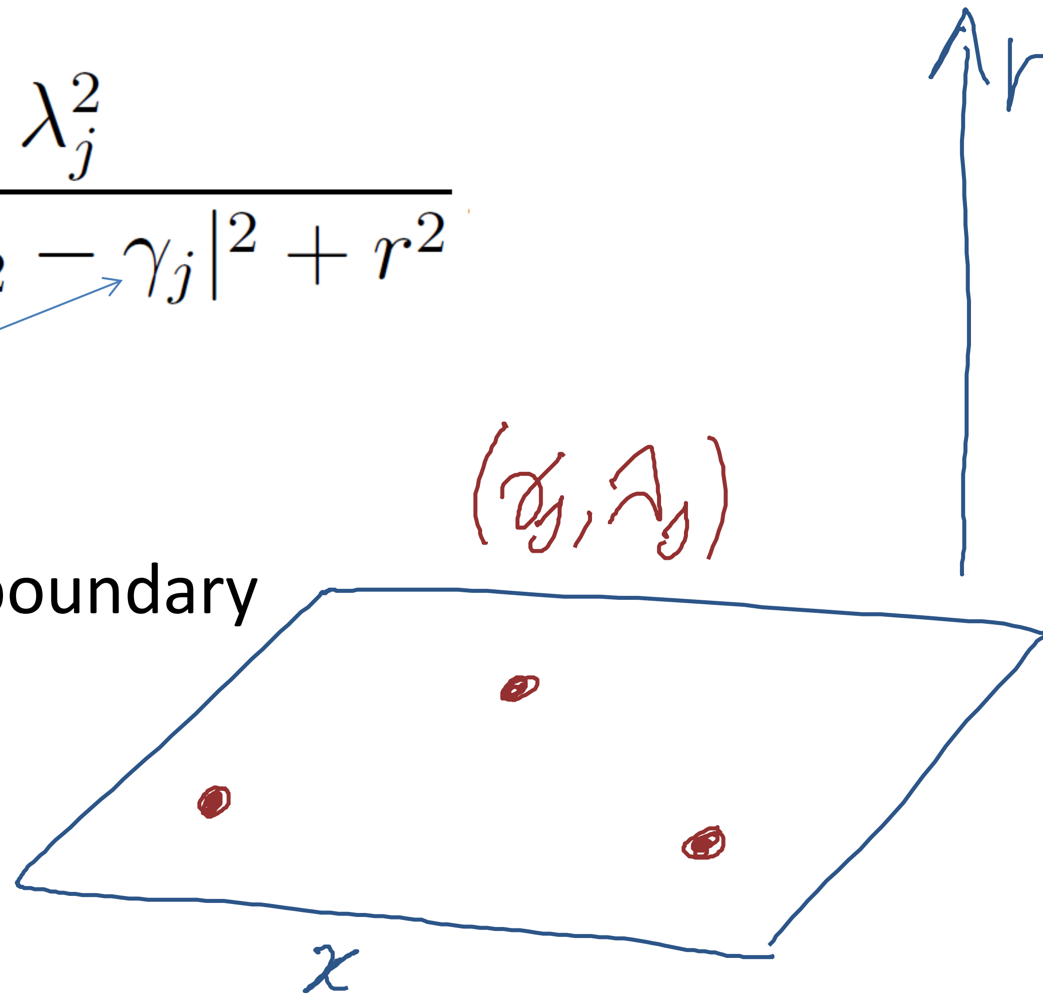
$$A_\mu = \frac{i}{2}\sigma_{\mu\nu}\partial_\nu \varrho \qquad \varrho = \log \psi$$

ψ is an arbitrary harmonic function

Hyperbolic monopoles from JNR instantons

$$\psi = \sum_{j=0}^{N} \frac{\lambda_j^2}{|x_1 + ix_2 - \gamma_j|^2 + r^2}$$

Poles are chosen on the boundary of the hyperbolic space



Manton-Sutcliffe

Explicit solution

Higgs field

$$|\Phi|^2 = \frac{r^2}{4\psi^2}\left(\left(\frac{\partial\psi}{\partial x_1}\right)^2 + \left(\frac{\partial\psi}{\partial x_2}\right)^2 + \left(\frac{\psi}{r} + \frac{\partial\psi}{\partial r}\right)^2\right)$$

Energy density

$$\mathcal{E} = \frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}g^{ij}\partial_j|\Phi|^2\right)$$

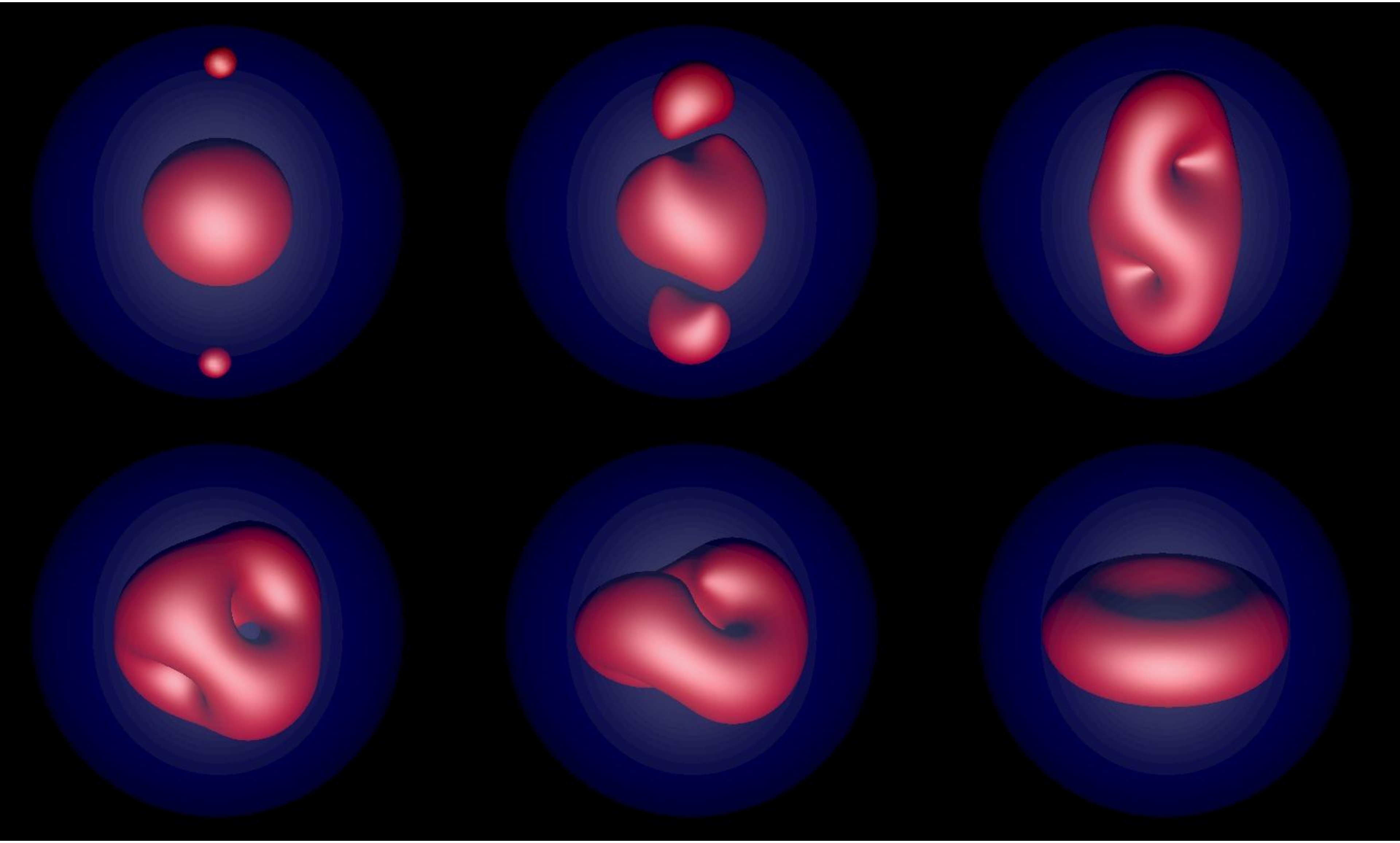
Two limitations

1) The Higgs vev is fixed by $v = 1/2$, and so $I = N$

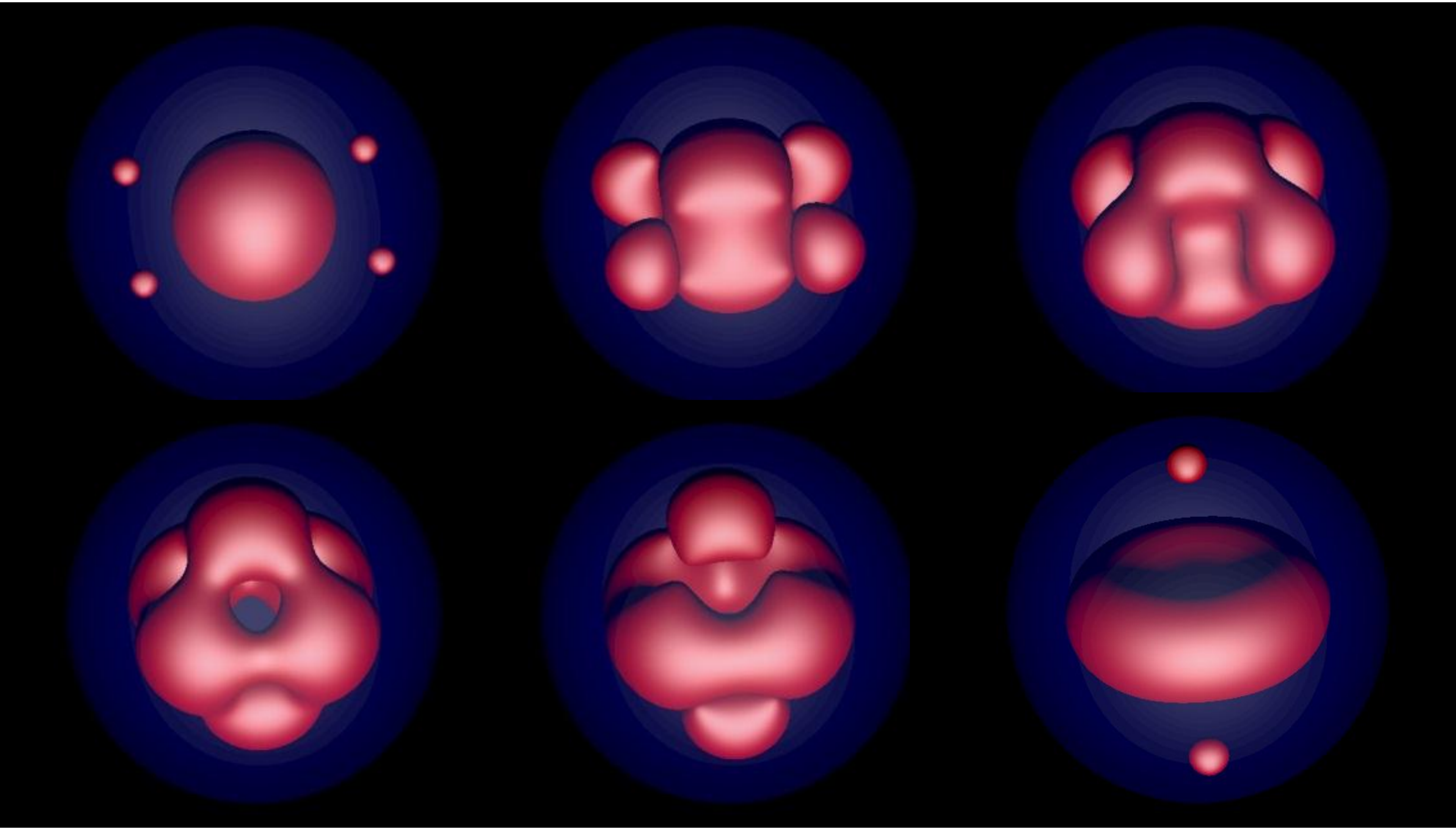
2) We can access only a subset of the full moduli

$$\dim(\mathbb{M}_N^{\mathrm{JNR}}) = 3N + 2 < 4N - 1 = \dim(\mathbb{M}_N)$$

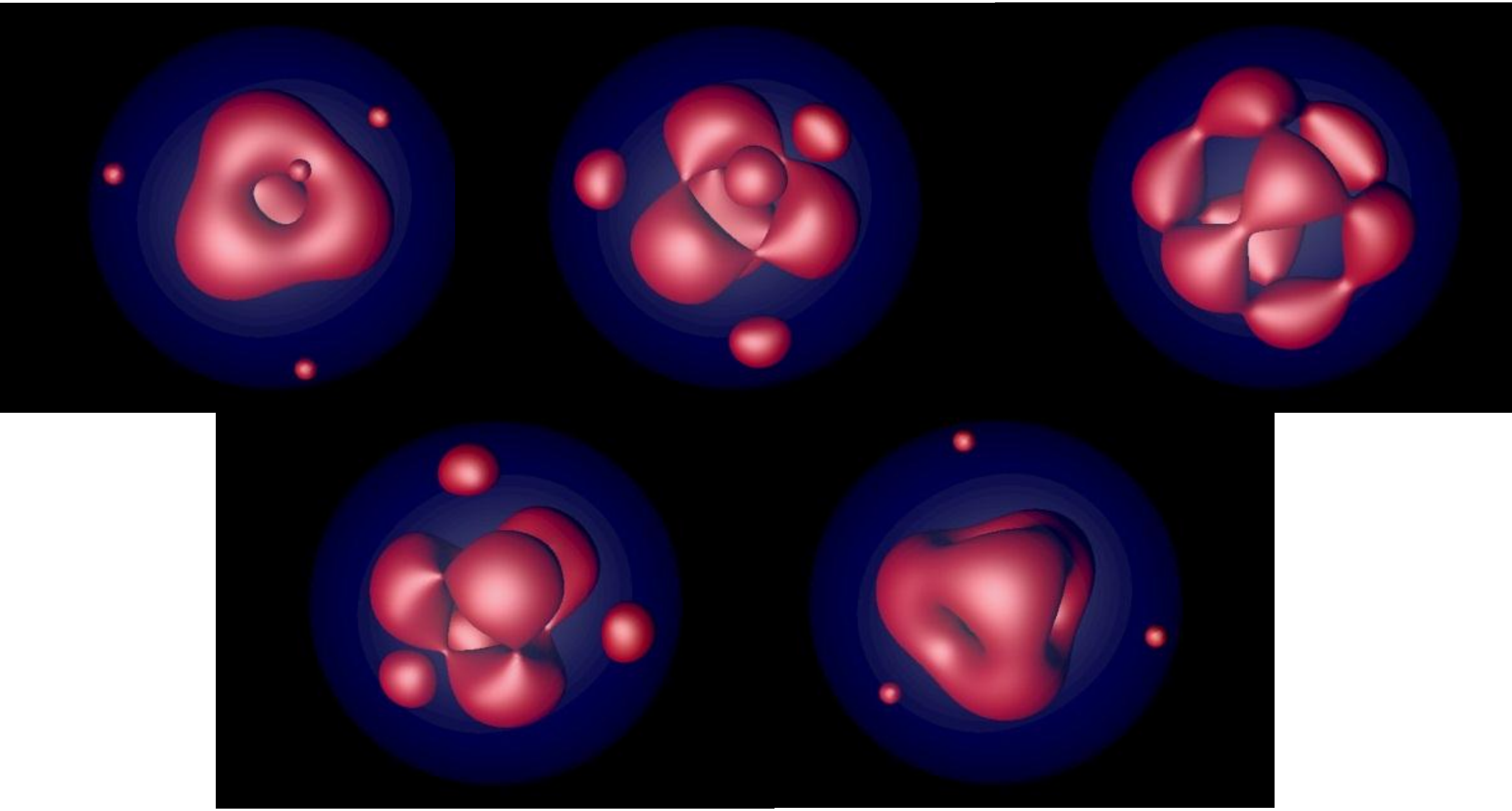
D2 three monopole



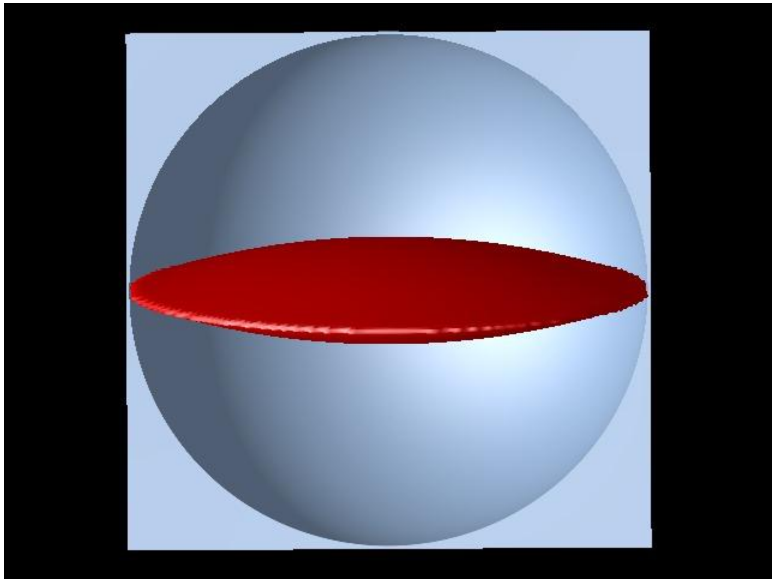
D4 five-monopole family



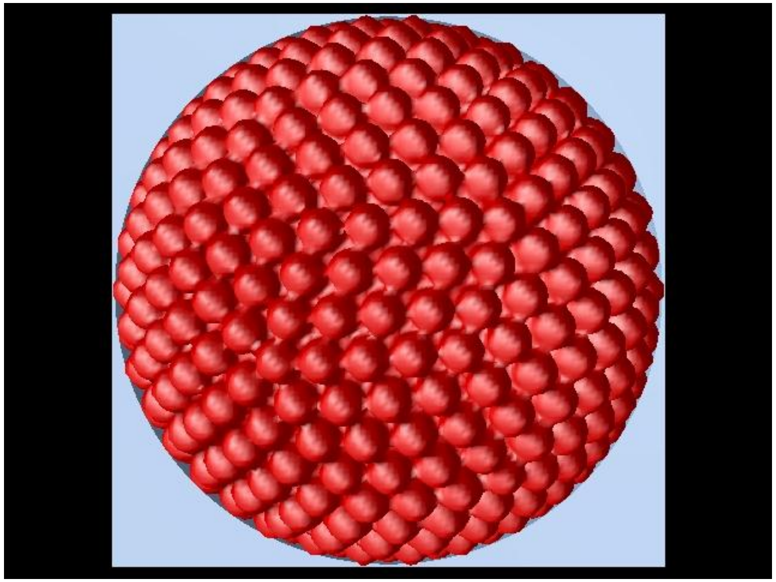
Tetrahedral seven monopole



Large N axial symmetric monopole



Large N spherical JNR-type monopole



Conclusion

- We considered solitons in holographic dual of large N theory
- Another large N (with N magnetic flux now) is useful for some physically interesting situations (e.g. finite density of QCD)
- Large N limit of monopoles still to be completely understood; large N limit of instanton just at the beginning…