Baryons at finite temperature

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Introduction

from hadronic to quark-gluon plasma

- thermodynamics: pressure, entropy, fluctuations
- symmetries: confinement, chiral symmetry

spectroscopy

- quarkonia
- light mesons
- baryons

real time

- transport
- far from equilibrium

Introduction

from hadronic to quark-gluon plasma

- thermodynamics: pressure, entropy, fluctuations
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- **BARYONS**

real time

- transport
- far from equilibrium

Mesons in a medium

mesons in a medium very well studied

- hadronic phase: thermal broadening, mass shift
- QGP: deconfinement/dissolution/melting
- quarkonia survival as thermometer
- transport: conductivity/dileptons from vector current
- chiral symmetry restoration

relatively easy on the lattice

- high-precision correlators

what about baryons?

Baryons in a medium

lattice studies of baryons at finite temperature very limited

- screening masses De Tar and Kogut 1987
- ... with a small chemical potential QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

not much more ...

holographic studies of baryons at finite temperature?

Outline

baryons across the deconfinement transition:

- some basic thermal field theory
- lattice QCD – FASTSUM collaboration
- baryon correlators
- in-medium effects below T_c
- parity doubling above T_c
- spectral functions

Baryons

correlators $$G^{\alpha\alpha'}(x) = \langle O^{\alpha}(x)\,\overline{O}^{\alpha'}(0)\rangle$$

in this work – N, Δ, Ω baryons

- $$O_N^{\alpha}(x) = \epsilon_{abc}\, u_a^{\alpha}(x) \left(d_b^T(x) C\gamma_5 u_c(x) \right)$$

- $$O_{\Delta,i}^{\alpha}(x) = \epsilon_{abc} \left[2u_a^{\alpha}(x) \left(d_b^T(x) C\gamma_i u_c(x) \right) + d_a^{\alpha}(x) \left(u_b^T(x) C\gamma_i u_c(x) \right) \right]$$

- $$O_{\Omega,i}^{\alpha}(x) = \epsilon_{abc}\, s_a^{\alpha}(x) \left(s_b^T(x) C\gamma_i s_c(x) \right)$$

Baryons

- essential difference with mesons: role of parity

$$\mathcal{P}O(\tau,\mathbf{x})\mathcal{P}^{-1} = \gamma_4 O(\tau,-\mathbf{x})$$

- positive/negative parity operators

$$O_\pm(x) = P_\pm O(x) \qquad\qquad P_\pm = \frac{1}{2}(1\pm\gamma_4)$$

- no parity doubling in Nature: nucleon ground state

positive parity: $m_+ = m_N = 0.939$ GeV

negative parity: $m_- = m_{N^*} = 1.535$ GeV

- thread: what happens as temperature increases?

Spectral properties in a medium

- euclidean correlators $G(x) = \langle O(x) O^\dagger(0) \rangle$
- dispersion relation

$$G(i\omega_n, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega, \mathbf{p})}{\omega - i\omega_n}$$

- imaginary part of retarded correlator

$$\rho(\omega, \mathbf{p}) = 2 \mathrm{Im}\, G(i\omega_n \to \omega + i\epsilon, \mathbf{p})$$

- back to euclidean time

$$G(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \mathbf{p})$$

Spectral properties: mesons/bosons

$$G(\tau,\mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega,\mathbf{p})$$

- bosonic operators $(\tilde{\tau} = \tau - 1/2T)$

$$K_{\rm boson}(\tau,\omega) = T\sum_n \frac{e^{-i\omega_n\tau}}{\omega - i\omega_n} = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)}$$

- kernel symmetric around $\tau = 1/2T$, odd in ω
- spectral decomposition

$$\rho_B(p) = \frac{1}{Z}\sum_{n,m}\left(e^{-k_n^0/T} - e^{-k_m^0/T}\right)|\langle n|O(0)|m\rangle|^2\,(2\pi)^4\delta^{(4)}(p+k_n-k_m)$$

- if $O^\dagger = \pm O \quad \Rightarrow \quad \omega\rho(\omega,\mathbf{p}) \geq 0$ positivity

Spectral properties: baryons/fermions

$$G^{\alpha\alpha'}(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho^{\alpha\alpha'}(\omega, \mathbf{p})$$

with

$$G^{\alpha\alpha'}(x - x') = \langle O^\alpha(x) \overline{O}^{\alpha'}(x') \rangle$$

$$\rho^{\alpha\alpha'}(x - x') = \langle \{ O^\alpha(x), \overline{O}^{\alpha'}(x') \} \rangle$$

- fermionic Matsubara frequencies

$$K(\tau, \omega) = T \sum_n \frac{e^{-i\omega_n \tau}}{\omega - i\omega_n} = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}} = e^{-\omega\tau} \left[1 - n_F(\omega)\right]$$

- kernel not symmetric, instead

$$K(1/T - \tau, \omega) = K(\tau, -\omega)$$

Kernels

- bosons

$$K_{\mathrm{boson}}(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)} = [1+n_B(\omega)]\, e^{-\omega\tau} + n_B(\omega)e^{\omega\tau}$$

- fermions: even and odd terms

$$K(\tau,\omega) = \frac{1}{2}\left[K_{\mathrm{e}}(\tau,\omega) + K_{\mathrm{o}}(\tau,\omega)\right],$$

$$K_{\mathrm{e}}(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1-n_F(\omega)]\, e^{-\omega\tau} + n_F(\omega)e^{\omega\tau}$$

$$K_{\mathrm{o}}(\tau,\omega) = -\frac{\sinh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1-n_F(\omega)]\, e^{-\omega\tau} - n_F(\omega)e^{\omega\tau}$$

- no singular behaviour $2T/\omega$ for fermions, no transport subtlety

Spectral decomposition: Positivity

$$\rho(x) = \sum \gamma_\mu \rho_\mu(x) + \mathbb{1}\rho_m(x)$$

- take trace with γ_4, $P_\pm = (\mathbb{1} \pm \gamma_4)/2$:

$$\rho_4(p) = \frac{1}{Z}\sum_{n,m,\alpha}\left(e^{-k_n^0/T} + e^{-k_m^0/T}\right)\frac{1}{4}\left|\langle n|O^\alpha(0)|m\rangle\right|^2 (2\pi)^4\delta^{(4)}(p+k_n-k_m)$$

$$\rho_\pm(p) = \frac{\pm 1}{Z}\sum_{n,m,\alpha}\left(e^{-k_n^0/T} + e^{-k_m^0/T}\right)\frac{1}{4}\left|\langle n|O_\pm^\alpha(0)|m\rangle\right|^2 (2\pi)^4\delta^{(4)}(p+k_n-k_m)$$

- $\rho_4(p), \pm\rho_\pm(p) \geq 0$ for all ω
- take trace with $\mathbb{1}$

$$\rho_m(p) = [\rho_+(p) + \rho_-(p)]/4$$

not sign definite

Charge conjugation

charge conjugation symmetry (at vanishing density):

$$G_\pm(\tau,\mathbf{p}) = -G_\mp(1/T-\tau,\mathbf{p}) \qquad \rho_\pm(-\omega,\mathbf{p}) = -\rho_\mp(\omega,\mathbf{p})$$

- relates pos/neg parity channels

using $G_+(\tau,\mathbf{p})$ and $\rho_+(\omega,\mathbf{p})$

- positive- (negative-) parity states propagate forward (backward) in euclidean time
- negative part of spectrum of $\rho_+ \leftrightarrow$ positive part of ρ_-

example: single state

$$G_+(\tau) = A_+e^{-m_+\tau} + A_-e^{-m_-(1/T-\tau)}$$

$$\rho_+(\omega)/(2\pi) = A_+\delta(\omega-m_+) + A_-\delta(\omega+m_-)$$

Chiral symmetry

- propagator

$$G(x) = \sum_\mu \gamma_\mu G_\mu(x) + \mathbb{1} G_m(x)$$

- chiral symmetry $\qquad \{\gamma_5, G\} = 0 \quad \Rightarrow \quad G_m = 0$
- hence

$$G_+(\tau, \mathbf{p}) = -G_-(\tau, \mathbf{p}) = G_+(1/T - \tau, \mathbf{p}) = 2G_4(\tau, \mathbf{p})$$

degeneracy of $\pm$ parity channels

$$\rho_+(p) = -\rho_-(p) = \rho_+(-p) = 2\rho_4(p)$$

- parity doubling
- in Nature at $T = 0$: no chiral symmetry/parity doubling

Baryons in a medium

questions:

- in-medium effects below T_c?
- relevant for heavy-ion phenomenology?
- emergence of parity doubling?
- connection to deconfinement transition, chiral symmetry?
- chiral symmetry $\Leftrightarrow$ parity doubling

FASTSUM

- anisotropic $N_f = 2+1$ Wilson-clover ensembles

FASTSUM collaboration

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Tim Harris (TCD->Mainz->Milan)
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This work

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Davide de Boni, Benjamin Jäger, Kristi Praki

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in preparation

FASTSUM ensembles

- $N_f = 2 + 1$ dynamical quark flavours, Wilson-clover
- many temperatures, below and above T_c
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices
- strange quark: physical value
- two light flavours: somewhat heavy $m_\pi = 384(4)$ MeV

N_s	24	24	24	24	24	24	24	24
N_τ	128	40	36	32	28	24	20	16
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
$N_{\rm cfg}$	140	500	500	1000	1000	1000	1000	1000
$N_{\rm src}$	16	4	4	2	2	2	2	2

- tuning and $N_\tau = 128$ data from HadSpec collaboration

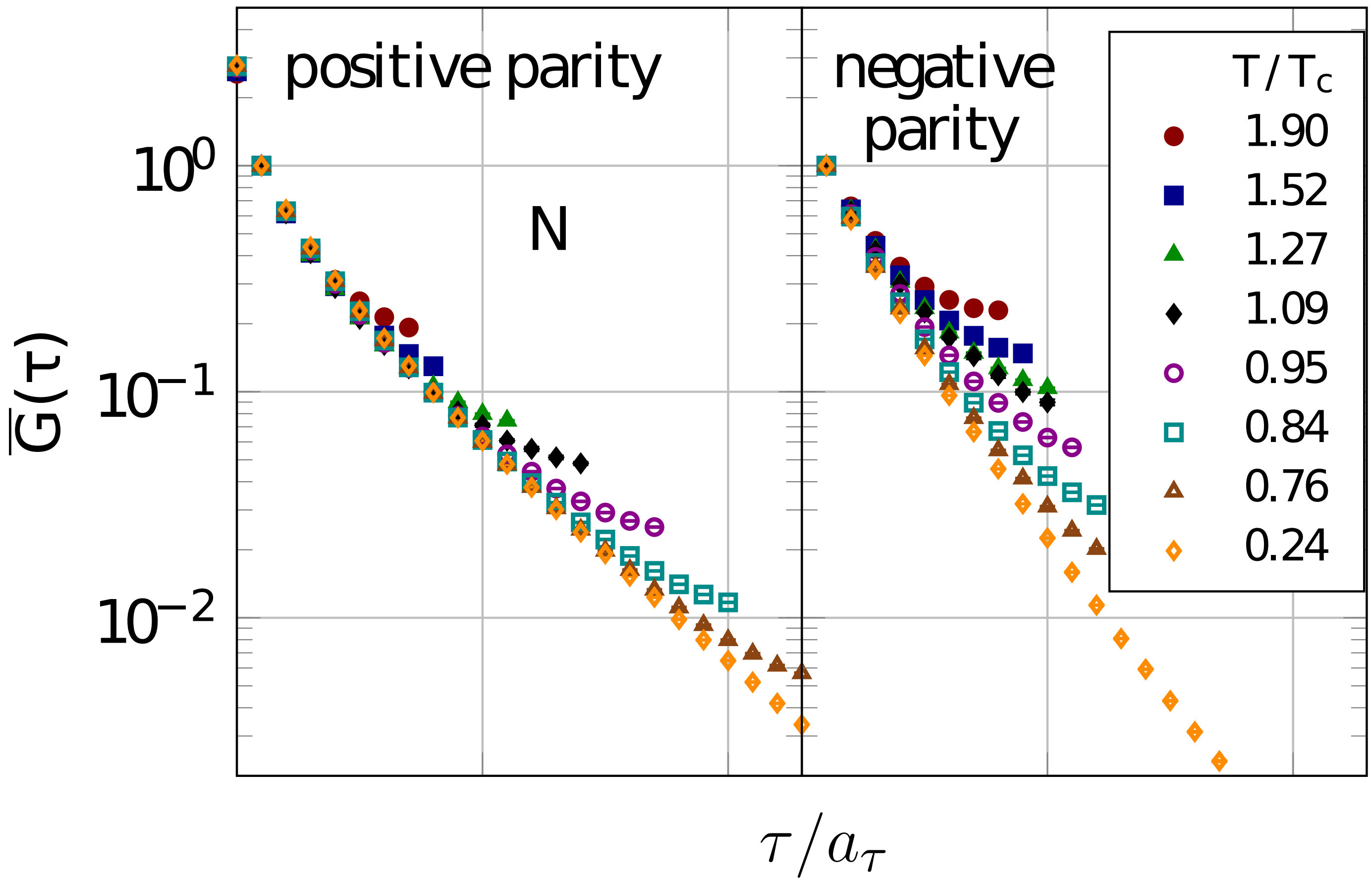
Baryons in a medium

technical remarks

- studied various interpolation operators
- Gaussian smearing for multiple sources and sinks
- same smearing parameters at all temperatures

Lattice correlators

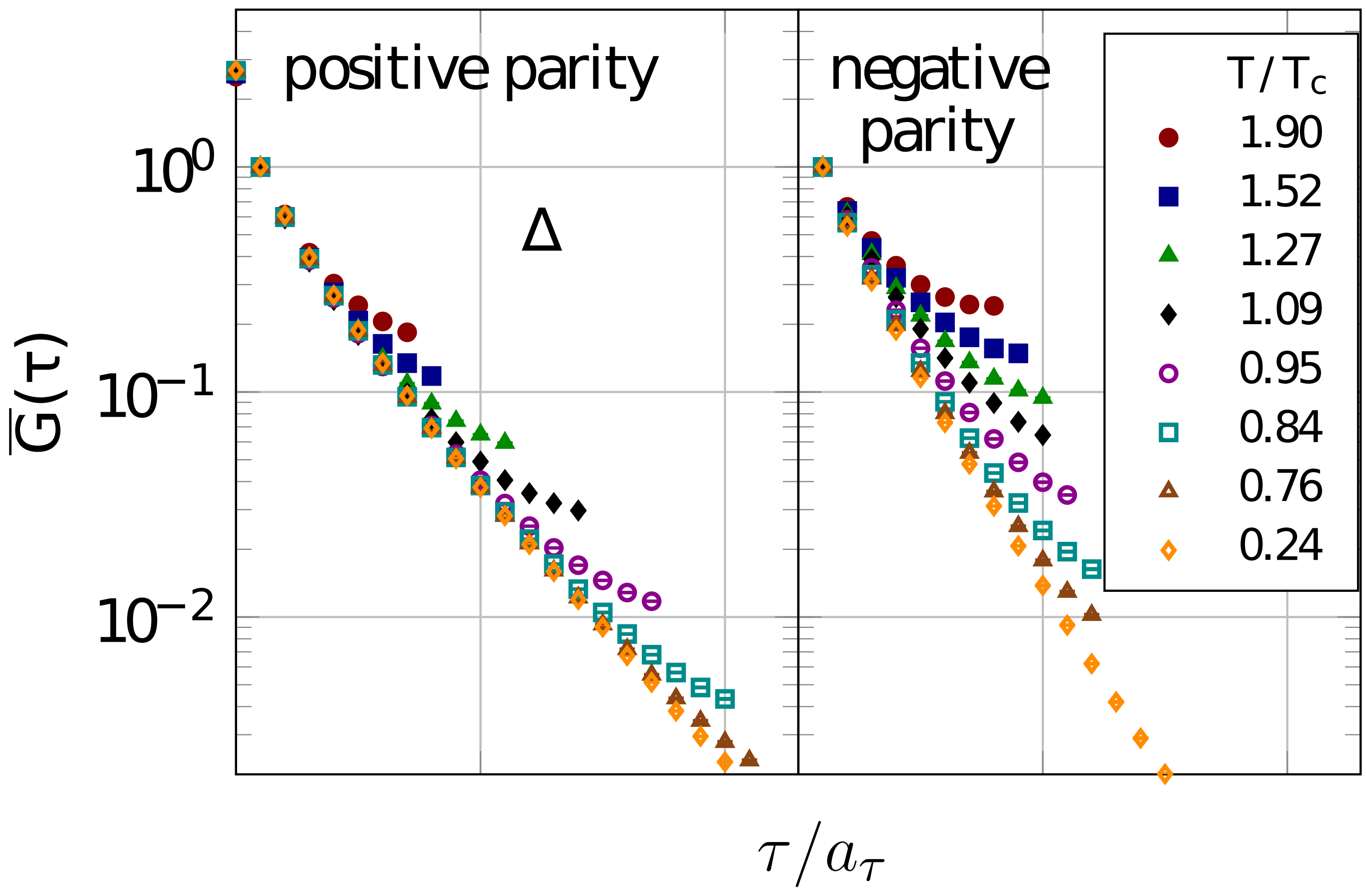
- nucleon



- pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

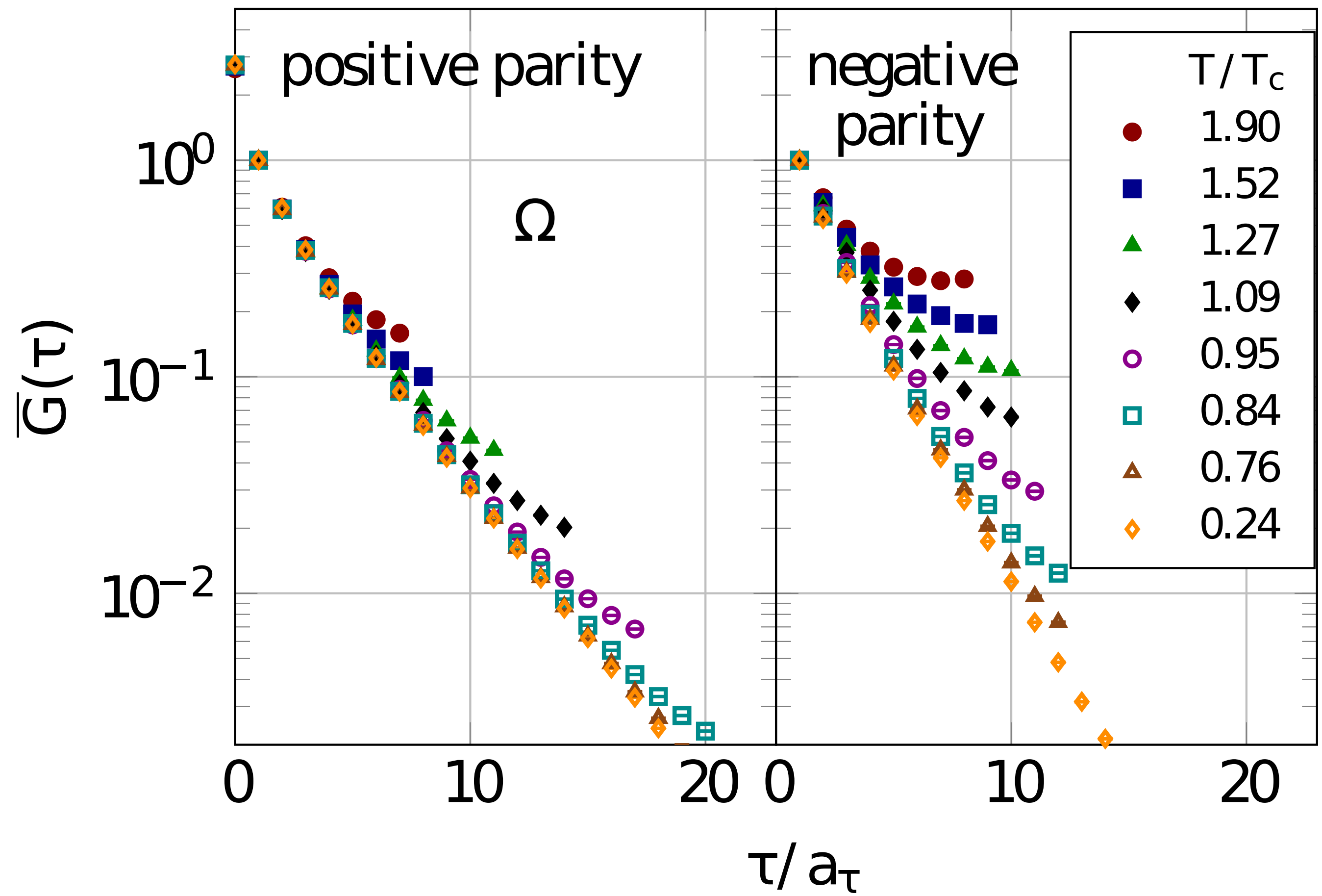
- Δ



- pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Lattice correlators

- Ω



- pos/neg parity channels nondegenerate
- more T dependence in negative-parity channel

Baryons in the hadronic phase

determine masses of pos/neg-parity groundstates

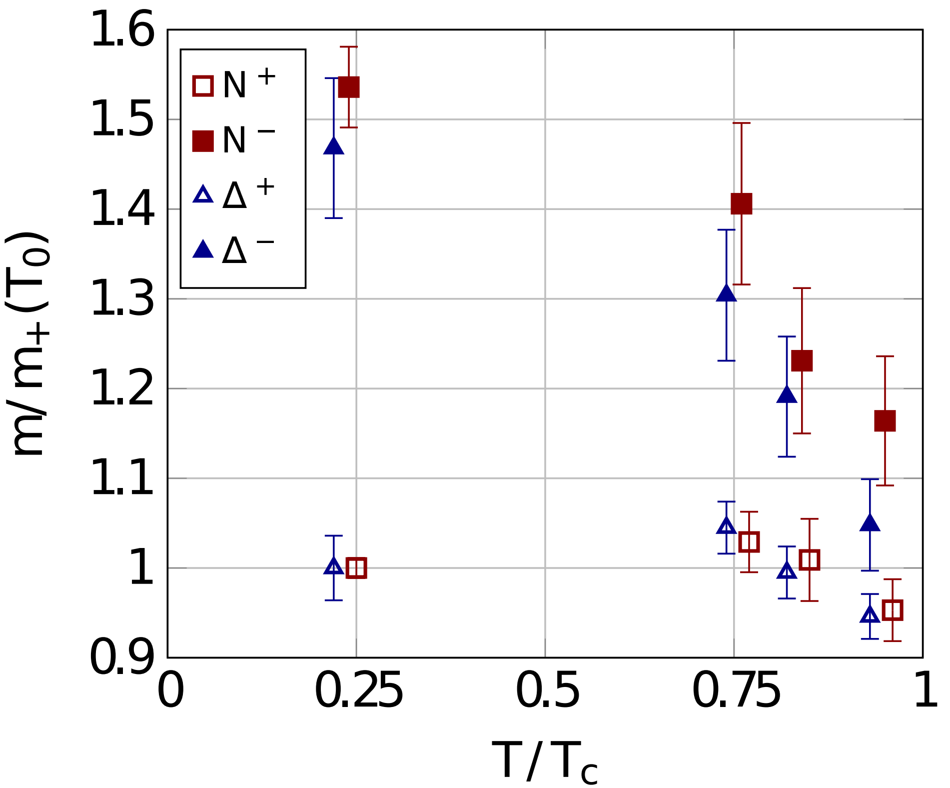
T/T_c	0.24	0.76	0.84	0.95	PDG ($T=0$)
m_+^N	1158(13)	1192(39)	1169(53)	1104(40)	939
m_-^N	1779(52)	1628(104)	1425(94)	1348(83)	1535(10)
m_+^Δ	1456(53)	1521(43)	1449(42)	1377(37)	1232(2)
m_-^Δ	2138(114)	1898(106)	1734(97)	1526(74)	1710(40)
m_+^Ω	1661(21)	1723(32)	1685(37)	1606(43)	1672.4(0.3)
m_-^Ω	2193(30)	2092(91)	1863(76)	1576(66)	2250–2380–2470
δ_N	0.212(15)	0.155(35)	0.099(40)	0.100(35)	0.241(1)
δ_Δ	0.190(31)	0.110(31)	0.089(31)	0.051(28)	0.162(14)
δ_Ω	0.138(9)	0.097(23)	0.050(23)	-0.009(25)	0.147–0.175–0.192

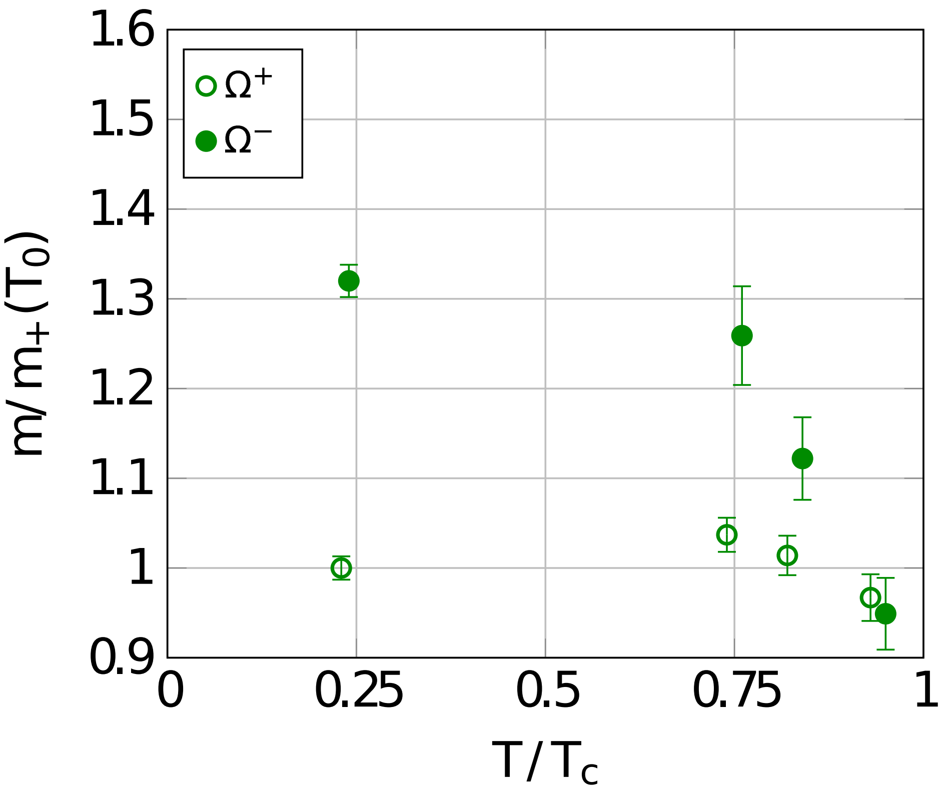
masses in MeV

$$\delta = \frac{m_- - m_+}{m_- + m_+}$$

Baryons in the hadronic phase

masses, normalised with m_+ at lowest temperature





- emerging degeneracy around T_c
- negative-parity masses reduced as T increases
- positive-parity masses nearly T independent

Baryons and parity partners

- distinct temperature dependence in hadronic phase
- relevant for heavy-ion phenomenology?

model studies of the role of chiral symmetry

example: parity doublet model `deTar & Kunihiro 89`

- chiral invariant contribution m_0 equal for N and N^*
- mass splitting due to chiral symmetry breaking
- degeneracy emerges as chiral symmetry is restored
- $m_0 \sim 500 - 800$ MeV

holographic predictions?

Baryon channels in QGP

- no clearly identifiable groundstates: baryons dissolved
- instead: parity doubling
- study correlator ratio

$$R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)}$$

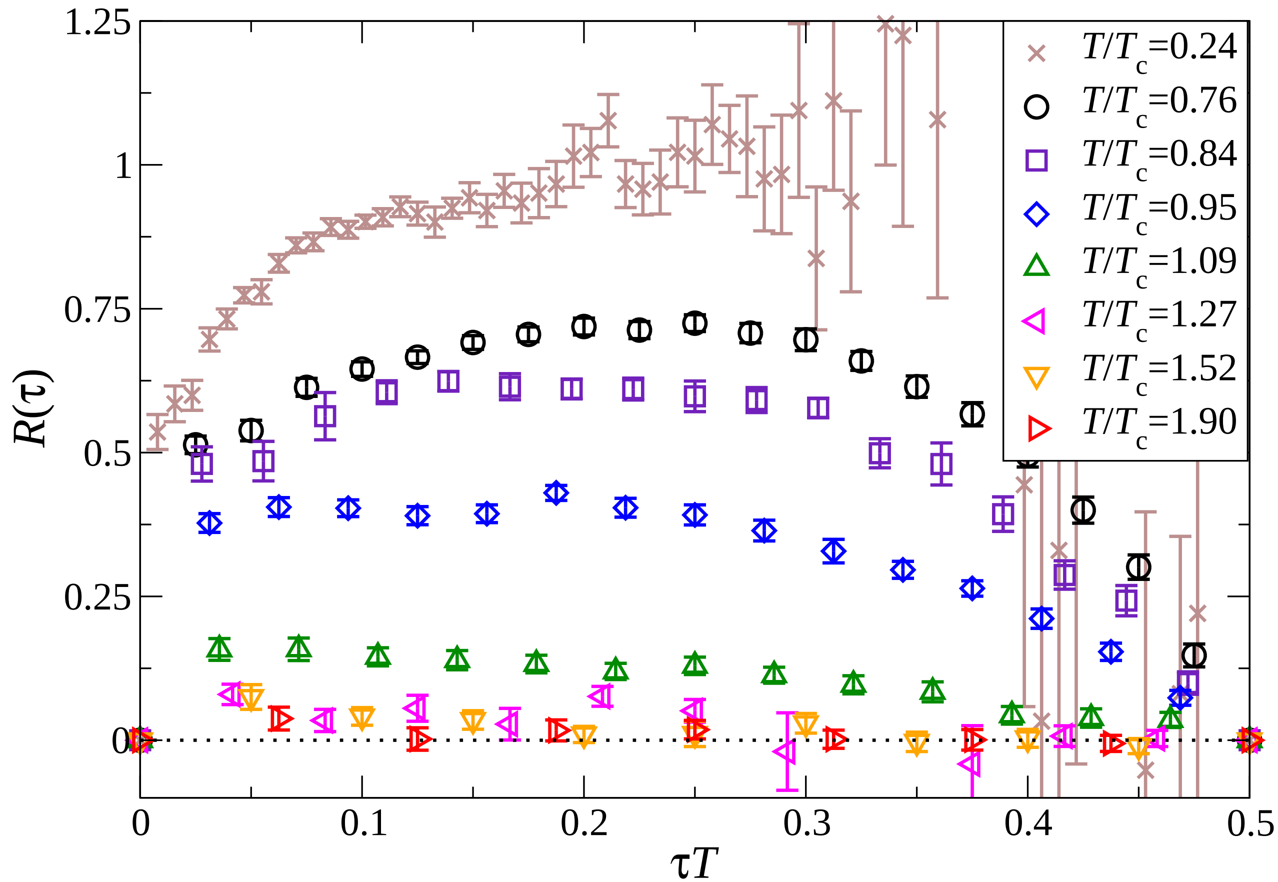
if

- no parity doubling and $m_- \gg m_+$: $R(\tau) = 1$
- parity doubling: $R(\tau) = 0$

note

- $R(1/T - \tau) = -R(\tau)$ and $R(1/2T) = 0$

Nucleon channel

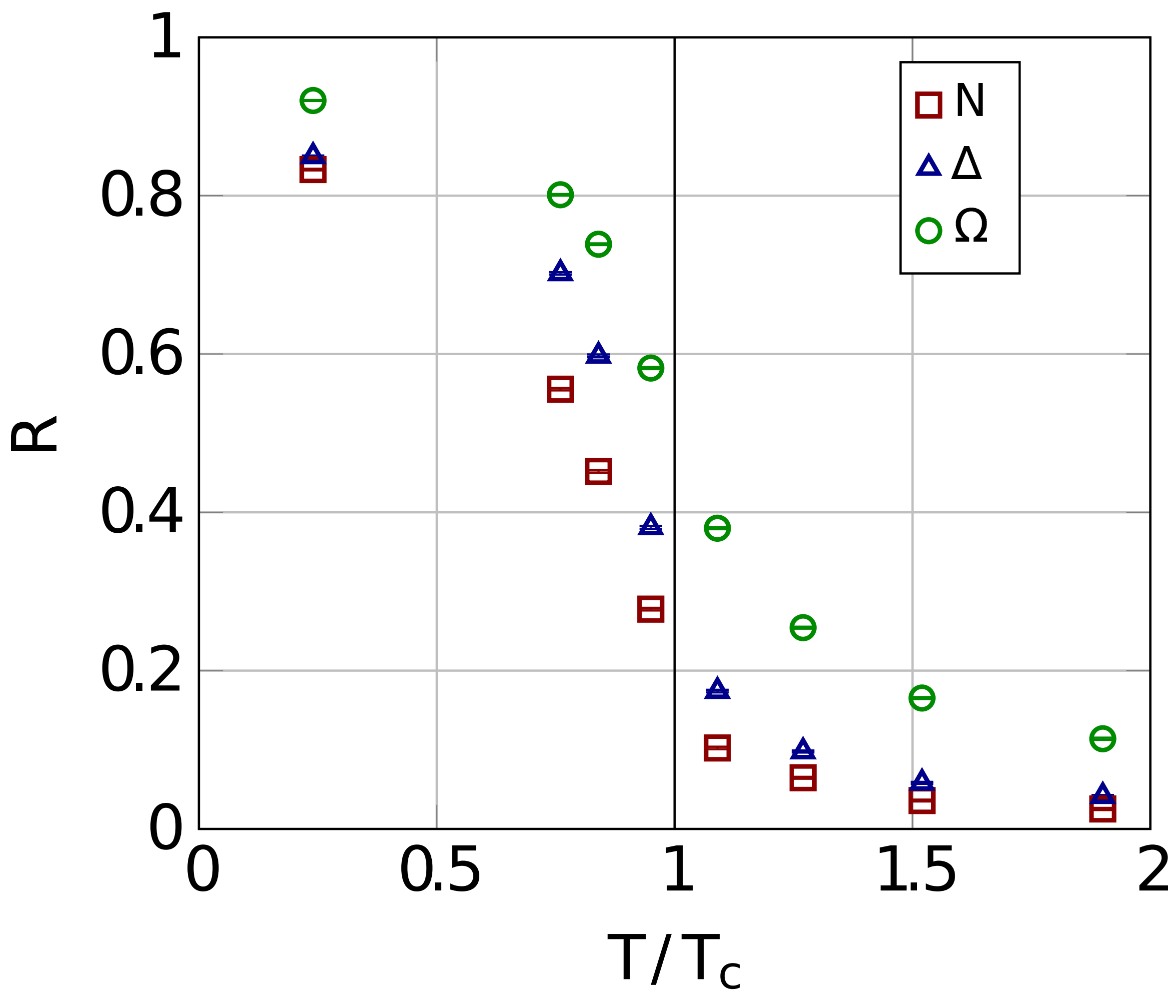


- ratio close to 1 below T_c, decreasing uniformly
- ratio close to 0 above T_c, parity doubling

Quasi-order parameter

- integrated ratio

$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$



- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: $m_q \neq 0$
- effect of heavier s quark visible

Parity doubling

- clear signal for parity doubling
  even with finite quark masses
- crossover behaviour, coinciding with transition to QGP
- visible effect of heavier s quark
  what about other strange baryons?

lattice technical remark:

- Wilson fermions break chiral symmetry at short distances
  what about chiral lattice fermions?

Spectral functions

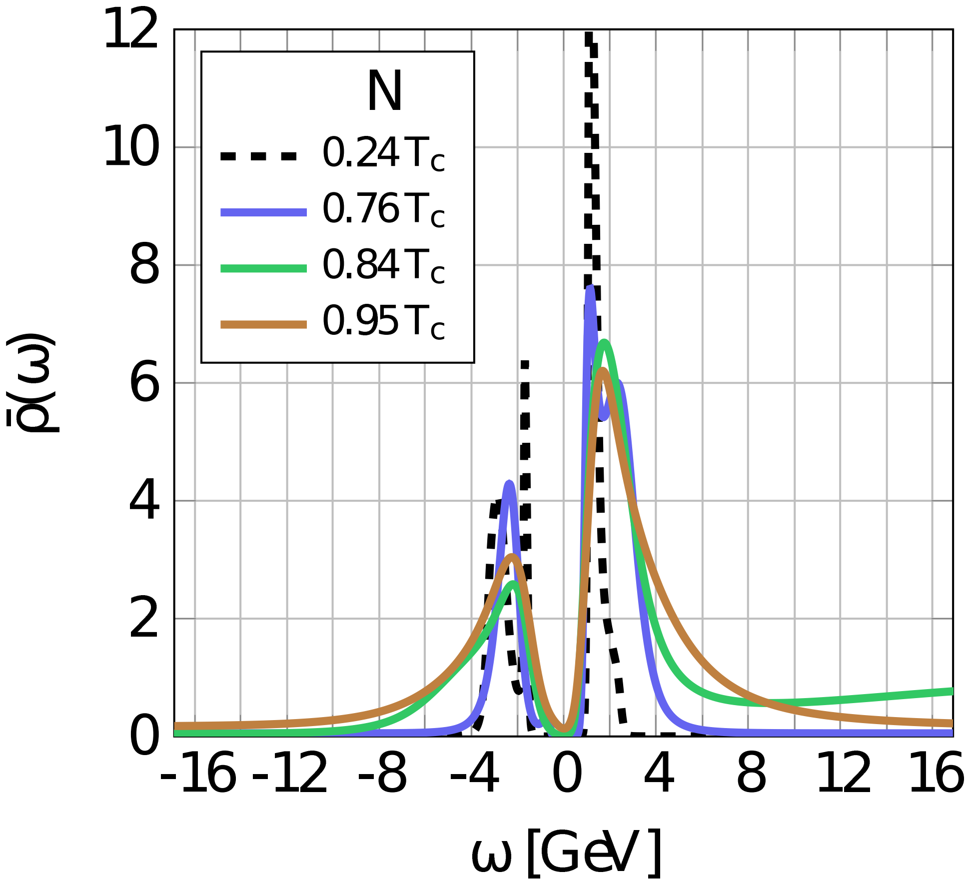
extract same information from spectral functions

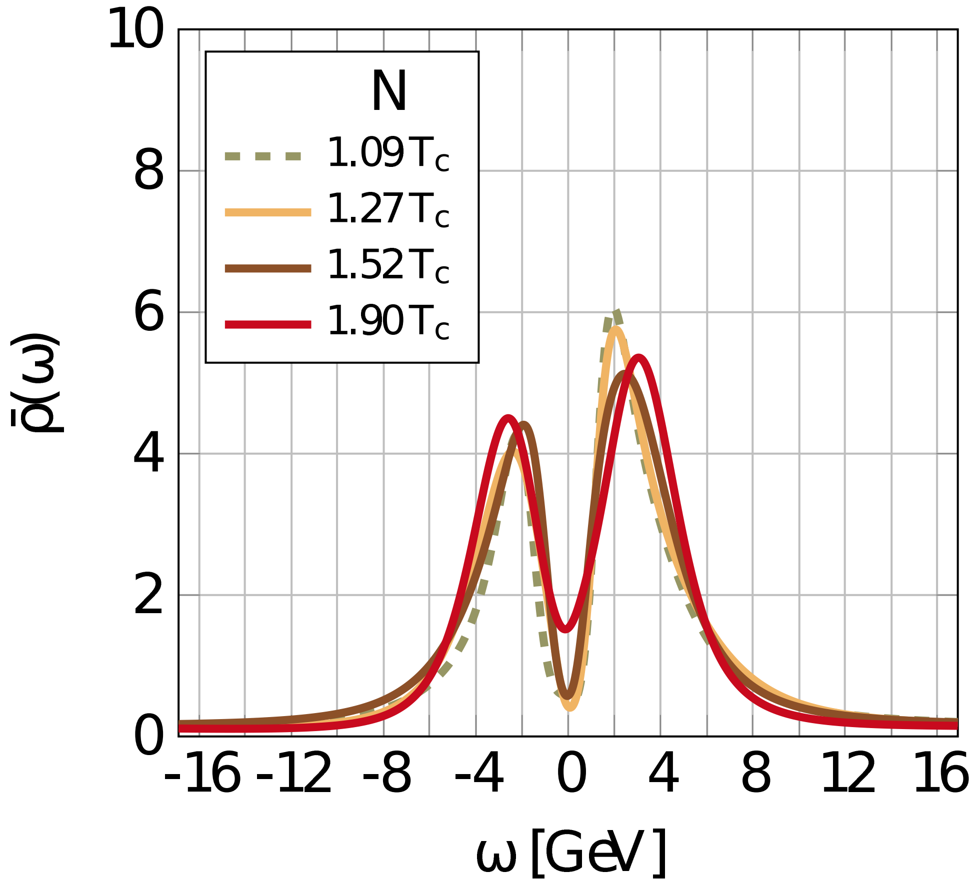
$$G_\pm(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\pm(\omega) \qquad K(\tau,\omega) = \frac{e^{-\omega\tau}}{1+e^{-\omega/T}}$$

- *ill-posed* inversion problem
- use Maximum Entropy Method (MEM)
- featureless default model
- construct $\rho_+(\omega) \geq 0$ for all ω
- $\rho_-(\omega) = -\rho_+(-\omega)$

Baryon spectral functions

- nucleon

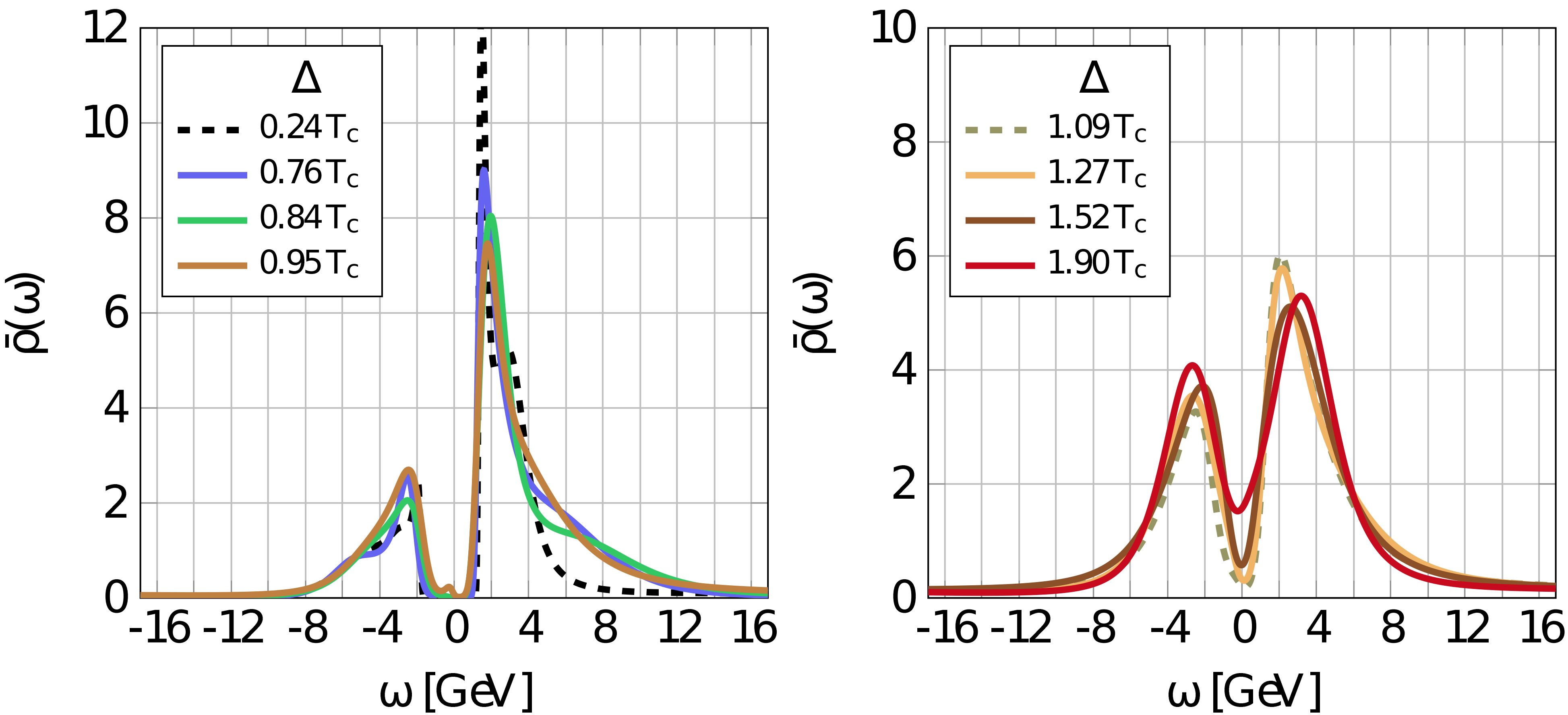




- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

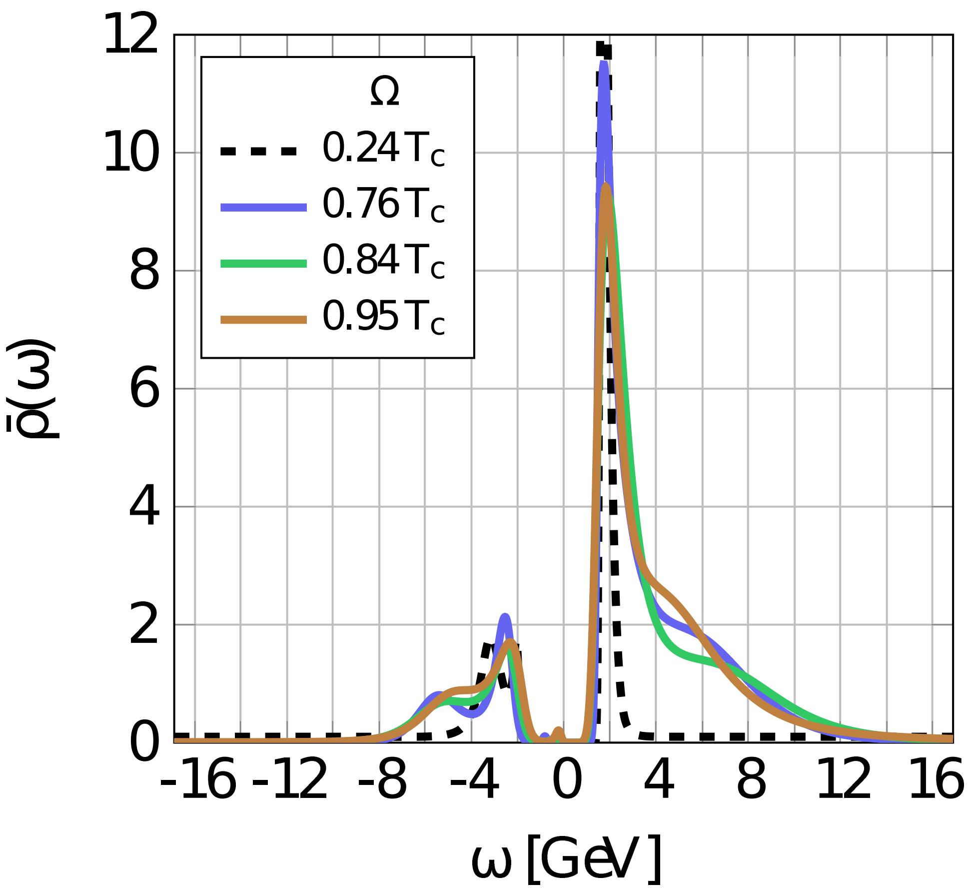
- Δ

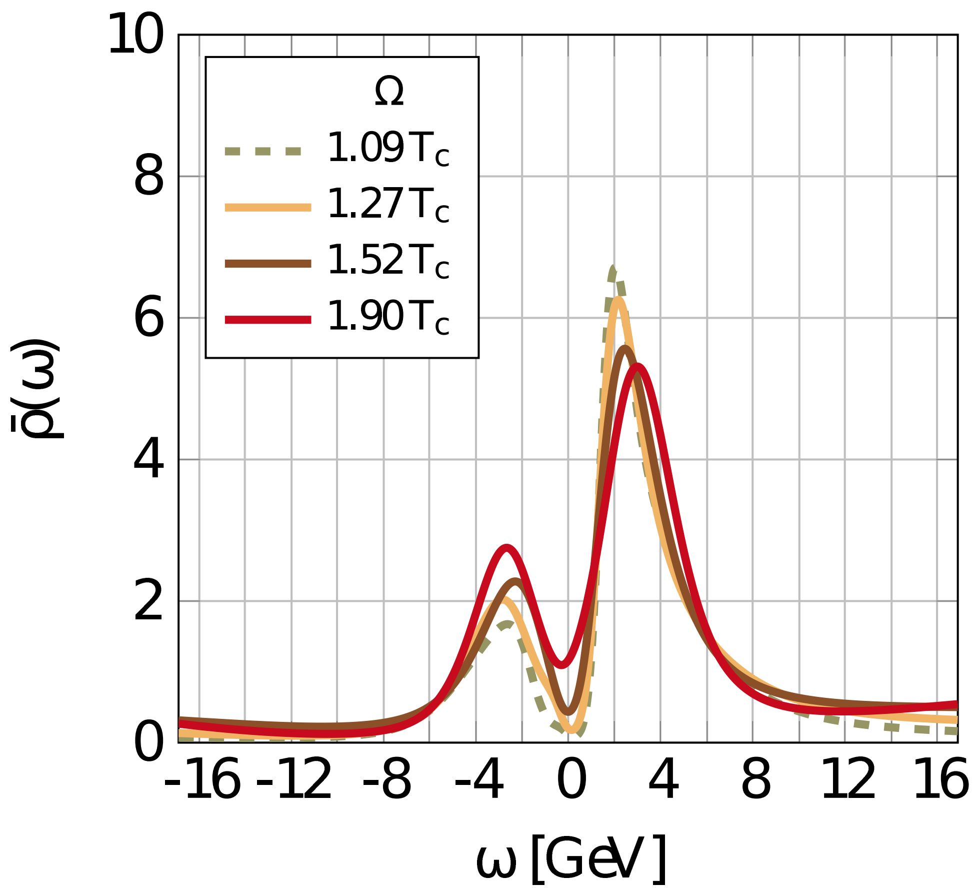


- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

- Ω

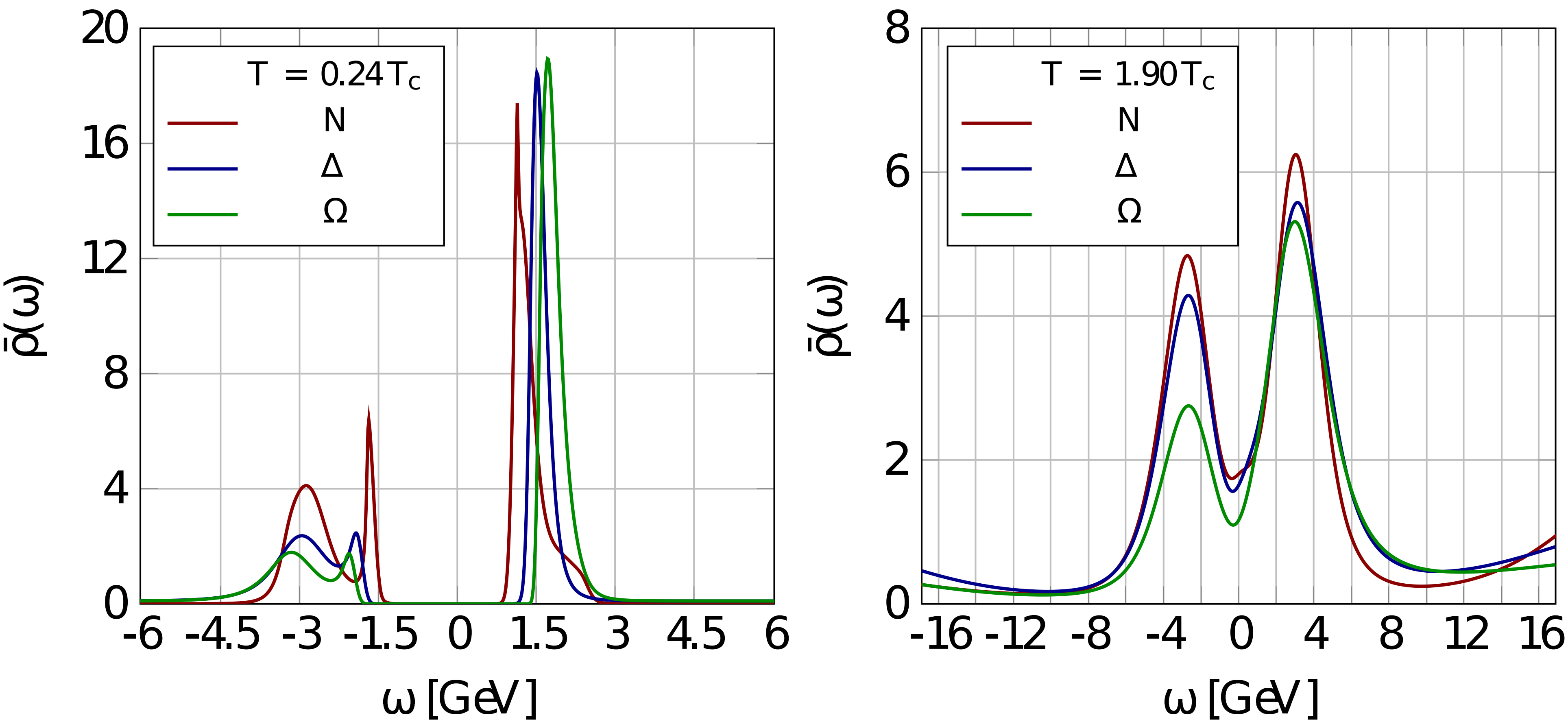




- groundstates below T_c
- degeneracy emerging above T_c, finite m_s

Baryon spectral functions

- all channels: low and high temperature



- groundstates below T_c
- degeneracy emerging above T_c

Baryon spectral functions

- results consistent with correlator analysis
- latter is on firmer ground, due to inversion uncertainties
- effect of heavier s quark visible

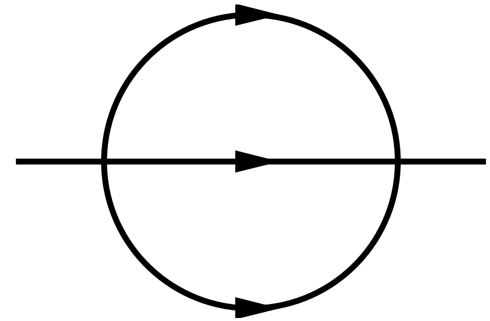
expectation at very high temperature

- compute baryon spectral functions at $g^2 \to 0$
- similar to computation of meson spectral functions

Karsch et al 03, GA & Martínez Resco 05

Free spectral functions

lowest order in perturbation theory



$G(x) = \langle O(x)\overline{O}(0)\rangle \qquad O(x) \sim u u^T C\gamma_5 d(x)$

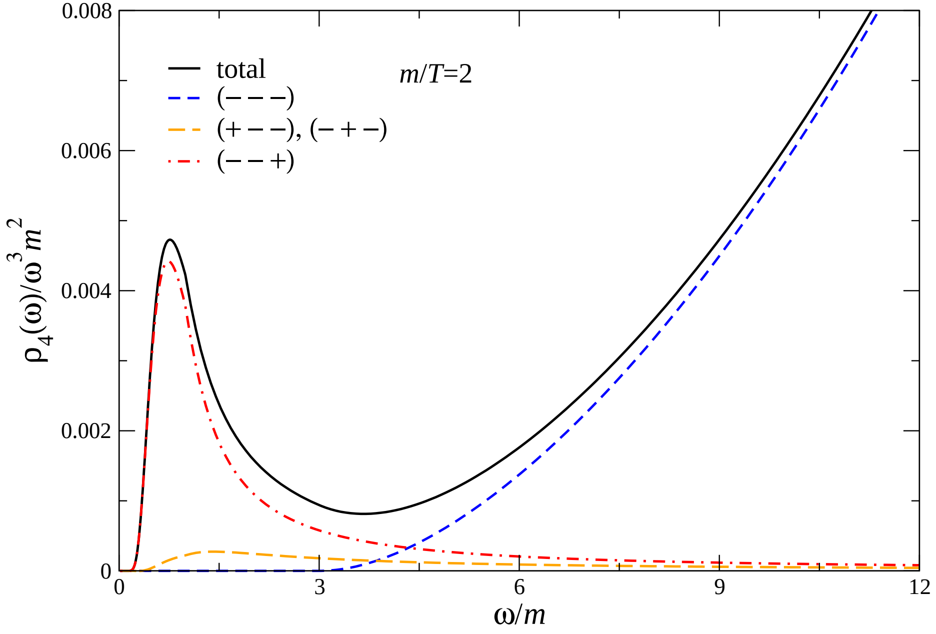
two-loop diagram $\qquad (c = 4, i, m)$

$$\rho_c(\omega) = 3\int_{\mathbf{k}_{1,2,3}} d\Phi_{123} \sum_{s_j=\pm} 2\pi\delta\left(\omega + \sum\nolimits_j s_j\omega_{\mathbf{k}_j}\right)[\text{stat.}]f_c(\omega, s_i, \mathbf{k}_i)$$

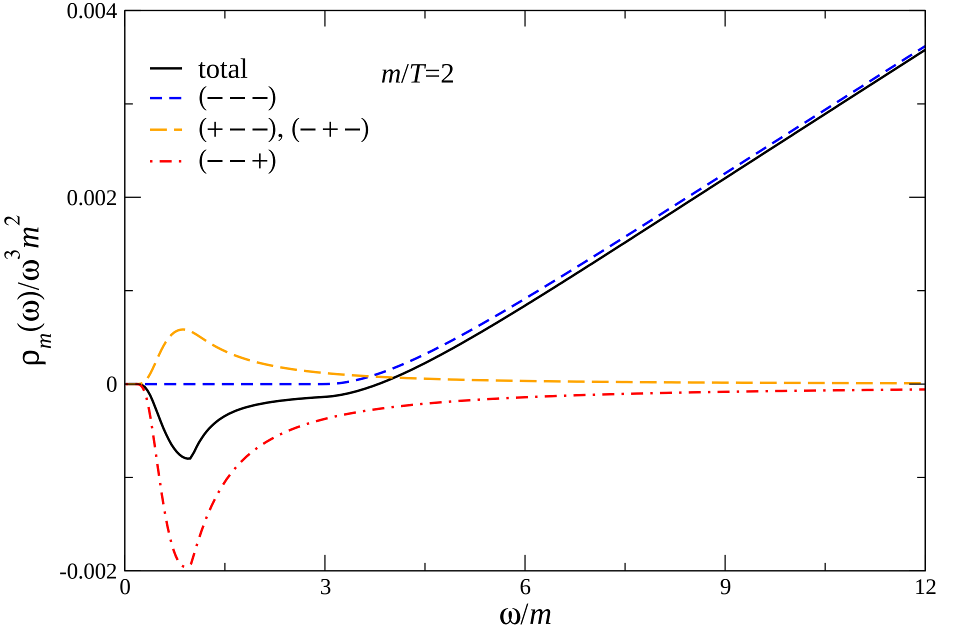
with
$$d\Phi_{123} = \prod_{j=1}^{3}\frac{d^3k_j}{(2\pi)^3 2\omega_{\mathbf{k}_j}}(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$[\text{stat.}] = n_F(s_1\omega_{\mathbf{k}_1})n_F(s_2\omega_{\mathbf{k}_3})n_F(s_3\omega_{\mathbf{k}_3}) + n_F(-s_1\omega_{\mathbf{k}_1})n_F(-s_2\omega_{\mathbf{k}_3})n_F(-s_3\omega_{\mathbf{k}_3})$$

Free spectral functions



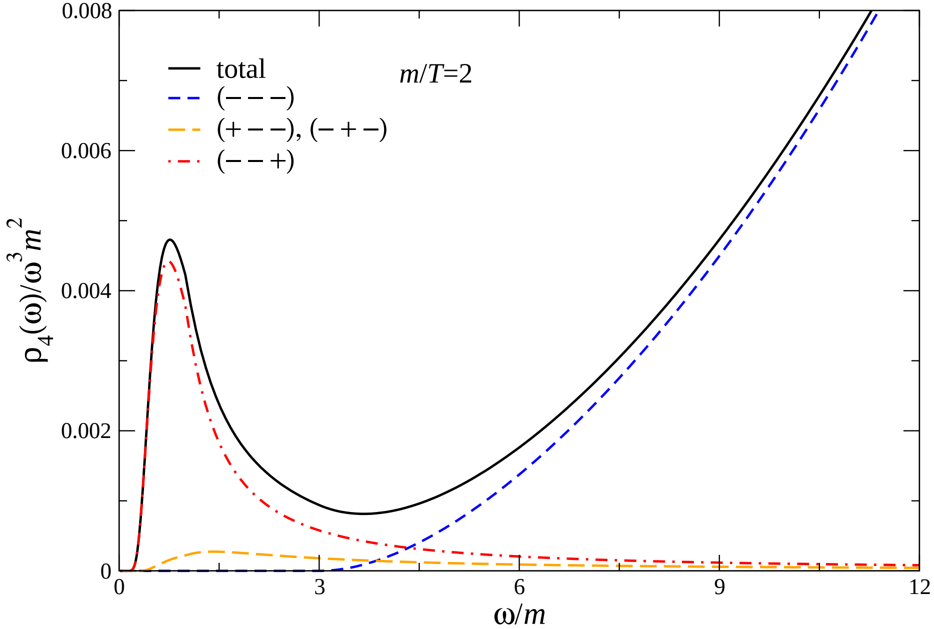
$\rho_4(\omega)$



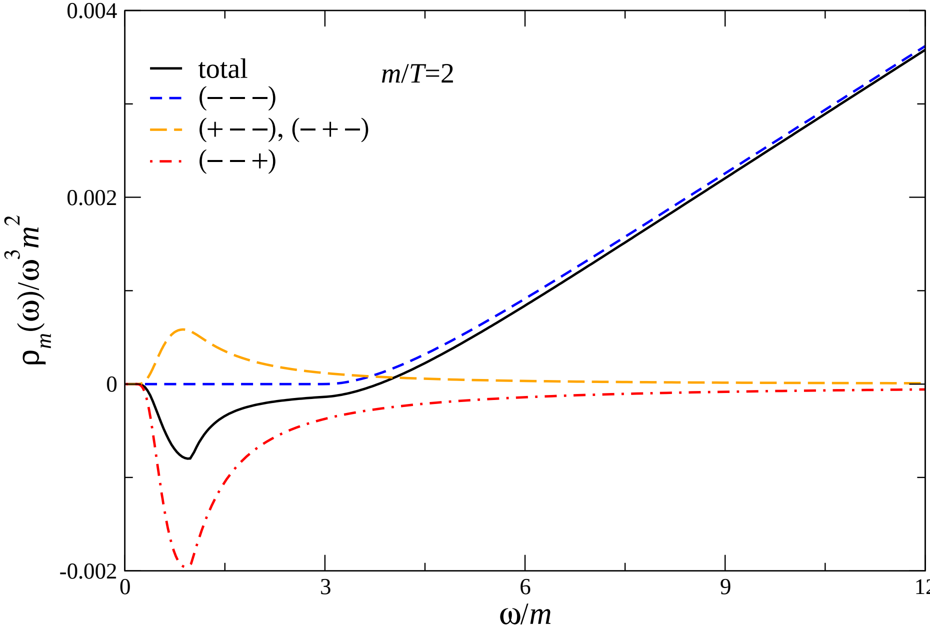
$\rho_m(\omega)$

- decay: $\omega > 3m$ with m quark mass
- at $T > 0$ scattering contributions for all ω
- large ω: thermal contributions suppressed
- $\rho_m(\omega)$ not positive definite

Free spectral functions



$\rho_4(\omega)$



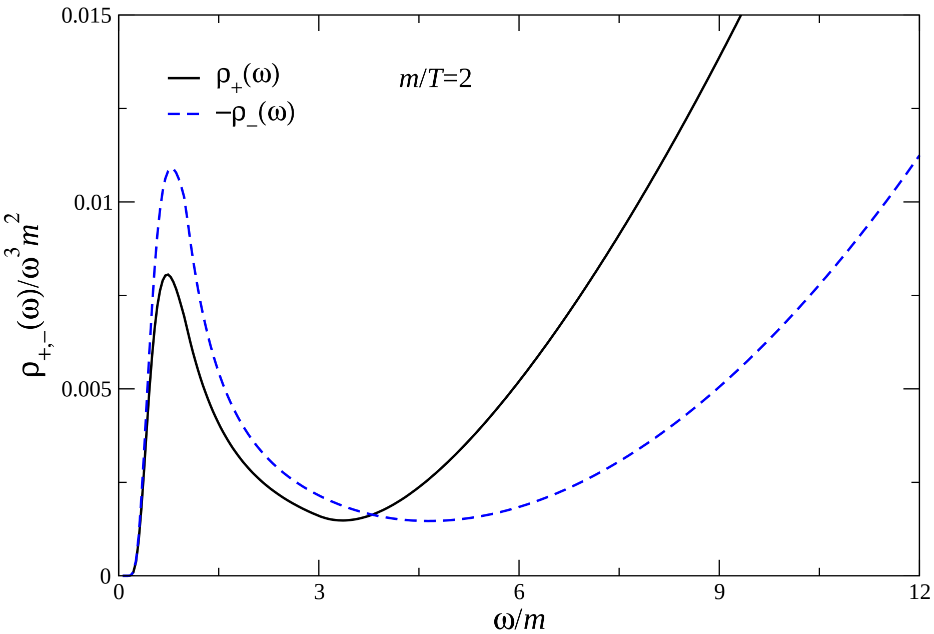
$\rho_m(\omega)$

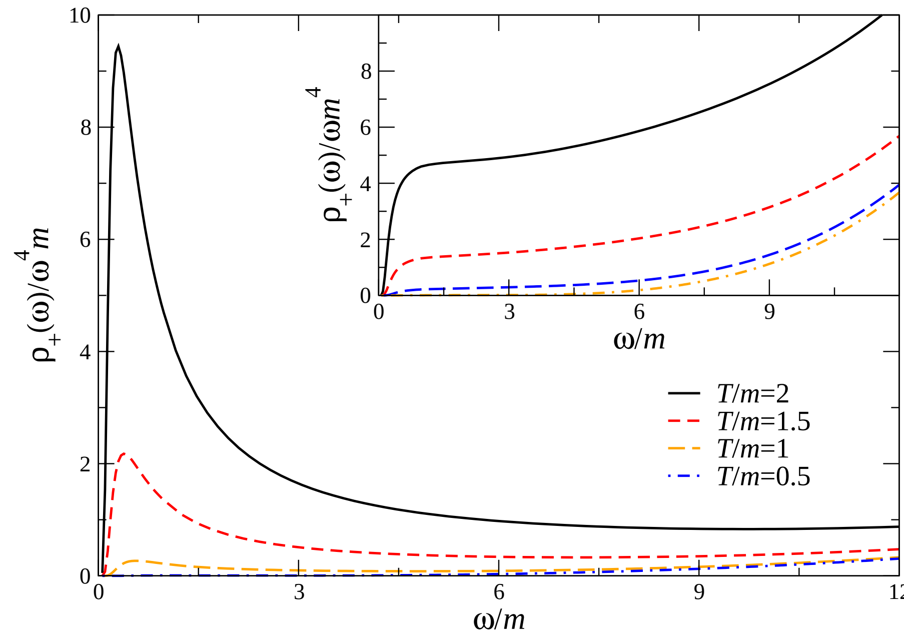
$$\omega \gg T \gg m$$

$$\rho_4(\omega) = \frac{5\omega^5}{2048\pi^3}\left(1 + \frac{112\pi^4}{3}\frac{T^4}{\omega^4} + \dots\right)$$

$$\rho_m(\omega) = \frac{7m\omega^4}{512\pi^3}\left(1 - 4\pi^2\frac{T^2}{\omega^2} + \dots\right)$$

Free spectral functions



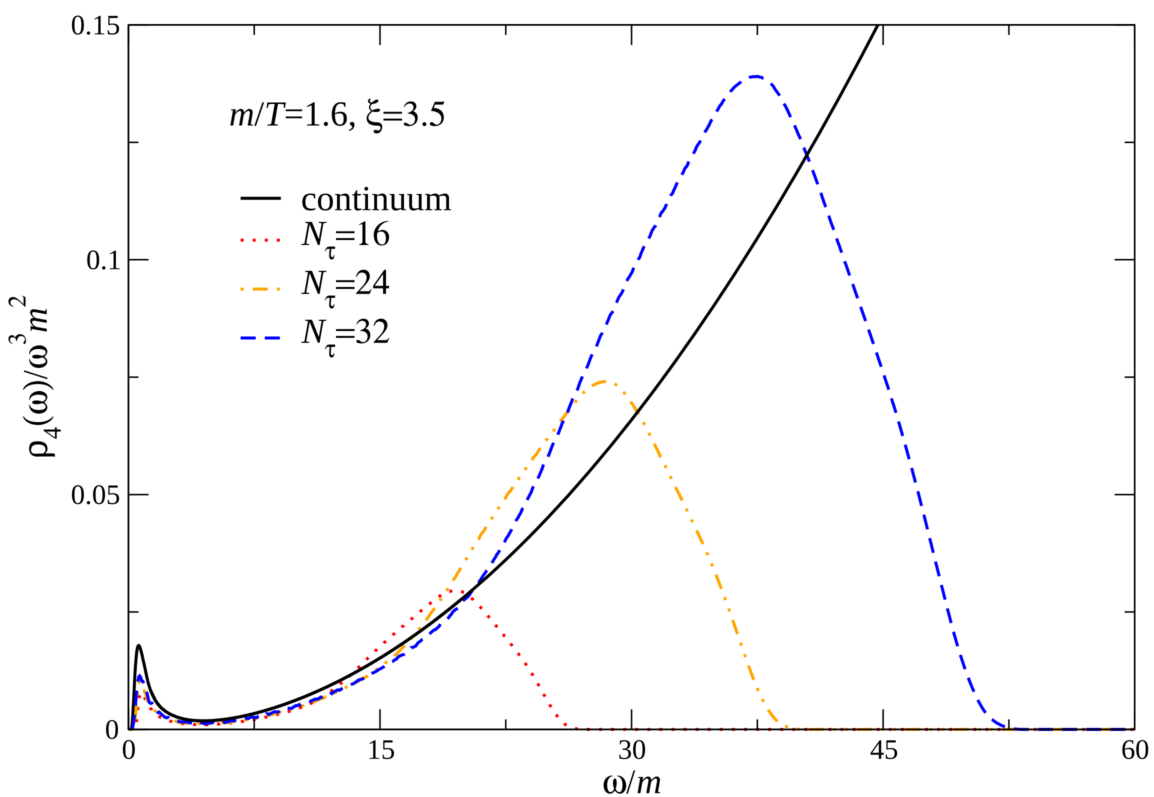


$$\rho_\pm(\omega) = \frac{1}{2}\left[\rho_m(\omega) \pm \rho_4(\omega)\right] \qquad\qquad \rho_+(\omega)$$

- thermal enhancement at $\omega \sim T \sim m$
- apparent peak depends on presentation/normalisation
- exponentially suppressed as $\omega \to 0$
- $\pm\rho_\pm(\omega) \geq 0 \qquad\qquad \rho_-(\omega) = -\rho_+(-\omega)$

Lattice free spectral functions

- lattice dispersion relation, sum over Brillouin zones
- maximal energy $\omega = 3\omega_{\mathbf{k},\max}$
- similar to mesons Karsch et al 03, GA & Martínez Resco 05
- no cusps due to two-loop Brillouin sum



Summary: baryons in medium

in hadronic phase

- pos-parity groundstates mostly T independent
- stronger T dependence in neg-parity groundstates
  reduction in mass, near degeneracy close to T_c
- relevant for heavy-ion phenomenology?

in quark-gluon plasma

- pos/neg parity channels degenerate: parity doubling
- linked to deconfinement transition and chiral symmetry restoration
- correlator and spectral function analysis consistent
- effect of heavier s quark noticeable

Outlook: baryons in medium

lattice

- Wilson fermions: no chiral symmetry at short distances
- manifestly chiral fermions?

physics

- strangeness dependence
- physical light quarks
- phenomenology

understanding

- models?
- holography?