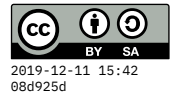


# Quiz 3

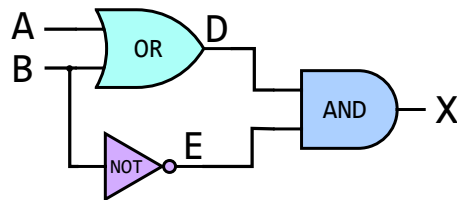
16 October 2019



## Solutions

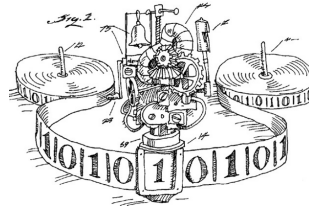
### Questions

1. Complete the truth table that corresponds to the following circuit. What would be the value of X in each row?



| A | B | $E=B'$   | $D=A+B$  | $X=D \cdot E$ |
|---|---|----------|----------|---------------|
| 0 | 0 | <u>1</u> | <u>0</u> | <u>0</u>      |
| 0 | 1 | <u>0</u> | <u>1</u> | <u>0</u>      |
| 1 | 0 | <u>1</u> | <u>1</u> | <u>1</u>      |
| 1 | 1 | <u>0</u> | <u>1</u> | <u>0</u>      |

2. This problem is about a program for a Turing Machine. Recall that a TM operates by reading and writing symbols on a tape that can be spooled to the left and right. For our program, each cell on the tape can contain either a zero (0), a one (1), or it can be blank (B).



The table below is a representation of a particular TM program. The TM keeps track of its current **state**, a small integer starting at 0. (This program only uses state 0.)

The first rule in the table says that if we're in state 0, and the symbol on the tape at the current position is a 0, we should write a 1 to that position, move the position to the **Right**, and stay in state **0**.

| rule number | current state | current symbol | write symbol | move to | next state |
|-------------|---------------|----------------|--------------|---------|------------|
| 1           | 0             | 0              | 1            | R       | 0          |
| 2           | 0             | 1              | 0            | R       | 0          |
| 3           | 0             | B              | B            | L       | halt       |

Simulate the execution of the above Turing Machine program on a tape containing a 4-bit number surrounded by blanks, as shown below. The starting position is underlined (it's the leftmost 1):

... B B 1 1 0 0 B B ...

What will be the contents of the tape when the machine halts? 0011 Please show your work below!

In tracing the TM, we'll use brackets [] to highlight the current position.

B B [1] 1 0 0 B B in state 0 so rule 2: write 0, move R  
 B B 0 [1] 0 0 B B in state 0 so rule 2: write 0, move R  
 B B 0 0 [0] 0 B B in state 0 so rule 1: write 1, move R  
 B B 0 0 1 [0] B B in state 0 so rule 1: write 1, move R  
 B B 0 0 1 1 [B] B in state 0 so rule 3: move L, then halt  
 B B 0 0 1 [1] B B

This program just takes the **complement** of a number – that is, it flips every bit to its opposite.