Generic Construction of Public-key Authenticated Encryption with Keyword Search Revisited: Stronger Security and Efficient Construction[*]

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Abstract

Public-key encryption with keyword search (PEKS) does not provide trapdoor privacy, i.e., keyword information is leaked through trapdoors. To prevent this information leakage, public key authenticated encryption with keyword search (PAEKS) has been proposed, where a sender's secret key is required for encryption, and a trapdoor is associated with not only a keyword but also the sender. Liu et al. (ASIACCS 2022) proposed a generic construction of PAEKS based on word-independent smooth projective hash functions (SPHFs) and PEKS. In this paper, we propose a new generic construction of PAEKS. The basic construction methodology is the same as that of the Liu et al. construction, where each keyword is converted into an extended keyword using SPHFs, and PEKS is used for extended keywords. Nevertheless, our construction is more efficient than Liu et al.'s in the sense that we only use one SPHF, but Liu et al. used two SPHFs. In addition, for consistency we considered a security model that is stronger than Liu et al.'s. Briefly, Liu et al. considered only keywords even though a trapdoor is associated with not only a keyword but also a sender. Thus, a trapdoor associated with a sender should not work against ciphertexts generated by the secret key of another sender, even if the same keyword is associated. Our consistency definition considers a multi-sender setting and captures this case. In addition, for indistinguishability against chosen keyword attack (IND-CKA) and indistinguishability against inside keyword guessing attack (IND-IKGA), we use a stronger security model defined by Qin et al. (ProvSec 2021), where an adversary is allowed to query challenge keywords to the encryption and trapdoor oracles. We also highlight several issues associated with the Liu et al. construction in terms of hash functions, e.g., their construction does not satisfy the consistency that they claimed to hold.

1 Introduction

For providing a search functionality against encrypted keyword, public key encryption with keyword search (PEKS) has been proposed by Boneh et al. [7]. As a feasibility result, PEKS can

[*] An extended abstract appeared at ACM APKC 2022 [16]. In this version, we give security proofs that were omitted in the proceedings version. Moreover, we reconsider the feasibility of the proposed generic construction according to the comment by Cheng and Meng [11], where the Benhamouda et al. word independent-SPHF construction, which was considered as a building block of the proposed generic construction, requires a super-polynomial modulus q to get statistical correctness. In this version, we explicitly mention that, at the expense of correctness error, we can employ the Li-Wang word-independent approximate SPHF [26] that provides approximate correctness.

be generically constructed from anonymous identity-based encryption (IBE) [1]. PEKS is briefly explained as follows. A sender encrypts a keyword using a receiver public key. A receiver generates a token to search for a keyword, called trapdoor, using the receiver's secret key. Then, based on the test algorithm, anyone can determine whether a ciphertext is an encryption of a keyword using a trapdoor. The test algorithm outputs 1 if the two keywords used for encryption and trapdoor generation are the same. In addition to this correctness, consistency is required, with the test algorithm outputting 0 if the two keywords used for encryption and trapdoor generation differ. Moreover, it is required that no information of keyword is leaked from ciphertexts. Unfortunately, PEKS does not provide trapdoor privacy, that is, information of keyword is leaked from trapdoors. More concretely, one can freely generate a ciphertext of an arbitrary-chosen keyword using the receiver's public key. Thus, when one obtains a trapdoor, one can check whether the trapdoor is associated to the keyword via the test algorithm.[1] One way to prevent the keyword guessing attack is to restrict searching, and another way is to restrict encryption.

The former approach is called designated-tester PEKS [3,17–19,21,35–37]. A server, who runs the test algorithm, also has a public key and a secret key pair. A sender, who generates a ciphertext, encrypts a keyword using both a receiver public key and the server public key. Then, the server secret key is required for running the test algorithm, in addition to a trapdoor. In the designated-tester setting, preventing the keyword guessing attack is relatively easy as mentioned by Chen [9], where a receiver encrypts a trapdoor by using the server public key and sends the ciphertext to the server (i.e., trapdoors are ciphertexts). The server still can run the test algorithm by decrypting the ciphertext and obtaining the trapdoor. This simple modification works well since no information of keyword is leaked owing to the encryption. As in Abdalla et al. generic construction of PEKS [1], designated-tester PEKS can be generically constructed from anonymous IBE [17, 35]. Though these generic constructions do not provide trapdoor privacy because a trapdoor is a secret key of the underlying IBE scheme, they can be easily modified to provide trapdoor privacy by employing the Chen observation above.

The latter approach is called public key authenticated encryption with keyword search (PAEKS) [10,12,22,28,29,29,31–34], and we focus on PAEKS in this paper. A sender, who generates a ciphertext, has a public key and a secret key pair. The sender encrypts a keyword using both a receiver's public key and the sender's secret key. A trapdoor is associated with both a keyword and a sender's public key. That is, the trapdoor only works against ciphertexts generated by the corresponding sender's secret key. Currently, two generic constructions of PAEKS have been proposed [28, 29]. Liu et al. [29] claimed that the construction proposed in [28] does not follow the syntax of PAEKS because it requires a trusted authority to assist users in generating their private keys. More precisely, the setup algorithm outputs a master secret key, and other sender/receiver key generations require the master secret key. Thus, we mainly consider the Liu et al. generic construction [29] in this paper. They employed word-independent smooth projective hash functions (SPHFs) [5, 25]. Each keyword is converted into an extended keyword using SPHF, and they employed PEKS for these extended keywords. We revisit their construction methodology in Section 4.

Our Contribution. In this paper, we propose a new generic construction of PAEKS from public key encryption (PKE), word-independent SPHF (WI-SPHF), pseudorandom function (PRF), and PEKS, and no random oracles are employed. The basic construction methodology is the same as that of the Liu et al. construction. Nevertheless, our construction is more efficient than the Liu et

[1] Boneh, Raghunathan, and Segev proposed function-private IBE [8]. They showed that function-private IBE (with anonymity) can be used for constructing PEKS schemes that are provably keyword private. Concretely, a trapdoor enables to identify encryptions of an underlying keyword, while not revealing any additional information about the keyword *beyond the minimum necessary*, as long as the keyword is sufficiently unpredictable. Owing to the search functionality of PEKS, it is inevitable to leak information of keyword as mentioned.

al. construction in the sense that we just employ one SPHF but Liu et al. employed two SPHFs. Moreover, for consistency we consider a stronger security model than that of Liu et al. Briefly, they just considered keywords while a trapdoor is associated with not only a keyword but also a sender. So, a trapdoor associated to a sender should not work against ciphertexts generated by a secret key of other sender, even if the same keyword is associated. Our definition considers a multi-sender setting, and captures this case. In addition, for indistinguishability against chosen keyword attack (IND-CKA) and indistinguishability against inside keyword guessing attack (IND-IKGA) (defined in Section 3.2), we employ a stronger security model defined by Qin et al. [34] where an adversary is allowed to query challenge keywords to the encryption and trapdoor oracles (though we modify it in accordance with our syntax).

We also point out there are several issues in the Liu et al. proposal regarding their construction and security models (See Section 4.2 in detail). The main issue is related to hash functions they employed. First, their construction does not satisfy the consistency requirement that they claimed to hold. Second, they assume that a hash value does not leak input information, but it is not guaranteed by the one-wayness of hash functions. These issues may be solved by introducing other consistency definitions and by assuming that the underlying hash functions are modeled as random oracles. Besides these issues, they considered a weak security model, as previously mentioned.

We introduce a designated-receiver setting, where the sender key generation algorithm takes as input a receiver's public key. The setting allows us to remove a trusted setup assumption. In other words, if we assume a trusted setup, we do not have to introduce the designated-receiver setting. See Section 3.1 in detail.

Disclaimer. We checked the ePrint version of Liu et al. paper, Version 3, posted on 23-Nov-2021 [27] that is the same as their AsiaCCS 2022 version [29] as they declared. According to my comments in [16], they have updated the ePrint paper and the latest version so far is Version 6, posted on 19-Apr-2022. We decline to check whether their update appropriately addressed my comments.

2 Preliminaries

Notations. For a positive integer $n \in \mathbb{N}$, we write $[1, n] = \{1, 2, \dots, n\}$. If A is a probabilistic algorithm, $y \leftarrow A(x; r)$ denotes the operation of running A on an input x and a randomness r, and letting y be the output. We omit r when it is not necessary to specify. $x \xleftarrow{\$} S$ denotes choosing an element x from a finite set S uniformly at random. For a security parameter λ, $\mathsf{negl}(\lambda)$ is a negligible function where for any $c > 0$, there exists an integer I such that $\mathsf{negl}(\lambda) < 1/\lambda^c$ for all $\lambda > I$.

2.1 Pseudorandom Functions (PRFs)

Let m and ℓ be polynomial and λ be a security parameter. A pseudorandom function (PRF) is a family of functions $\mathsf{PRF} = \{F_K : \{0,1\}^{m(\lambda)} \rightarrow \{0,1\}^{\ell(\lambda)}\}$ where $K \in \{0,1\}^\lambda$.

Definition 1 (Pseudo-randomness). *We say that* PRF *is pseudo-random if for all probabilistic polynomial-time (PPT) adversaries* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{pseudo\text{-}random}}_{\mathsf{PRF},\mathcal{A}}(\lambda) := |\Pr[K \xleftarrow{\$} \{0,1\}^\lambda : \mathcal{A}^{F_K(\cdot)}(1^\lambda) = 1] - \Pr[R \xleftarrow{\$} \mathcal{RF} : \mathcal{A}^{R(\cdot)}(1^\lambda) = 1]|$ *is negligible in the security parameter* λ *where* $\mathcal{RF}$ *is a set of random functions mapping* $m(\lambda)$ *bits to* $\ell(\lambda)$ *bits.*

2.2 Public Key Encryption (PKE)

A PKE scheme PKE consists of three algorithms $(\mathsf{PKE.KeyGen}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$. The key generation algorithm $\mathsf{PKE.KeyGen}$ takes a security parameter λ as input, and outputs a public key $\mathsf{pk}_{\mathsf{PKE}}$ and a secret key $\mathsf{dk}_{\mathsf{PKE}}$. The encryption algorithm takes $\mathsf{pk}_{\mathsf{PKE}}$ and a plaintext $M \in \mathcal{MS}$ as input, where $\mathcal{MS}$ is a message space, and outputs a ciphertext $\mathsf{ct}_{\mathsf{PKE}}$. When we need to explicitly treat the randomness ρ for encryption, we denote $\mathsf{ct}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, M; \rho)$. The decryption algorithm $\mathsf{PKE.Dec}$ takes $\mathsf{dk}_{\mathsf{PKE}}$ and $\mathsf{ct}_{\mathsf{PKE}}$ as input, and outputs M. For correctness, it is required that for any security parameter λ, any $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$ and any plaintext $M \in \mathcal{MS}$, $\Pr[\mathsf{PKE.Dec}(\mathsf{dk}_{\mathsf{PKE}}, \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, M)) = M] = 1 - \mathsf{negl}(\lambda)$ holds. We also require that the standard indistinguishability against chosen plaintext attack (IND-CPA) holds. Let state be state information that $\mathcal{A}$ can preserve any information, and state is used for transferring state information to the other stage.

Definition 2 (IND-CPA). *For all PPT adversaries $\mathcal{A}$, we define the following experiment:*

$$\begin{array}{l}
\mathsf{Exp}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{A}}(\lambda): \\
(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda) \\
(M_0^*, M_1^*, \mathsf{state}) \leftarrow \mathcal{A}(\mathsf{pk}_{\mathsf{PKE}}) \ \textit{s.t.}\ M_0^*, M_1^* \in \mathcal{MS} \wedge M_0^* \neq M_1^* \wedge |M_0^*| = |M_1^*| \\
b \xleftarrow{\$} \{0,1\};\ \mathsf{ct}^*_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, M_b^*) \\
b' \leftarrow \mathcal{A}(\mathsf{ct}^*_{\mathsf{PKE}}, \mathsf{state}) \\
\textit{If } b = b' \textit{ then output } 1 \textit{ and } 0 \textit{ otherwise.}
\end{array}$$

We say that a PKE scheme PKE is IND-CPA secure if the advantage

$$\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{A}}(\lambda) := |\Pr[\mathsf{Exp}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{A}}(\lambda) = 1] - 1/2| < \mathsf{negl}(\lambda)$$

2.3 Word-independent Smooth Projective Hash Functions (WI-SPHFs)

Cramer and Shoup proposed hash proof systems (HPSs) [13], which are special kind of non-interactive zero-knowledge proof systems for a language, for constructing PKE. Later, several applications of HPSs have been considered such as password-based authenticated key exchange (PAKE) (e.g., [5, 25]). In this paper, we employ word-independent smooth projective hash functions (WI-SPHFs) (which are also called KV-SPHF [5] in reference to [25]). We introduce the definition given by Benhamouda et al. [6].

Definition 3 (Languages). *Let $\mathsf{Setup.lpar}$ be a PPT algorithm that takes a security parameter λ as input, and outputs $(\mathsf{lpar}, \mathsf{ltrap})$ where lpar is a parameter and ltrap is a trapdoor. $\mathscr{L}_{\mathsf{lpar},\mathsf{ltrap}}$ is a language indexed by $(\mathsf{lpar}, \mathsf{ltrap})$ together with an NP language indexed by lpar $\widetilde{\mathscr{L}}_{\mathsf{lpar}}$, with witness relation $\widetilde{\mathscr{R}}_{\mathsf{lpar}}$ such that $\widetilde{\mathscr{L}}_{\mathsf{lpar}} = \{\chi \in \mathcal{X}_{\mathsf{lpar}} \mid \exists w \ s.t.\ \widetilde{\mathscr{R}}_{\mathsf{lpar}}(\chi, w) = 1\} \subseteq \mathscr{L}_{\mathsf{lpar},\mathsf{ltrap}} \subseteq \mathcal{X}_{\mathsf{lpar}}$. We denote $(\widetilde{\mathscr{L}}_{\mathsf{lpar}}, \mathscr{L}_{\mathsf{lpar},\mathsf{ltrap}}, \mathcal{X}_{\mathsf{lpar}})_{\mathsf{lpar},\mathsf{ltrap}}$ as a family of languages.*

Next, we define languages of ciphertexts. Benhamouda et al. [6] introduced languages of ciphertexts for a labeled PKE scheme. More precisely, Benhamouda et al. defined the languages for a IND-CCA2 secure labeled PKE scheme that is converted from (a simplified variant of) the tag-IND-CCA2 secure Micciancio-Peikert PKE scheme [30] using the Dolev-Dwork-Naor (DDN) transformation [14]. Then, they also showed that the tag PKE scheme is IND-CPA secure when the tag of the ciphertext is known in advance or is constant, and proposed a $\mathsf{WI\text{-}SPHF}$ for IND-CPA ciphertexts. Since we need to employ the $\mathsf{WI\text{-}SPHF}$, we focus on their languages of IND-CPA ciphertexts. Since the tag/label can be a constant value, it can be included in $\mathsf{pk}_{\mathsf{PKE}}$ in advance, and then we do not have to consider tag/label anymore. Thus, we define languages of ciphertexts for a standard PKE scheme (defined in Section 2.2). Moreover, Benhamouda et al. focused on the languages of ciphertexts of 0, we also consider such ciphertexts as follows.

Definition 4 (Languages of Ciphertexts [6])**.** *Let* $\mathsf{PKE} = (\mathsf{PKE.KeyGen}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$ *be a PKE scheme. The* $\mathsf{Setup.lpar}$ *algorithm is set as* $\mathsf{PKE.KeyGen}$*, and* $(\mathsf{lpar}, \mathsf{ltrap}) = (\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}})$*. Then, languages of ciphertexts are defined as* $\widetilde{\mathscr{L}}_{\mathsf{pk}_{\mathsf{PKE}}} = \{\mathsf{ct}_{\mathsf{PKE}} \mid \exists \rho \text{ s.t. } \mathsf{ct}_{\mathsf{PKE}} = \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho)\} \subseteq \mathscr{L}_{\mathsf{pk}_{\mathsf{PKE}},\mathsf{dk}_{\mathsf{PKE}}} = \{\mathsf{ct}_{\mathsf{PKE}} \mid \mathsf{PKE.Dec}(\mathsf{dk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}}) = 0\}$ *where* $\mathscr{X}_{\mathsf{pk}_{\mathsf{PKE}}}$ *is a set of valid ciphertexts generated by* $\mathsf{pk}_{\mathsf{PKE}}$*. Here,* $\widetilde{\mathscr{R}}_{\mathsf{lpar}}(\mathsf{ct}_{\mathsf{PKE}}, \rho) = 1$ *iff* $\mathsf{ct}_{\mathsf{PKE}} = \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho)$*.*

In the actual Benhamouda et al. construction, dual-Regev ciphertexts of 0 is considered: $\mathbf{c} = \mathbf{A}\mathbf{s} + \mathbf{e} \in \mathbb{Z}_q^m$ where $\mathbf{A} \in \mathbb{Z}_q^{m \times n}$ is a public matrix, $\mathbf{s} \in \mathbb{Z}_q^n$ and $\mathbf{e} \in \mathbb{Z}_q^m$ are the randomness. If $\mathbf{e}$ is supposed to be small, then $\mathbf{c}$ is close to the q-ary lattice Λ generated by $\mathbf{A}$. On the other hand, if $\mathbf{c}$ is an encryption of 1, then $\mathbf{c}$ is far from the q-ary lattice Λ generated by $\mathbf{A}$. Thus, whether a word $\mathbf{c}$ belongs to $\widetilde{\mathscr{L}}_{\mathsf{pk}_{\mathsf{PKE}}}$ (i.e., a ciphertext of 0) or $\mathscr{X}_{\mathsf{pk}_{\mathsf{PKE}}} \setminus \mathscr{L}_{\mathsf{pk}_{\mathsf{PKE}}}$ (i.e., a ciphertext whose decryption result is 1) is indistinguishable owing to the IND-CPA security of PKE. Obviously, the membership in $\mathscr{L}_{\mathsf{pk}_{\mathsf{PKE}},\mathsf{dk}_{\mathsf{PKE}}}$ can be checked in polynomial time by $\mathsf{dk}_{\mathsf{PKE}}$.

Next, we define the syntax of WI-SPHF. Here, the term "word independent" means that the ProjKG algorithm does not take a word $\chi \in \mathscr{X}_{\mathsf{lpar}}$ as an input, and the smoothness (defined below) holds even if the word is chosen adaptively after seeing the projection key (thus, the smoothness is called adaptive smoothness).

Definition 5 (Syntax of Word-independent SPHF [6])**.** *Let* $(\widetilde{\mathscr{L}}_{\mathsf{lpar}}, \mathscr{L}_{\mathsf{lpar},\mathsf{ltrap}}, \mathscr{X}_{\mathsf{lpar}})_{\mathsf{lpar},\mathsf{ltrap}}$ *be a family of languages. A word-independent SPHF* WI-SPHF *for these languages consists of four algorithms* (HashKG, ProjKG, Hash, ProjHash) *defined as follows. Let* λ *be a security parameter used for running the* Setup.lpar *algorithm.*

HashKG**:** *The hash key generation algorithm takes* lpar *as input, and outputs a hashing key* hk*.*

ProjKG**:** *The projection key derivation algorithm takes* hk *and* lpar *as input, and outputs a projection key* hp*.*

Hash**:** *The hash algorithm takes* hk*,* lpar*, and a word* $\chi \in \mathscr{X}_{\mathsf{lpar}}$ *as input, and outputs a hash value* $\mathsf{H} \in \{0,1\}^\nu$ *for some positive integer* $\nu = \Omega(\lambda)$*.*

ProjHash**:** *The projected hash algorithm takes* hp*,* lpar*,* χ*, and the witness* w *for the word* $\chi \in \widetilde{\mathscr{L}}_{\mathsf{lpar}}$ *as input, and outputs a projected hash value* $\mathsf{pH} \in \{0,1\}^\nu$*.*

Definition 6 (Approximate Correctness [6])**.** *For any security parameter* λ*, let* $(\mathsf{lpar}, \mathsf{ltrap}) \leftarrow \mathsf{Setup.lpar}(1^\lambda)$*. With overwhelming probability over the randomness of* Setup.lpar*, for any* $\chi \in \widetilde{\mathscr{L}}_{\mathsf{lpar}}$ *and associated witness* w*,* $\mathsf{H} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{lpar}, \chi)$ *is approximately determined by* $\mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{lpar})$ *relative to the Hamming distance* HD *where* $\Pr[\mathsf{HD}(\mathsf{Hash}(\mathsf{hk}, \mathsf{lpar}, \chi), \mathsf{ProjHash}(\mathsf{hp}, \mathsf{lpar}, \chi, w)) > \epsilon \cdot \nu] = \mathsf{negl}(\lambda)$*. Here, the probability is taken over the choice of* $\mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{lpar})$ *and the randomness of* Hash *and* ProjHash*.*[2] *Then, we say that* WI-SPHF *is approximately* ϵ*-correct. Moreover, we say that* WI-SPHF *is statistically correct if it is 0-correct.*

We remark that Benhamouda et al. first constructed a word-independent bit-PHF for languages of IND-CPA ciphertexts, where the hash value is just a bit (i.e., $\nu = 1$). Next they showed that

[2] As mentioned by Benhamouda et al., they considered probabilistic Hash and ProjHash algorithms.

a $\mathsf{WI\text{-}SPHF}$, with the output length $\{0,1\}^\nu$ where $\nu = \Omega(\lambda)$, can be constructed from a word-independent bit-PHF generically (Lemma B.4 [6]). Here, the original word-independent bit-PHF is supposed to be statistically correct (if it is approximate ϵ-correct, then the converted SPHF is not word-independent due to additional error correcting codes, even the underlying bit-PHF is word-independent). Benhamouda et al. showed how to construct a bit-PHF with statistical correctness (Lemma 4.1 [6]). Li and Wang [26] also proposed a word-independent approximate SPHF without employing error correcting codes which can also be employed in our construction.

Definition 7 (Adaptive Smoothness [6])**.** *For any security parameter λ, let $(\mathsf{lpar}, \mathsf{ltrap}) \leftarrow \mathsf{Setup.lpar}(1^\lambda)$. With overwhelming probability over the randomness of $\mathsf{Setup.lpar}$, for all functions f onto $\mathcal{X}_{\mathsf{lpar}} \setminus \mathscr{L}_{\mathsf{lpar}}$, the following two distributions have statistical distance negligible in λ: $\{(\mathsf{lpar}, f(\mathsf{hp}), \mathsf{hp}, \mathsf{H}) | \mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{lpar}), \mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{lpar}), \mathsf{H} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{lpar}, f(\mathsf{hp}))\}$ and $\{(\mathsf{lpar}, f(\mathsf{hp}), \mathsf{hp}, \mathsf{H}) | \mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{lpar}), \mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{lpar}), \mathsf{H} \xleftarrow{\$} \{0,1\}^\nu\}$.*

2.4 Public-key Encryption with Keyword Search (PEKS)

In this section, we introduce the definitions of PEKS given in [1].

Definition 8 (Syntax of PEKS [1])**.** *A PEKS scheme* PEKS *consists of four algorithms* $(\mathsf{PEKS.KG}, \mathsf{PEKS.Enc}, \mathsf{PEKS.Trapdoor}, \mathsf{PEKS.Test})$ *defined as follows.*

$\mathsf{PEKS.KG}$**:** *The key generation algorithm takes a security parameter λ as input, and outputs a public key $\mathsf{pk}_{\mathsf{PEKS}}$ and a secret key $\mathsf{sk}_{\mathsf{PEKS}}$. We assume that $\mathsf{pk}_{\mathsf{PEKS}}$ implicitly contains the keyword space $\mathcal{KS}$.*

$\mathsf{PEKS.Enc}$**:** *The keyword encryption algorithm takes $\mathsf{pk}_{\mathsf{PEKS}}$ and a keyword $kw \in \mathcal{KS}$ as input, and outputs a ciphertext $\mathsf{ct}_{\mathsf{PEKS}}$.*

$\mathsf{PEKS.Trapdoor}$**:** *The trapdoor algorithm takes $\mathsf{pk}_{\mathsf{PEKS}}$, $\mathsf{sk}_{\mathsf{PEKS}}$, and a keyword $kw \in \mathcal{KS}$ as input, and outputs a trapdoor td_{kw}.*

$\mathsf{PEKS.Test}$**:** *The test algorithm takes $\mathsf{ct}_{\mathsf{PEKS}}$ and td_{kw} as input, and outputs 1 or 0.*

In accordance with the definition given in [1], correctness and consistency are separately defined. Briefly, for a ciphertext of a keyword kw and a trapdoor of a keyword kw', the former guarantees that the $\mathsf{PEKS.Test}$ algorithm outputs 1 if $kw = kw'$, and the latter guarantees that the $\mathsf{PEKS.Test}$ algorithm outputs 0 if $kw \neq kw'$. We emphasize that we employ computational consistency, i.e., consistency holds against computationally bounded adversaries.

Definition 9 (Correctness [1])**.** *For any security parameter λ, any key pairs $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$ and any keyword $kw \in \mathcal{KS}$, let $\mathsf{ct}_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, kw)$ and $\mathsf{td}_{kw} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}, kw)$. Then $\Pr[\mathsf{PEKS.Test}(\mathsf{ct}_{\mathsf{PEKS}}, \mathsf{td}_{kw}) = 1] = 1 - \mathsf{negl}(\lambda)$ holds.*

Definition 10 (Computational Consistency [1])**.** *For all PPT adversaries $\mathcal{A}$, we define the following experiment:*

$$\begin{aligned}
&\mathsf{Exp}^{\mathsf{consist}}_{\mathsf{PEKS},\mathcal{A}}(\lambda):\\
&(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)\\
&(kw, kw') \leftarrow \mathcal{A}(\mathsf{pk}_{\mathsf{PEKS}}) \ \textit{s.t.}\ kw, kw' \in \mathcal{KS} \wedge kw \neq kw'\\
&\mathsf{ct}_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, kw)\\
&\mathsf{td}_{kw'} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}, kw')\\
&\textit{If}\ \mathsf{PEKS.Test}(\mathsf{ct}_{\mathsf{PEKS}}, \mathsf{td}_{kw'}) = 1\ \textit{then output}\ 1\ \textit{and}\ 0\ \textit{otherwise.}
\end{aligned}$$

We say that a PEKS scheme PEKS *is consistent if the advantage*

$$\mathsf{Adv}^{\mathsf{consist}}_{\mathsf{PEKS},\mathcal{A}}(\lambda) := \Pr[\mathsf{Exp}^{\mathsf{consist}}_{\mathsf{PEKS},\mathcal{A}}(\lambda) = 1]$$

is negligible in the security parameter λ.

Next, we define ciphertext indistinguishability against chosen keyword attack (IND-CKA)[3] which guarantees that no information of keyword is leaked from ciphertexts.

Definition 11 (IND-CKA [1]). *For all PPT adversaries* $\mathcal{A}$*, we define the following experiment:*

$$\begin{aligned}
&\mathsf{Exp}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PEKS},\mathcal{A}}(\lambda):\\
&(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)\\
&(kw_0^*, kw_1^*, \mathsf{state}) \leftarrow \mathcal{A}^{\mathcal{O}_T(\mathsf{pk}_{\mathsf{PEKS}},\mathsf{sk}_{\mathsf{PEKS}},\cdot)}(\mathsf{pk}_{\mathsf{PEKS}}) \ \textit{s.t.}\ kw_0^*, kw_1^* \in \mathcal{KS} \wedge kw_0^* \neq kw_1^*\\
&b \overset{\$}{\leftarrow} \{0,1\};\ \mathsf{ct}^*_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, kw_b^*)\\
&b' \leftarrow \mathcal{A}^{\mathcal{O}_T(\mathsf{pk}_{\mathsf{PEKS}},\mathsf{sk}_{\mathsf{PEKS}},\cdot)}(\mathsf{state}, \mathsf{ct}^*_{\mathsf{PEKS}})\\
&\textit{If } b = b' \textit{ then output } 1 \textit{ and } 0 \textit{ otherwise.}
\end{aligned}$$

$\mathcal{O}_T$ *takes* $kw \in \mathcal{KS}$ *as input, and returns* $\mathsf{td}_{kw} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}, kw)$*. Here* $kw \notin \{kw_0^*, kw_1^*\}$*. We say that a PEKS scheme* PEKS *is IND-CKA secure if the advantage*

$$\mathsf{Adv}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PEKS},\mathcal{A}}(\lambda) := |\Pr[\mathsf{Exp}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PEKS},\mathcal{A}}(\lambda) = 1] - 1/2| < \mathsf{negl}(\lambda)$$

is negligible in the security parameter λ.

We emphasize that PEKS does not provide trapdoor privacy, i.e., information of kw is leaked from td_{kw}. Actually, for some trapdoor td_{kw}, anyone can compute $\mathsf{ct}_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, kw')$ for any $kw' \in \mathcal{KS}$, and then anyone can check whether $kw = kw'$ or not by running $\mathsf{PEKS.Test}(\mathsf{ct}_{\mathsf{PEKS}}, \mathsf{td}_{kw})$.

3 Definitions of Designated-Receiver Multi-Sender PAEKS

3.1 Designated-Receiver Setting

In the previous definition [10, 12, 22, 29, 31–34], a setup algorithm is defined that takes a security parameter as input, and outputs a public parameter pp. Then, two key generation algorithms, $\mathsf{PAEKS.KG_R}$ and $\mathsf{PAEKS.KG_S}$, are defined which take as input pp, and output a key pair, respectively. In our definition, the sender key generation algorithm $\mathsf{PAEKS.KG_S}$ takes a receiver public key $\mathsf{pk_R}$ as input, i.e., our definition captures a designated-receiver setting.

One may think that this setting restricts the flexibility of key generations. However, a sender needs to designate a receiver before the sender encrypts a keyword. Then, due to the functionality of PAEKS, the sender needs to use its own secret key $\mathsf{sk_S}$. Since the sender has designated the receiver, the designated-receiver key generation does not restrict encryption. Of course, the designated-receiver setting restricts the order of searching, i.e., no trapdoor can be generated before the corresponding sender's public key is generated. Beyond this restriction, there is a merit of the designated-receiver setting that can avoid introducing a trusted setup. It is required in the Liu et al. construction [29] that no one knows a decryption key $\mathsf{dk_{PKE}}$ of the underlying PKE scheme because $\mathsf{dk_{PKE}} = \mathsf{Itrap}$ can be used to break the underlying membership problem. Thus, the setup

[3] In [1], this security notion is called PEKS-IND-CPA.

algorithm outputs $\mathsf{pk}_{\mathsf{PKE}}$ only. Liu et al. introduced a hash function that generates $\mathsf{pk}_{\mathsf{PKE}}$ (it seems to guarantee that there is no corresponding decryption key). However, how to construct such a hash function is unclear (we will discuss it in detail in Section 4.2). In the designated-receiver setting, the $\mathsf{PAEKS.KG_R}$ algorithm simply runs $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$ and $\mathsf{dk}_{\mathsf{PKE}}$ can be regarded as a secret key of the receiver. Then, we do not have to consider how to erase $\mathsf{dk}_{\mathsf{PKE}}$ anymore (we will show that giving $\mathsf{dk}_{\mathsf{PKE}}$ to a receiver does not affect security). In other words, if we assume a trusted setup, where a setup algorithm runs $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$, erase $\mathsf{dk}_{\mathsf{PKE}}$ and outputs $\mathsf{pk}_{\mathsf{PKE}}$ only as a public parameter, we do not have to introduce the designated-receiver setting. However, it would be better to clarify who can know $\mathsf{dk}_{\mathsf{PKE}}$. In this sense, our setting is more desirable.

3.2 Definitions of PAEKS

Owing to the functionality of PAEKS, a trapdoor is associated with not only a keyword kw but also a sender by indicating the sender's public key $\mathsf{pk_S}$. Thus, we explicitly denote a trapdoor $\mathsf{td}_{\mathsf{S},kw}$. Because we consider multiple senders, the $\mathsf{PAEKS.KG_S}$ algorithm is run by each sender. For simplicity, we assume that there are n senders, and we denote the i-th sender's key pair as $(\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]})$ where $i \in [1, n]$ is the sender index, and denote $\mathsf{td}_{\mathsf{S}[i],kw}$ as a trapdoor generated by indicating $\mathsf{pk}_{\mathsf{S}[i]}$ and kw. We use $(\mathsf{pk_S}, \mathsf{sk_S})$ and $\mathsf{td}_{\mathsf{S},kw}$ when no sender index is explicitly appeared.

Definition 12 (Syntax of Designated-Receiver Multi-Sender PAEKS). *A PAEKS scheme* PAEKS *consists of five algorithms* $(\mathsf{PAEKS.KG_R}, \mathsf{PAEKS.KG_S}, \mathsf{PAEKS.Enc}, \mathsf{PAEKS.Trapdoor}, \mathsf{PAEKS.Test})$ *defined as follows.*

$\mathsf{PAEKS.KG_R}$**:** *The receiver key generation algorithm takes a security parameter* λ *as input, and outputs a public key* $\mathsf{pk_R}$ *and a secret key* $\mathsf{sk_R}$*. We assume that* $\mathsf{pk_R}$ *implicitly contains the keyword space* $\mathcal{KS}$*.*

$\mathsf{PAEKS.KG_S}$**:** *The sender key generation algorithm takes* $\mathsf{pk_R}$ *as input, and outputs a public key* $\mathsf{pk_S}$ *and a secret key* $\mathsf{sk_S}$*.*

$\mathsf{PAEKS.Enc}$**:** *The keyword encryption algorithm takes* $\mathsf{pk_R}$*,* $\mathsf{pk_S}$*,* $\mathsf{sk_S}$*, and a keyword* $kw \in \mathcal{KS}$ *as input, and outputs a ciphertext* $\mathsf{ct}_{\mathsf{PAEKS}}$*.*

$\mathsf{PAEKS.Trapdoor}$**:** *The trapdoor algorithm takes* $\mathsf{pk_R}$*,* $\mathsf{pk_S}$*,* $\mathsf{sk_R}$*, and a keyword* $kw \in \mathcal{KS}$ *as input, and outputs a trapdoor* $\mathsf{td}_{\mathsf{S},kw}$*.*

$\mathsf{PAEKS.Test}$**:** *The test algorithm takes* $\mathsf{ct}_{\mathsf{PAEKS}}$ *and* $\mathsf{td}_{\mathsf{S},kw}$ *as input, and outputs 1 or 0.*

Definition 13 (Correctness). *For any security parameter* λ*, any key pairs* $(\mathsf{pk_R}, \mathsf{sk_R}) \leftarrow \mathsf{PAEKS.KG_R}(1^\lambda)$ *and* $(\mathsf{pk_S}, \mathsf{sk_S}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$*, and any keyword* $kw \in \mathcal{KS}$*, let* $\mathsf{ct}_{\mathsf{PAEKS}} \leftarrow \mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk_S}, \mathsf{sk_S}, kw)$ *and* $\mathsf{td}_{\mathsf{S},kw} \leftarrow \mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk_S}, \mathsf{sk_R}, kw)$*. Then* $\Pr[\mathsf{PAEKS.Test}(\mathsf{ct}_{\mathsf{PAEKS}}, \mathsf{td}_{\mathsf{S},kw}) = 1] = 1 - \mathsf{negl}(\lambda)$ *holds.*

Next, we define consistency. Previous works employ the following definition: as in the same setting of the correctness, except for two keywords $kw, kw' \in \mathcal{KS}$,

$$\Pr[\mathsf{PAEKS.Test}(\mathsf{ct}_{\mathsf{PAEKS}}, \mathsf{td}_{\mathsf{S},kw'}) = 0] = 1 - \mathsf{negl}(\lambda)$$

holds where $\mathsf{ct}_{\mathsf{PAEKS}}$ is a ciphertext of kw, $\mathsf{td}_{\mathsf{S},kw'}$ is a trapdoor of kw', and $kw \neq kw'$. Before giving our definition, we point out two problems of this previous definition.

1. It captures statistical consistency where it requires to hold even against computationally unbounded adversaries. However, the previous schemes do not satisfy this definition because they used a collision-resistant hash function for providing consistency (i.e., if $kw \neq kw'$, then its hash values are supposed to be different). That is, there "exists" kw and kw' such that its hash values are the same but $kw \neq kw'$ and computationally unbounded adversaries can find them. Thus, we need to consider computationally bounded adversaries. Of course, there is a room for providing statistical consistency as in a PEKS scheme given by Abdalla et al. [1]. We remark that the construction methodology of the statistically consistent Abdalla et al. PEKS scheme completely contradicts trapdoor privacy because a keyword itself is contained in a trapdoor (this setting does not contradict to provide IND-CKA). Thus it seems difficult to construct a statistically consistent PAEKS scheme with trapdoor privacy (though we do not exclude the possibility). Thus, in this paper we define computational consistency that considers PPT adversaries.

2. The previous definitions only considered keywords, however, a trapdoor is associated with not only a keyword kw but also a sender by indicating the sender's public key $\mathsf{pk_S}$. Thus, for $\mathsf{ct_{PAEKS}} \leftarrow \mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk_{S[0]}}, \mathsf{sk_{S[0]}}, kw)$ and $\mathsf{td}_{\mathsf{S}[1],kw'} \leftarrow \mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk_{S[1]}}, \mathsf{sk_R}, kw')$, $\mathsf{PAEKS.Test}(\mathsf{ct_{PAEKS}}, \mathsf{td}_{\mathsf{S}[1],kw'}) = 0$ should hold even if $kw = kw'$. This case is not captured in the previous definition. We consider this case by setting $(kw, i) \neq (kw', j)$ in the experiment.

Definition 14 (Computational Consistency)**.** *For all PPT adversaries $\mathcal{A}$, we define the following experiment:*

$\mathsf{Exp}^{\mathsf{consist}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda)$:
$(\mathsf{pk_R}, \mathsf{sk_R}) \leftarrow \mathsf{PAEKS.KG_R}(1^\lambda)$
$(\mathsf{pk_{S[0]}}, \mathsf{sk_{S[0]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$; $(\mathsf{pk_{S[1]}}, \mathsf{sk_{S[1]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$
$(kw, kw', i, j) \leftarrow \mathcal{A}(\mathsf{pk_R}, \mathsf{pk_{S[0]}}, \mathsf{pk_{S[1]}})$ *s.t.* $kw, kw' \in \mathcal{KS} \wedge i, j \in \{0,1\} \wedge (kw, i) \neq (kw', j)$
$\mathsf{ct_{PAEKS}} \leftarrow \mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}, kw)$; $\mathsf{td}_{\mathsf{S}[j],kw'} \leftarrow \mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[j]}, \mathsf{sk_R}, kw')$
If $\mathsf{PAEKS.Test}(\mathsf{ct_{PAEKS}}, \mathsf{td}_{\mathsf{S}[j],kw'}) = 1$ *then output* 1 *and* 0 *otherwise.*

We say that a PAEKS scheme PAEKS *is consistent if the advantage*

$$\mathsf{Adv}^{\mathsf{consist}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda) := \Pr[\mathsf{Exp}^{\mathsf{consist}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda) = 1]$$

is negligible in the security parameter λ.

Next, we define two indistinguishability notions. First, we define ciphertext privacy by formalizing indistinguishability against chosen keyword attack (IND-CKA) which guarantees that no information of keyword is leaked from ciphertexts (as in PEKS). In the previous IND-CKA definition, two oracles are defined, an encryption oracle $\mathcal{O}_C$ and a trapdoor oracle $\mathcal{O}_T$. Then, two keywords, kw_0^* and kw_1^* are chosen by the adversary $\mathcal{A}$, and the challenge ciphertext $\mathsf{ct}^*_{\mathsf{PAEKS}}$ is computed by either kw_0^* or kw_1^*. Here, we need to consider what values can be input to these oracles. To exclude the trivial case, we need to restrict $\mathcal{A}$ to obtain a trapdoor that can be used for distinguishing between kw_0^* and kw_1^*. In other words, $\mathcal{A}$ should be allowed to obtain any trapdoor excluding the above case, e.g., $\mathsf{td}_{\mathsf{S},kw_0^*} \leftarrow \mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk_S}, \mathsf{sk_R}, kw_0^*)$ if $\mathsf{sk_S}$ is not used for generating the challenge ciphertext. In almost previous definition, $\mathcal{A}$ is not allowed to input kw_0^* and kw_1^* to $\mathcal{O}_C$ and $\mathcal{O}_T$. Qin et al. [34] have improved this restriction where kw_0^* and kw_1^* can be inputs. They call this security notion fully CI-security (Cipher-keyword Indistinguishability). For $\mathcal{O}_T$, the receiver is fixed and it matches the multi-sender setting. For $\mathcal{O}_C$, the sender is fixed, and it does not consider the multi-sender setting. Thus, we modify the definition of $\mathcal{O}_C$ following the multi-sender setting. Since $\mathcal{O}_C$ has no restriction, our definition implies fully CI-security.

Definition 15 (IND-CKA)**.** *For all PPT adversaries $\mathcal{A}$, we define the following experiment:*

$$
\begin{aligned}
&\mathsf{Exp}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n):\\
&\ (\mathsf{pk_R}, \mathsf{sk_R}) \leftarrow \mathsf{PAEKS.KG_R}(1^\lambda)\\
&\ \textit{For } i \in [1,n],\ (\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})\\
&\ (kw_0^*, kw_1^*, i^*, \mathsf{state}) \leftarrow \mathcal{A}^{\mathcal{O}_C(\mathsf{pk_R},\cdot,\cdot),\mathcal{O}_T(\mathsf{pk_R},\cdot,\mathsf{sk_R},\cdot)}(\mathsf{pk_R}, \{\mathsf{pk}_{\mathsf{S}[i]}\}_{i\in[1,n]})\\
&\quad s.t.\ kw_0^*, kw_1^* \in \mathcal{KS} \wedge\ kw_0^* \neq kw_1^* \wedge i^* \in [1,n]\\
&\ b \xleftarrow{\$} \{0,1\};\ \mathsf{ct}^*_{\mathsf{PAEKS}} \leftarrow \mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[i^*]}, \mathsf{sk}_{\mathsf{S}[i^*]}, kw_b^*)\\
&\ b' \leftarrow \mathcal{A}^{\mathcal{O}_C(\mathsf{pk_R},\cdot,\cdot),\mathcal{O}_T(\mathsf{pk_R},\cdot,\mathsf{sk_R},\cdot)}(\mathsf{state}, \mathsf{ct}^*_{\mathsf{PAEKS}})\\
&\ \textit{If } b = b' \textit{ then output } 1 \textit{ and } 0 \textit{ otherwise.}
\end{aligned}
$$

$\mathcal{O}_C$ takes $kw \in \mathcal{KS}$ and $i \in [1,n]$ as input, and returns the result of $\mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}, kw)$. Here, there is no restriction. $\mathcal{O}_T$ takes $kw \in \mathcal{KS}$ and $i \in [1,n]$ as input, and returns the result of $\mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk_R}, kw)$. Here $(kw, i) \notin \{(kw_0^*, i^*), (kw_1^*, i^*)\}$. We say that a PAEKS scheme PAEKS is IND-CKA secure if the advantage

$$\mathsf{Adv}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n) := |\Pr[\mathsf{Exp}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n) = 1] - 1/2|$$

is negligible in the security parameter λ.

Next, we define trapdoor privacy by formalizing indistinguishability against inside keyword guessing attack (IND-IKGA) which guarantees that no information of keyword is leaked from trapdoors. To exclude the trivial case, we need to restrict that $\mathcal{A}$ obtains a ciphertext that can be used for distinguishing kw_0^* or kw_1^*. Again we revisit Qin et al.'s definition [34]. For $\mathcal{O}_T$, the receiver is fixed, and it matches the multi-sender setting. For $\mathcal{O}_C$ the sender is fixed and it does not capture the multi-sender setting. So, we modify the definition of $\mathcal{O}_C$ in accordance with the multi-sender setting. Cheng and Meng [11] introduced an IND-IKGA notion where $\mathcal{O}_T$ has no restriction, i.e., an adversary is allowed to obtain a trapdoor for $(kw, i) \in \{(kw_0^*, i^*), (kw_1^*, i^*)\}$. They call this security notion fully TI-security (Trapdoor Indistinguishability). Since $\mathcal{O}_T$ has a restriction $(kw, i) \notin \{(kw_0^*, i^*), (kw_1^*, i^*)\}$ in our definition, our IND-IKGA does not imply fully TI-security.

Definition 16 (IND-IKGA)**.** *For all PPT adversaries $\mathcal{A}$, we define the following experiment:*

$$
\begin{aligned}
&\mathsf{Exp}^{\mathsf{IND\text{-}IKGA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n):\\
&\ (\mathsf{pk_R}, \mathsf{sk_R}) \leftarrow \mathsf{PAEKS.KG_R}(1^\lambda)\\
&\ \textit{For } i \in [1,n],\ (\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})\\
&\ (kw_0^*, kw_1^*, i^*, \mathsf{state})\\
&\quad \leftarrow \mathcal{A}^{\mathcal{O}_C(\mathsf{pk_R},\cdot,\cdot),\mathcal{O}_T(\mathsf{pk_R},\cdot,\mathsf{sk_R},\cdot)}(\mathsf{pk_R}, \{\mathsf{pk}_{\mathsf{S}[i]}\}_{i\in[1,n]})\ s.t.\ kw_0^*, kw_1^* \in \mathcal{KS} \wedge\ kw_0^* \neq kw_1^* \wedge i^* \in [1,n]\\
&\ b \xleftarrow{\$} \{0,1\}\\
&\ \mathsf{td}^*_{\mathsf{S}[i^*],kw_b^*} \leftarrow \mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk}_{\mathsf{S}[i^*]}, \mathsf{sk_R}, kw_b^*)\\
&\ b' \leftarrow \mathcal{A}^{\mathcal{O}_C(\mathsf{pk_R},\cdot,\cdot),\mathcal{O}_T(\mathsf{pk_R},\cdot,\mathsf{sk_R},\cdot)}(\mathsf{state}, \mathsf{td}^*_{\mathsf{S}[i^*],kw_b^*})\\
&\ \textit{If } b = b' \textit{ then output } 1 \textit{ and } 0 \textit{ otherwise.}
\end{aligned}
$$

$\mathcal{O}_C$ takes $kw \in \mathcal{KS}$ and $i \in [1,n]$ as input, and returns the result of $\mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_{S[\mathit{i}]}}, kw)$. *Here, $(kw, i) \notin \{(kw_0^*, i^*), (kw_1^*, i^*)\}$. $\mathcal{O}_T$ takes $kw \in \mathcal{KS}$ and $i \in [1,n]$ as input, and returns the result of* $\mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_R}, kw)$. *Here $(kw, i) \notin \{(kw_0^*, i^*), (kw_1^*, i^*)\}$. We say that a PAEKS scheme* PAEKS *is IND-IKGA secure if the advantage*

$$\mathsf{Adv}^{\mathsf{IND\text{-}IKGA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n) := |\Pr[\mathsf{Exp}^{\mathsf{IND\text{-}IKGA}}_{\mathsf{PAEKS},\mathcal{A}}(\lambda, n) = 1] - 1/2|$$

is negligible in the security parameter λ.

3.3 Relation among our definitions, fully CI/TI-security, and multi-ciphertext/multi-trapdoor indistinguishability

As mentioned before, our IND-CKA definition implies fully CI-security since challenge keywords can be input to the encryption oracle. On the contrary, our IND-IKGA definition does not imply fully TI-security. That is, if $\mathcal{A}$ is allowed to send either (kw_0^*, i^*) or (kw_1^*, i^*) to the trapdoor oracle (which is allowed in the fully TI-security model), two trapdoors may be linked, i.e., whether these trapdoors are for the same keyword can be checked.

Qin et al. [33] considered multi-ciphertext indistinguishability (MCI) where in the IND-CKA experiment $\mathcal{A}$ declares two keyword vectors $(kw_{0,1}^*, \dots, kw_{0,N}^*)$ and $(kw_{1,1}^*, \dots, kw_{1,N}^*)$ for some N, and the challenger returns the challenge ciphertexts of $kw_{b,i}^*$ for $i \in [1, N]$. As mentioned in [34], if the encryption oracle $\mathcal{O}_C$ has no restriction (i.e., any input is allowed), then IND-CKA implies MCI. Thus, our IND-CKA definition provides MCI security.

Similarly, Pan and Li [32] considered multi-trapdoor indistinguishability (MTI) where in the IND-IKGA experiment $\mathcal{A}$ declares two keyword vectors $(kw_{0,1}^*, \dots, kw_{0,N}^*)$ and $(kw_{1,1}^*, \dots, kw_{1,N}^*)$ for some N, and the challenger returns the challenge trapdoors of $kw_{b,i}^*$ for $i \in [1, N]$. Our IND-IKGA definition does not directly imply MTI since it does not imply fully TI-security. As mentioned by Qin et al. [33], a trapdoor generation algorithm must be probabilistic to provide MTI. Because Benhamouda et al. [6] introduced a probabilistic rounding function, our construction could provide fully TI-security and MTI. However, adaptive smoothness guarantees that when $(\mathsf{lpar}, \chi, \mathsf{hp})$ is fixed, $\mathsf{H} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{lpar}, f(\mathsf{hp}))$ is statistically close to uniform over $\{0,1\}^\nu$, but it does not directly guarantee unlinkability of two hash values, i.e., information whether two hash values are computed by the same input or not might be leaked. We leave how to provide fully TI-security and MTI in a generic construction as a future work.

4 Analysis of Liu et al. Generic Construction

In this section, we analyze the Liu et al. generic construction [29] from the viewpoint of security, security models, efficiency, and instantiability of the generic construction.

4.1 Core Idea of Liu et al. Construction

First, we revisit the Liu et al. construction methodology as follows. Basically, they employed the Katz-Vaikuntanathan password-based authenticated key exchange (PAKE) construction [25] which is explained as follows. Let $\mathsf{pk_{PKE}}$ be a common public key where no one knows the corresponding $\mathsf{dk_{PKE}}$. A client (a sender in the PAEKS context) setups $\mathsf{hk}_c \leftarrow \mathsf{HashKG}(\mathsf{pk_{PKE}})$ and $\mathsf{hp}_c \leftarrow \mathsf{ProjKG}(\mathsf{hk}_c, \mathsf{pk_{PKE}})$, and a server (a receiver in the PAEKS context) setups $\mathsf{hk}_s \leftarrow \mathsf{HashKG}(\mathsf{pk_{PKE}})$ and $\mathsf{hp}_s \leftarrow \mathsf{ProjKG}(\mathsf{hk}_s, \mathsf{pk_{PKE}})$, respectively. Here, hp_c and hp_s are public keys and hk_c and hk_s are secret keys, respectively. The client generates an encryption C of the password pw using $\mathsf{pk_{PKE}}$, and the server also generates an encryption C' of the password pw using

$\mathsf{pk}_{\mathsf{PKE}}$. Then, the shared key is $\mathsf{shard\text{-}key} := \mathsf{Hash}(\mathsf{hk}_c, \mathsf{pk}_{\mathsf{PKE}}, C') \oplus \mathsf{ProjHash}(\mathsf{hp}_s, \mathsf{pk}_{\mathsf{PKE}}, C, pw) = \mathsf{Hash}(\mathsf{hk}_s, \mathsf{pk}_{\mathsf{PKE}}, C) \oplus \mathsf{ProjHash}(\mathsf{hp}_c, \mathsf{pk}_{\mathsf{PKE}}, C', pw)$. The left-side of the equation can be computed by the client using its secret key hk_c and the witness pw, and the right-side of the equation can be computed by the server using its secret key hk_s and the witness pw. The equation holds owing to the (approximate) correctness of SPHF. Moreover, thanks to the word-independency, projected keys, hp_c and hp_s, can be public keys before seeing words (ciphertexts).

The basic idea of the Liu et al. PAEKS construction is to prepare an extended keyword $\mathsf{der}\text{-}kw$ from a keyword kw by $\mathsf{der}\text{-}kw \leftarrow \mathsf{HF}(kw, \mathsf{shard\text{-}key})$ where HF is a secure hash function (we intentionally use "secure hash function" in accordance with the Liu et al. description). Then, a ciphertext of $\mathsf{der}\text{-}kw$ is computed by the $\mathsf{PEKS.Enc}$ algorithm, and a trapdoor of $\mathsf{der}\text{-}kw$ is computed by the $\mathsf{PEKS.Trapdoor}$ algorithm.

4.2 Issues of the Liu et al. Construction

Security. Here, we highlight two issues regarding the underlying hash function to derive an extended keyword $\mathsf{der}\text{-}kw \leftarrow \mathsf{HF}(kw, \mathsf{shard\text{-}key})$. We note that they claimed that their construction is secure in the standard model. That is, HF is not modeled as a random oracle. Liu et al. required that if $kw \neq kw'$, then $\mathsf{der}\text{-}kw \neq \mathsf{der}\text{-}kw'$. Thus, they implicitly assumed that HF is collision resistant. However, there "exists" kw and kw' such that $kw \neq kw'$ and $\mathsf{der}\text{-}kw = \mathsf{der}\text{-}kw'$. Of course we can assume that no PPT adversary can find them, but obviously the construction does not provide statistical consistency. The first issue can be easily fixed by introducing computational consistency. However, the second issue is more important. Since a PAEKS trapdoor for kw is a PEKS trapdoor for $\mathsf{der}\text{-}kw$, information of $\mathsf{der}\text{-}kw$ is leaked from the trapdoor (because PEKS does not provide trapdoor privacy). They required that if $\mathsf{shard\text{-}key}$ is random, then $\mathsf{der}\text{-}kw$ is also random, and assumed that a hash value does not leak input information. However, since HF is not modeled as a random oracle, this claim does not hold even if HF provides one-wayness. These issues seem to be fixed by assuming that HF is a random oracle.

Regarding a hash function, we highlight that their setup algorithm is also problematic. The setup algorithm runs $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$, chooses a plaintext $M_{\mathsf{PKE}} \in \mathcal{MS}$, and generates a common public key $\mathsf{mpk} \leftarrow \mathsf{HF}(\mathsf{pk}_{\mathsf{PKE}}, M_{\mathsf{PKE}})$ where HF is a secure hash function (here we omit label from the input). Then, mpk is set as lpar of the underlying SPHF, and it is required that no one knows the corresponding decryption key. First, since $\mathsf{Setup.lpar} = \mathsf{PKE.KeyGen}$, it is not directly guaranteed that mpk works as lpar. Even if mpk is identical from a public key generated by the $\mathsf{PKE.KeyGen}$ algorithm, second, how to switch a ciphertext (a word in the SPHF context) from $\widetilde{\mathscr{L}}_{\mathsf{lpar}}$ to $\mathcal{X}_{\mathsf{lpar}} \setminus \mathscr{L}_{\mathsf{lpar}}$ in the security proof is unclear. This step is mandatory to utilize smoothness.[4] Owing to the IND-CPA security of PKE, this can be switched (as in our security proof); however, to do so, mpk must be generated by the challenger of the PKE scheme and set as the common public key. Since mpk is a hash value in the Liu et al. construction, how to set mpk as the hash value must be considered. This issue can be solved by assuming that HF is a random oracle (i.e., using the programmability of the random oracle). Alternatively, specifying $\mathsf{pk}_{\mathsf{PKE}}$ as a common public key is sufficient (even though the corresponding decryption key needs to be erased, which requires a trusted setup).

Security Model. In the definition of consistency (besides statistical or computational), they did not consider the case "$kw = kw'$ and senders are different", which is considered in our consistency

[4] Precisely, Liu et al. did not directly employ smoothness. They assumed that a hash value H of SPHF for a word $\chi \in \widetilde{\mathscr{L}}_{\mathsf{lpar}}$ is random (they called it pseudo-randomness), and they switched H to be random without switching the word from $\widetilde{\mathscr{L}}_{\mathsf{lpar}}$ to $\mathcal{X}_{\mathsf{lpar}} \setminus \mathscr{L}_{\mathsf{lpar}}$. Even if this argument is correct, still $\mathsf{mpk} = \mathsf{lpar}$ needs to be set as a hash value $\mathsf{mpk} \leftarrow \mathsf{HF}(\mathsf{pk}_{\mathsf{PKE}}, M_{\mathsf{PKE}})$.

definition. Since a trapdoor is associated with not only a keyword but also a sender, considering this case is important. In addition, regarding IND-CKA and IND-IKGA, they used a weak model, where an adversary $\mathcal{A}$ is not allowed to send challenge keywords to two oracles: an encryption oracle $\mathcal{O}_C$ and a trapdoor oracle $\mathcal{O}_T$.

Liu et al. also claimed that their construction provides multi-trapdoor indistinguishability (MTI) if the trapdoor algorithm of the underlying PEKS scheme is probabilistic (Theorem 5.3. [29]). Besides the probabilistic algorithm, they implicitly assumed that two trapdoors are unlinkable, i.e., it hides whether two trapdoors are generated for the same keyword or not. Then, for simulating MTI game in the proof of Theorem 3.3. [29], a simulator just responses random values as the challenge trapdoors. However, even if the trapdoor algorithm is probabilistic, it does not directly guarantee the unlinkability.

Efficiency. They employed two SPHFs as in the Katz-Vaikuntanathan PAKE construction. In PAKE, this is mandatory because both a client and a server need to show the possession of witness (password). However, this is not the case in PAEKS: a receiver does not have to prove the possession of witness. In our construction, a sender is required to show the possession of witness (randomness of a ciphertext), and a receiver just uses a hash key for computing the hash value. This improves the efficiency of the construction.

Instantiability. Finally, we discuss the instantiability of the Liu et al. generic construction (besides hash functions discussed above). They employed a labeled IND-CCA2 PKE scheme that defines languages of ciphertexts of SPHF. However, Benhamouda et al. [6] proposed a $\mathsf{WI\text{-}SPHF}$ for a IND-CPA PKE scheme, and mentioned that their SPHF construction for the IND-CCA2 PKE scheme is not word-independent. Moreover, Liu et al. employed an IND-CCA1 PKE scheme in their implementation. Although the Liu et al. construction may work well since IND-CCA2 implies IND-CPA, however, it is not clear whether the underlying SPHF is word-independent (at least in their implementation). In the next section, we show that the underlying PKE is required to be IND-CPA secure, and no CCA2/CCA1 security is required.

We also remark that one of the main goals of Liu et al. was to construct a post-quantum PAEKS scheme by instantiating their generic construction from lattices. Thus, even if the above-mentioned issues regarding hash functions can be solved by introducing random oracles, it would be better to determine whether these issues can be solved, even in the quantum random oracle model because there exists a scheme which is secure in the random oracle model but is not secure in the quantum random oracle model [39].

5 Our PAEKS Construction

5.1 Proposed Generic Construction

High-level Description. We also employ the methodology of Katz-Vaikuntanathan PAKE construction. As an important difference from the Liu et al. construction, we introduce one SPHF where a receiver publishes a projected key hp, and a sender generates a ciphertext $\mathsf{ct}_{\mathsf{PKE}}$ of 0 and sets it as a public key $\mathsf{pk}_{\mathsf{S}} = \mathsf{ct}_{\mathsf{PKE}}$. Its randomness ρ_S is set as a secret key sk_{S}. For running the $\mathsf{ProjHash}$ algorithm in the $\mathsf{PAEKS.Enc}$ algorithm, $\mathsf{sk}_{\mathsf{S}} = \rho_S$ is used as the witness. That is, for generating a ciphertext of a keyword, a secret key $\mathsf{sk}_{\mathsf{S}} = \rho_S$ is required, whereas for generating a trapdoor, a public key $\mathsf{pk}_{\mathsf{S}} = \mathsf{ct}_{\mathsf{PKE}}$ is enough in addition to sk_{R}. This setting matches the syntax of PAEKS. Since a word $\mathsf{ct}_{\mathsf{PKE}}$ is generated after seeing hp, the underlying SPHF needs to be word-independent. Here, the public key of PKE $\mathsf{pk}_{\mathsf{PKE}}$ is generated by the receiver, and the sender needs to know $\mathsf{pk}_{\mathsf{PKE}}$ for running the $\mathsf{PAEKS.KG}_{\mathsf{S}}$ algorithm. Thus, our construction employs a designated-receiver setting. For deriving an extended keyword $\mathsf{der}\text{-}kw$, we employ a PRF where a PRF key is a hash value of SPHF. Intuitively, after utilizing adaptive smoothness, a hash value of SPHF is random. Then, owing to the pseudo-randomness of PRF, $\mathsf{der}\text{-}kw$ is also random. For extended keywords, we employ a PEKS scheme. As in the usual PEKS setting, the receiver runs $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$, and $\mathsf{sk}_{\mathsf{PEKS}}$ is used for generating a trapdoor for $\mathsf{der}\text{-}kw$.

Let $\mathsf{PRF} = \{F_K : \mathcal{KS} \rightarrow \mathcal{KS}\}$ be a family of PRFs where $K \in \{0,1\}^\nu$, $\mathsf{PEKS} = (\mathsf{PEKS.KG}, \mathsf{PEKS.Enc}, \mathsf{PEKS.Trapdoor}, \mathsf{PEKS.Test})$ be a PEKS scheme with a keyword space $\mathcal{KS}$, $\mathsf{PKE} = (\mathsf{PKE.KeyGen}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$ be a PKE scheme, and $\mathsf{WI\text{-}SPHF} = (\mathsf{HashKG}, \mathsf{ProjKG}, \mathsf{Hash}, \mathsf{ProjHash})$ be a $\mathsf{WI\text{-}SPHF}$ with the output length $\{0,1\}^\nu$. We remark that, because we employ a hash value of SPHF as a PRF key, we assume that $K \in \{0,1\}^\nu$, and because a PRF takes a keyword as input and outputs an extended keyword, we assume that $F_K : \mathcal{KS} \rightarrow \mathcal{KS}$. We also assume that $2^{-\nu}$ is negligible in the security parameter λ to guarantee that for randomly chosen two values $\mathsf{H}, \mathsf{H}' \xleftarrow{\$} \{0,1\}^\nu$, $\mathsf{H} \neq \mathsf{H}'$ holds with overwhelming probability.

Proposed Generic Construction

$\mathsf{PAEKS.KG_R}(1^\lambda)$**:** Run $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$, $\mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{pk}_{\mathsf{PKE}})$, $\mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}})$, and $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$. Set $\mathsf{pk_R} = (\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{pk}_{\mathsf{PEKS}})$ and $\mathsf{sk_R} = (\mathsf{hk}, \mathsf{dk}_{\mathsf{PKE}}, \mathsf{sk}_{\mathsf{PEKS}})$, and output $(\mathsf{pk_R}, \mathsf{sk_R})$.

$\mathsf{PAEKS.KG_S}(\mathsf{pk_R})$**:** Parse $\mathsf{pk_R} = (\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{pk}_{\mathsf{PEKS}})$. Run $\mathsf{ct}_{\mathsf{PKE}} = \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho_S)$, set $\mathsf{pk_S} = \mathsf{ct}_{\mathsf{PKE}}$ and $\mathsf{sk_S} = \rho_S$, and output $(\mathsf{pk_S}, \mathsf{sk_S})$. If we explicitly describe the sender index i, then we denote $\mathsf{pk}_{\mathsf{S}[i]} = \mathsf{ct}^{(i)}_{\mathsf{PKE}}$ and $\mathsf{sk}_{\mathsf{S}[i]} = \rho^{(i)}_S$.

$\mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk_S}, \mathsf{sk_S}, kw)$**:** Parse $\mathsf{pk_R} = (\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{pk}_{\mathsf{PEKS}})$, $\mathsf{pk_S} = \mathsf{ct}_{\mathsf{PKE}}$, and $\mathsf{sk_S} = \rho_S$. Compute $\mathsf{pH} \leftarrow \mathsf{ProjHash}(\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}}, \rho_S)$ and run $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{pH}}(kw)$. Compute $\mathsf{ct}_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{der}\text{-}kw)$, set $\mathsf{ct}_{\mathsf{PAEKS}} = \mathsf{ct}_{\mathsf{PEKS}}$, and output $\mathsf{ct}_{\mathsf{PAEKS}}$.

$\mathsf{PAEKS.Trapdoor}(\mathsf{pk_R}, \mathsf{pk_S}, \mathsf{sk_R}, kw)$**:** Parse $\mathsf{pk_R} = (\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{pk}_{\mathsf{PEKS}})$, $\mathsf{pk_S} = \mathsf{ct}_{\mathsf{PKE}}$, and $\mathsf{sk_R} = (\mathsf{hk}, \mathsf{dk}_{\mathsf{PKE}}, \mathsf{sk}_{\mathsf{PEKS}})$. Compute $\mathsf{H} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}})$ and run $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{H}}(kw)$. Compute $\mathsf{td}_{\mathsf{der}\text{-}kw} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}, \mathsf{der}\text{-}kw)$, set $\mathsf{td}_{\mathsf{S},kw} = \mathsf{td}_{\mathsf{der}\text{-}kw}$, and output $\mathsf{td}_{\mathsf{S},kw}$.

$\mathsf{PAEKS.Test}(\mathsf{ct}_{\mathsf{PAEKS}}, \mathsf{td}_{\mathsf{S},kw})$**:** Parse $\mathsf{ct}_{\mathsf{PAEKS}} = \mathsf{ct}_{\mathsf{PEKS}}$ and $\mathsf{td}_{\mathsf{S},kw} = \mathsf{td}_{\mathsf{der}\text{-}kw}$. Output the result of $\mathsf{PEKS.Test}(\mathsf{ct}_{\mathsf{PEKS}}, \mathsf{td}_{kw})$.

If PKE is correct, then $\mathsf{ct}_{\mathsf{PKE}} \in \widetilde{\mathscr{L}}_{\mathsf{pk}_{\mathsf{PKE}}}$ where $\mathsf{ct}_{\mathsf{PKE}} = \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho_S)$. Moreover, if $\mathsf{WI\text{-}SPHF}$ is correct, then $\mathsf{pH} = \mathsf{H}$ holds with overwhelming probability where $\mathsf{pH} \leftarrow \mathsf{ProjHash}(\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}}, \rho_S)$ and $\mathsf{H} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}})$. Moreover, if PEKS is correct, then $\mathsf{PAEKS.Test}(\mathsf{ct}_{\mathsf{PAEKS}}, \mathsf{td}_{\mathsf{S},kw}) = 1$ where $\mathsf{ct}_{\mathsf{PAEKS}} = \mathsf{ct}_{\mathsf{PEKS}}$. Thus, the proposed construction provides correctness if these building blocks provide correctness.

5.2 Non Designated-Receiver Setting

As previously mentioned, if we assume a trusted setup, then we do not have to introduce the designated-receiver setting. Concretely, a setup algorithm takes a security parameter λ as input, runs $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$, erases $\mathsf{dk}_{\mathsf{PKE}}$, and outputs $\mathsf{pk}_{\mathsf{PKE}}$ only as a public parameter. However, it would be better to clarify who can know $\mathsf{dk}_{\mathsf{PKE}}$ since $\mathsf{dk}_{\mathsf{PKE}} = \mathsf{ltrap}$ can be used for breaking the underlying membership problem. In this sense, our setting is more desirable.

5.3 Feasibility of our Generic Construction

In our construction, we employ $\mathsf{WI\text{-}SPHF}$, PKE, PRF, and PEKS as building blocks. Any PRF can be employed, e.g., HMAC-SHA-256. Since PEKS can be generically constructed from anonymous IBE [1], and lattice-based IBE is basically anonymous, our generic construction yields post-quantum PAEKS. For example, we can employ the Yamada IBE scheme [38] or the Jager-Kurek-Niehues IBE scheme [23]. We can also employ the Gentry-Peikert-Vaikuntanathan (GPV) IBE scheme [20] because Katsumata et al. [24] showed that the GPV IBE scheme is secure in the quantum random oracle model. Alternatively, Behnia et al. [4] gave implementations of lattice-based PEKS schemes when Agrawal-Boneh-Boyen IBE [2] or Ducas-Lyubashevsky-Prest IBE [15] are employed as the underlying anonymous IBE scheme. These PEKS schemes may be attractive in terms of efficiency.

The last piece is $\mathsf{WI\text{-}SPHF}$. Since Benhamouda et al. [6] proposed a $\mathsf{WI\text{-}SPHF}$ for a lattice-based IND-CPA PKE scheme, our construction is feasible. We remark that the Benhamouda et al. construction requires a super-polynomial modulus q to get statistical correctness. At the expense of correctness error, we can employ the Li-Wang word-independent approximate SPHF [26] that provides approximate correctness. In the Li-Wang construction, a word is a ciphertext and plaintext pair (c, m) of the labeled CCA1 Micciancio-Peikert PKE scheme, and the witness is the randomness used for encryption. Because the hash value is a bit, we need to amplify the length by sampling several independent hash keys and concatenating the output of all the corresponding hash results (Lemma B.2 [6]). We remark that no CCA security is required in the proposed construction although Li and Wang employed a CCA1 PKE scheme.

5.4 Security Analysis

Theorem 1. *Our construction is computationally consistent if* PEKS *is computationally consistent,* PKE *is IND-CPA secure,* $\mathsf{WI\text{-}SPHF}$ *provides (approximate) correctness and adaptive smoothness, and* PRF *is pseudo-random.*

Proof Overview. Since the underlying PEKS scheme PEKS is computationally consistent, what we need to show is as follows: for $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{pH}^{(i)}}(kw)$ (computed in the $\mathsf{PAEKS.Enc}$ algorithm in the experiment) and $\mathsf{der}\text{-}kw' \leftarrow F_{\mathsf{H}^{(j)}}(kw')$ (computed in the $\mathsf{PAEKS.Trapdoor}$ algorithm in the experiment), $\mathsf{der}\text{-}kw \neq \mathsf{der}\text{-}kw'$ holds if $(kw, i) \neq (kw', j)$, where $\mathsf{pH}^{(i)} \leftarrow \mathsf{ProjHash}(\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}^{(i)}_{\mathsf{PKE}}, \rho^{(i)}_S)$ and $\mathsf{H}^{(j)} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}^{(j)}_{\mathsf{PKE}})$. Our proof strategy is explained as follows. First, the computation of $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{pH}^{(i)}}(kw)$ is switched to $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{H}^{(i)}}(kw)$ where $\mathsf{H}^{(i)} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}^{(i)}_{\mathsf{PKE}})$. Owing to the (approximate) correctness of $\mathsf{WI\text{-}SPHF}$, this modification is indistinguishable. Next, $\mathsf{pk}_{\mathsf{S}[i]} = \mathsf{ct}^{(i)}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho^{(i)}_S)$ computed in the $\mathsf{PAEKS.KG_S}$ algorithm is switched to $\mathsf{ct}^{(i)}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 1; \rho^{(i)}_S)$. In the SPHF-context, a word $\mathsf{ct}^{(i)}_{\mathsf{PKE}} \in \widetilde{\mathscr{L}}_{\mathsf{pk}_{\mathsf{PKE}}}$ is switched to $\mathsf{ct}^{(i)}_{\mathsf{PKE}} \in \mathscr{X}_{\mathsf{pk}_{\mathsf{PKE}}} \setminus \mathscr{L}_{\mathsf{pk}_{\mathsf{PKE}}}$. Owing to the IND-CPA security of PKE, this modification is indistinguishable. We remark that, in this game the simulator does not know $\rho^{(i)}_S$ since it is the randomness of the challenge ciphertext of PKE. However, due to the modification in the previous game, the simulator computes $\mathsf{H}^{(i)}$ without using $\rho^{(i)}_S$. Then we can utilize adaptive smoothness, i.e., for computing $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{H}^{(i)}}(kw)$, $\mathsf{H}^{(i)}$ is randomly chosen from $\{0,1\}^{\nu}$. $\mathsf{pk}_{\mathsf{S}[j]} = \mathsf{ct}^{(j)}_{\mathsf{PKE}}$ is also switched as well, and for $\mathsf{der}\text{-}kw' \leftarrow F_{\mathsf{H}^{(j)}}(kw')$, $\mathsf{H}^{(j)}$ is randomly chosen from $\{0,1\}^{\nu}$ (if $i = j$, then two values are the same). Then, PRF keys $\mathsf{H}^{(i)}$ and $\mathsf{H}^{(j)}$ are random. Thus, we can replace $F_{\mathsf{H}^{(i)}}$ and $F_{\mathsf{H}^{(j)}}$ with random functions owing to the pseudo-randomness of PRF. If $i = j$, then $kw \neq kw'$. Then, $\mathsf{der}\text{-}kw$ and $\mathsf{der}\text{-}kw'$ are randomly chosen, respectively, and $\mathsf{der}\text{-}kw \neq \mathsf{der}\text{-}kw'$ holds with overwhelming probability. If $kw = kw'$, then $i \neq j$. Then, $F_{\mathsf{H}^{(i)}}$ and $F_{\mathsf{H}^{(j)}}$ are different random functions. Thus $\mathsf{der}\text{-}kw$ and $\mathsf{der}\text{-}kw'$ are randomly chosen, respectively (even the input is the same), and $\mathsf{der}\text{-}kw \neq \mathsf{der}\text{-}kw'$ holds with overwhelming probability. Finally, owing to the computational consistency of PEKS, our construction provides computational consistency.

Proof. We prove the theorem via sequences of games $\mathsf{Game}_0, \ldots, \mathsf{Game}_9$. Let W_k denote an event that $\mathcal{A}$ wins in Game_k ($k \in \{0, 1, \ldots, 9\}$). Without loss of generality, $(i, j) \in \{0, 1\} \times \{0, 1\}$ is determined before generating $(\mathsf{pk_R}, \mathsf{pk_{S[0]}}, \mathsf{pk_{S[1]}})$ with the probability 1/4.

Game_0**:** This game is the same as the original computational consistency game in Definition 14.

Game_1**:** This game is the same as Game_0 except that the computation of $\mathsf{ct_{PAEKS}} \leftarrow \mathsf{PAEKS.Enc}(\mathsf{pk_R}, \mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_{S[\mathit{i}]}}, kw)$ is changed as follows. Let $\mathsf{sk_R} = (\mathsf{hk}, \mathsf{dk_{PKE}}, \mathsf{sk_{PEKS}})$. Instead of computing $\mathsf{pH}^{(i)} \leftarrow \mathsf{ProjHash}(\mathsf{hp}, \mathsf{pk_{PKE}}, \mathsf{ct}^{(i)}_{\mathsf{PKE}}, \rho^{(i)}_S)$, compute $\mathsf{H}^{(i)} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk_{PKE}}, \mathsf{ct}^{(i)}_{\mathsf{PKE}})$ and run $\mathsf{der}\text{-}kw \leftarrow F_{\mathsf{H}^{(i)}}(kw)$. $\mathcal{B}$ computes $\mathsf{ct_{PAEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk_{PEKS}}, \mathsf{der}\text{-}kw)$. If WI-SPHF provides (approximate) correctness, then $\mathsf{pH}^{(i)} = \mathsf{H}^{(i)}$ holds with overwhelming probability. Thus, $|\Pr[W_0] - \Pr[W_1]|$ is negligible in the security parameter λ.

Game_2**:** This game is the same as Game_1 except that $(\mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_{S[\mathit{i}]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$ is changed as $\mathsf{pk_{S[\mathit{i}]}} = \mathsf{ct}^{(i)}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk_{PKE}}, 1; \rho^{(i)}_S)$. We show that $|\Pr[W_1] - \Pr[W_2]|$ is negligible if PKE is IND-CPA secure as follows. Let $\mathcal{C}$ be the challenger of the IND-CPA game. We construct an algorithm $\mathcal{B}$ as follows. $\mathcal{C}$ runs $(\mathsf{pk_{PKE}}, \mathsf{dk_{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$ and sends $\mathsf{pk_{PKE}}$ to $\mathcal{B}$. $\mathcal{B}$ runs $\mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{pk_{PKE}})$, $\mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{pk_{PKE}})$, and $(\mathsf{pk_{PEKS}}, \mathsf{sk_{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$, and sets $\mathsf{pk_R} = (\mathsf{hp}, \mathsf{pk_{PKE}}, \mathsf{pk_{PEKS}})$. For running $(\mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_{S[\mathit{i}]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$, $\mathcal{B}$ sends $(M^*_0, M^*_1) = (0, 1)$ to $\mathcal{C}$. $\mathcal{C}$ chooses $b \xleftarrow{\$} \{0, 1\}$, computes $\mathsf{ct}^*_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk_{PKE}}, M^*_b)$, and sends $\mathsf{ct}^*_{\mathsf{PKE}}$ to $\mathcal{B}$. $\mathcal{B}$ runs $(\mathsf{pk_{S[\bar{\mathit{i}}]}}, \mathsf{sk_{S[\bar{\mathit{i}}]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$ as usual, where $\bar{i} = 0$ if $i = 1$ and $\bar{i} = 1$ if $i = 0$, and sends $(\mathsf{pk_R}, \mathsf{pk_{S[0]}}, \mathsf{pk_{S[1]}})$ to $\mathcal{A}$. We note that $\rho^{(i)}_S$ is not used for running the $\mathsf{PAEKS.Enc}$ algorithm due to the modification in Game_1. If $b = 0$, then $\mathcal{B}$ simulates Game_1 and if $b = 1$, $\mathcal{B}$ simulates Game_2. Thus, $|\Pr[W_1] - \Pr[W_2]| \leq \mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{B}}(\lambda)$ holds.

Game_3**:** This game is the same as Game_2 except that $\mathsf{H}^{(i)} \xleftarrow{\$} \{0, 1\}^\nu$. Since $\mathsf{ct}^{(i)}_{\mathsf{PKE}} \in \mathscr{X}_{\mathsf{pk_{PKE}}} \setminus \mathscr{L}_{\mathsf{pk_{PKE}}}$, $|\Pr[W_2] - \Pr[W_3]|$ is negligible if WI-SPHF provides adaptive smoothness.

Game_4**:** This game is the same as Game_3 except that $(\mathsf{pk_{S[\mathit{i}]}}, \mathsf{sk_{S[\mathit{i}]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$ is changed as $\mathsf{pk_{S[\mathit{i}]}} = \mathsf{ct}^{(i)}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk_{PKE}}, 0; \rho^{(i)}_S)$. As in Game_2, $|\Pr[W_3] - \Pr[W_4]| \leq \mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{B}}(\lambda)$ holds.

Game_5**:** This game is the same as Game_4 except that $(\mathsf{pk_{S[\mathit{j}]}}, \mathsf{sk_{S[\mathit{j}]}}) \leftarrow \mathsf{PAEKS.KG_S}(\mathsf{pk_R})$ is changed as $\mathsf{pk_{S[\mathit{j}]}} = \mathsf{ct}^{(j)}_{\mathsf{PKE}} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk_{PKE}}, 1; \rho^{(j)}_S)$. As in Game_2, $|\Pr[W_4] - \Pr[W_5]|$ is negligible if PKE is IND-CPA secure. If $i = j$, then skip this game.

Game_6**:** This game is the same as Game_5 except that $\mathsf{H}^{(j)} \xleftarrow{\$} \{0, 1\}^\nu$. As in Game_3, $|\Pr[W_5] - \Pr[W_6]|$ is negligible if WI-SPHF provides adaptive smoothness. If $i = j$, then skip this game.

Game_7**:** This game is the same as Game_6 except that $\mathsf{der}\text{-}kw \xleftarrow{\$} \mathcal{KS}$. We show that $|\Pr[W_6] - \Pr[W_7]|$ is negligible if PRF is pseudo-random as follows. Let $\mathcal{C}$ be the challenger of PRF. We construct an algorithm $\mathcal{B}$ as follows. $\mathcal{B}$ prepares all keys as in Game_6. When $\mathcal{B}$ prepares $\mathsf{der}\text{-}kw$, $\mathcal{B}$ sends kw to $\mathcal{C}$. Then, $\mathcal{C}$ returns either $F_K(kw)$ or a random R. In the former case, $\mathcal{B}$ simulates Game_6 (implicitly set $K = \mathsf{H}^{(i)}$), and in the latter case, $\mathcal{B}$ simulates Game_7. Thus, $|\Pr[W_6] - \Pr[W_7]| \leq \mathsf{Adv}^{\mathsf{pseudo\text{-}random}}_{\mathsf{PRF},\mathcal{B}}(\lambda)$ holds.

Game_8**:** This game is the same as Game_7 except that $\mathsf{der}\text{-}kw' \xleftarrow{\$} \mathcal{KS}$. As in Game_7, $|\Pr[W_7]-\Pr[W_8]|$ is negligible if PRF is pseudo-random.

Game_9**:** We show that the probability that $\mathcal{A}$ wins in this game is negligible as follows. Let $\mathcal{C}$ be the challenger of computational consistency of PEKS. We construct an algorithm $\mathcal{B}$ as follows. $\mathcal{C}$ runs $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$ and sends $\mathsf{pk}_{\mathsf{PEKS}}$ to $\mathcal{B}$. $\mathcal{B}$ prepares $(\mathsf{pk}_{\mathsf{R}}, \mathsf{pk}_{\mathsf{S}[i]}, \mathsf{pk}_{\mathsf{S}[j]})$ in accordance with the previous game, except that $\mathcal{B}$ contains $\mathsf{pk}_{\mathsf{PEKS}}$ to pk_{R}. When $\mathcal{A}$ sends $(kw, kw', i.j)$ to $\mathcal{B}$, $\mathcal{B}$ randomly chooses $\mathsf{der}\text{-}kw, \mathsf{der}\text{-}kw' \xleftarrow{\$} \mathcal{KS}$, and sends $(\mathsf{der}\text{-}kw, \mathsf{der}\text{-}kw')$ to $\mathcal{C}$. Here, the probability of $\mathsf{der}\text{-}kw = \mathsf{der}\text{-}kw'$ is $2^{-\nu}$, and is negligible. $\mathcal{C}$ runs $\mathsf{ct}_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{der}\text{-}kw)$ and $\mathsf{td}_{\mathsf{der}\text{-}kw'} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}, \mathsf{der}\text{-}kw')$. Since PEKS provides computational consistency, the probability that $\mathsf{PEKS.Test}(\mathsf{ct}_{\mathsf{PEKS}}, \mathsf{td}_{\mathsf{der}\text{-}kw'}) = 1$ holds is negligible. This concludes the proof. □

Theorem 2. *Our construction is IND-CKA secure if* PEKS *is IND-CKA secure,* PKE *is IND-CPA secure,* $\mathsf{WI\text{-}SPHF}$ *provides (approximate) correctness and adaptive smoothness, and* PRF *is pseudo-random.*

Proof Overview. Basically, the theorem holds if PEKS is IND-CKA secure since a PAEKS ciphertext is a PEKS ciphertext in our construction. What we need to care is: $\mathcal{O}_T$ in the PAEKS experiment needs to be simulated by $\mathcal{O}_T$ in the PEKS experiment. We remark that if $\mathcal{A}$ sends kw to $\mathcal{O}_T$ in the PAEKS experiment, the keyword to be input to $\mathcal{O}_T$ in the PEKS experiment is not kw, is $\mathsf{der}\text{-}kw$. That is, if there exists $kw \notin \{kw_0^*, kw_1^*\}$ such that its extended keyword $\mathsf{der}\text{-}kw \in \{\mathsf{der}\text{-}kw_0^*, \mathsf{der}\text{-}kw_1^*\}$ where $\mathsf{der}\text{-}kw_0^*$ and $\mathsf{der}\text{-}kw_1^*$ are extended keywords of kw_0^* and kw_1^*, respectively, then the simulation fails (because the challenge ciphertext is computed by $\mathsf{der}\text{-}kw_b^*$ for $b \xleftarrow{\$} \{0,1\}$). To exclude this case, we show that all extended keywords are random using the same strategy of the proof of computational consistency. First, $\mathsf{pH}^{(i)}$ is switched to $\mathsf{H}^{(i)}$ owing to the (approximate) correctness of $\mathsf{WI\text{-}SPHF}$. Second, $\mathsf{ct}_{\mathsf{PKE}}^{(i)}$ is switched to a ciphertext of 1 owing to the IND-CPA security of PKE. Third, $\mathsf{H}^{(i)}$ is switched to a random value owing to the adaptive smoothness of $\mathsf{WI\text{-}SPHF}$. For all $i \in [1,n]$, run the above procedures. Then, all extended keywords are randomly chosen because of the pseudo-randomness of PRF. Here, the maximum number of extended keywords is at most $q_C + q_T + 2$ where q_C is the number of encryption queries, q_T is the number of trapdoor queries, and 2 is for generating the challenge ciphertext, i.e,, for (kw_0^*, kw_1^*, i^*) that $\mathcal{A}$ declares, and two extended keywords $\mathsf{der}\text{-}kw_0^*$ and $\mathsf{der}\text{-}kw_1^*$ are defined. Finally, owing to the IND-CKA security of PEKS, our construction is IND-CKA secure. We remark that $\mathcal{O}_C$ in the PAEKS experiment can be simulated during extended keyword randomization phases as in the same strategy of the proof of computational consistency that also prepares a ciphertext. To simulate $\mathcal{O}_T$ during extended keyword randomization phases, the simulator runs $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$, and uses $\mathsf{sk}_{\mathsf{PEKS}}$ to compute a trapdoor.

Proof. We prove the theorem via sequences of games $\mathsf{Game}_0, \dots, \mathsf{Game}_6$. Let W_k denote an event that $\mathcal{A}$ wins in Game_k ($k \in \{0,6\}$) and $W_{k,i}$ denote an event that $\mathcal{A}$ wins in $\mathsf{Game}_{k,i}$ ($k \in \{1,2,3,4\}$, $i \in \{1,\dots,n\}$) and in $\mathsf{Game}_{5,j}$ ($j \in \{1,\dots,N\}$) where N is defined later. .

Game_0**:** This game is the same as the original IND-CKA game in Definition 15.

Next, we introduce subgames $\mathsf{Game}_{1,1}, \mathsf{Game}_{2,1}, \mathsf{Game}_{3,1}, \mathsf{Game}_{4,1}, \mathsf{Game}_{1,2}, , \dots \mathsf{Game}_{4,2}, \dots, \mathsf{Game}_{1,n}, \dots, \mathsf{Game}_{4,n}$.

$\mathsf{Game}_{1,i}$**:** Set $\mathsf{Game}_{4,0} = \mathsf{Game}_0$. We describe $\mathsf{Game}_{1,i}$ ($i \in [1,n]$) as follows. This game is the same as $\mathsf{Game}_{4,i-1}$ except that compute $\mathsf{H}^{(i)} \leftarrow \mathsf{Hash}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}}^{(i)})$ instead of computing

$\mathsf{pH}^{(i)} \leftarrow \mathsf{ProjHash}(\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{ct}_{\mathsf{PKE}}^{(i)}, \rho_S^{(i)})$. If WI-SPHF provides (approximate) correctness, then $\mathsf{pH}^{(i)} = \mathsf{H}^{(i)}$ holds with overwhelming probability. Thus, $|\Pr[W_{4,i-1}] - \Pr[W_{1,i}]|$ is negligible in the security parameter λ.

$\mathsf{Game}_{2,i}$: We describe $\mathsf{Game}_{2,i}$ $(i \in [1,n])$ as follows. This game is the same as $\mathsf{Game}_{1,i}$ except that $(\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}) \leftarrow \mathsf{PAEKS.KG}_{\mathsf{S}}(\mathsf{pk}_{\mathsf{R}})$ is changed as $\mathsf{pk}_{\mathsf{S}[i]} = \mathsf{ct}_{\mathsf{PKE}}^{(i)} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 1; \rho_S^{(i)})$. We show that $|\Pr[W_{1,i}] - \Pr[W_{2,i}]|$ is negligible if PKE is IND-CPA secure as follows. Let $\mathcal{C}$ be the challenger of the IND-CPA game. We construct an algorithm $\mathcal{B}$ as follows. $\mathcal{C}$ runs $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{dk}_{\mathsf{PKE}}) \leftarrow \mathsf{PKE.KeyGen}(1^\lambda)$ and sends $\mathsf{pk}_{\mathsf{PKE}}$ to $\mathcal{B}$. $\mathcal{B}$ runs $\mathsf{hk} \leftarrow \mathsf{HashKG}(\mathsf{pk}_{\mathsf{PKE}})$, $\mathsf{hp} \leftarrow \mathsf{ProjKG}(\mathsf{hk}, \mathsf{pk}_{\mathsf{PKE}})$, and $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$, and sets $\mathsf{pk}_{\mathsf{R}} = (\mathsf{hp}, \mathsf{pk}_{\mathsf{PKE}}, \mathsf{pk}_{\mathsf{PEKS}})$. $\mathcal{B}$ generates $(\mathsf{pk}_{\mathsf{S}[1]}, \mathsf{sk}_{\mathsf{S}[1]}), \dots, (\mathsf{pk}_{\mathsf{S}[i-1]}, \mathsf{sk}_{\mathsf{S}[i-1]})$ as in $\mathsf{Game}_{1,i}$. For running $(\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}) \leftarrow \mathsf{PAEKS.KG}_{\mathsf{S}}(\mathsf{pk}_{\mathsf{R}})$, $\mathcal{B}$ sends $(M_0^*, M_1^*) = (0,1)$ to $\mathcal{C}$. $\mathcal{C}$ chooses $b \xleftarrow{\$} \{0,1\}$, computes $\mathsf{ct}_{\mathsf{PKE}}^* \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, M_b^*)$, and sends $\mathsf{ct}_{\mathsf{PKE}}^*$ to $\mathcal{B}$. $\mathcal{B}$ generates $(\mathsf{pk}_{\mathsf{S}[i+1]}, \mathsf{sk}_{\mathsf{S}[i+1]}), \dots, (\mathsf{pk}_{\mathsf{S}[n]}, \mathsf{sk}_{\mathsf{S}[n]})$ as usual and sends $\mathsf{pk}_{\mathsf{R}}, \{\mathsf{pk}_{\mathsf{S}[i]}\}_{i\in[1,n]}$ to $\mathcal{A}$. For an encryption query (kw, i), B uses $\mathsf{H}^{(i)}$ and it does not require $\rho_S^{(i)}$. So, even if $\mathsf{pk}_{\mathsf{S}[i]} = \mathsf{ct}_{\mathsf{PKE}}^{(i)}$ is switched to a ciphertext of 1, $\mathcal{B}$ can answer the query. For a trapdoor query (kw, i), B generates a trapdoor $\mathsf{td}_{\mathsf{S}[i],kw}$ using $\mathsf{sk}_{\mathsf{PEKS}}$. If $b = 0$, then $\mathcal{B}$ simulates $\mathsf{Game}_{1,i}$ and if $b = 1$, $\mathcal{B}$ simulates $\mathsf{Game}_{2,i}$. Thus, $|\Pr[W_{1,i}] - \Pr[W_{2,i}]| \leq \mathsf{Adv}_{\mathsf{PKE},\mathcal{B}}^{\mathsf{IND\text{-}CPA}}(\lambda)$ holds.

$\mathsf{Game}_{3,i}$: We describe $\mathsf{Game}_{3,i}$ $(i \in [1,n])$ as follows. This game is the same as $\mathsf{Game}_{2,i}$ except that $\mathsf{H}^{(i)} \xleftarrow{\$} \{0,1\}^\nu$. $|\Pr[W_{2,i}] - \Pr[W_{3,i}]|$ is negligible if WI-SPHF provides adaptive smoothness.

$\mathsf{Game}_{4,i}$: We describe $\mathsf{Game}_{4,i}$ $(i \in [1,n])$ as follows. This game is the same as $\mathsf{Game}_{3,i}$ except that $(\mathsf{pk}_{\mathsf{S}[i]}, \mathsf{sk}_{\mathsf{S}[i]}) \leftarrow \mathsf{PAEKS.KG}_{\mathsf{S}}(\mathsf{pk}_{\mathsf{R}})$ is changed as $\mathsf{pk}_{\mathsf{S}[i]} = \mathsf{ct}_{\mathsf{PKE}}^{(i)} \leftarrow \mathsf{PKE.Enc}(\mathsf{pk}_{\mathsf{PKE}}, 0; \rho_S^{(i)})$. As in $\mathsf{Game}_{2,i}$, $|\Pr[W_{3,i}] - \Pr[W_{4,i}]|$ is negligible if PKE is IND-CPA secure.

Next, we introduce subgames $\mathsf{Game}_{5,0}, \dots, \mathsf{Game}_{5,N}$ where $N \leq q_C + q_T + 2$ is the number of extended keywords appeared in the game. Here, q_C is the number of encryption queries and q_T is the number of trapdoor queries.

$\mathsf{Game}_{5,i}$: Set $\mathsf{Game}_{5,0} = \mathsf{Game}_{4,n}$. Let $\{\mathsf{der}\text{-}kw_1, \dots, \mathsf{der}\text{-}kw_N\}$ be the set of distinct extended keywords. This game is the same as $\mathsf{Game}_{5,i-1}$ except that $\mathsf{der}\text{-}kw_i \xleftarrow{\$} \mathcal{KS}$. We show that $|\Pr[W_{5,i-1}] - \Pr[W_{5,i}]|$ is negligible if PRF is pseudo-random. Let $\mathcal{C}$ be the challenger of PRF. We construct an algorithm $\mathcal{B}$ as follows. When $\mathcal{B}$ prepares $\mathsf{der}\text{-}kw_i$ for the i-th query (kw, j)[5] for some $j \in [1,n]$, $\mathcal{B}$ sends kw to $\mathcal{C}$. Then, $\mathcal{C}$ returns either $F_K(kw)$ or a random value. In the former case, $\mathcal{B}$ simulates $\mathsf{Game}_{4,i-1}$ (implicitly set $K = \mathsf{H}^{(j)}$), and in the latter case, $\mathcal{B}$ simulates $\mathsf{Game}_{4,i}$. Thus, $|\Pr[W_{5,i-1}] - \Pr[W_{5,i}]| \leq \mathsf{Adv}_{\mathsf{PRF},\mathcal{B}}^{\mathsf{pseudo\text{-}random}}(\lambda)$ holds.

Next, we show that the probability that $\mathcal{A}$ wins in the final game is negligible if PEKS is IND-CKA secure.

Game_6: Set $\mathsf{Game}_{5,N} = \mathsf{Game}_6$. Let $\mathcal{C}$ be the challenger of the IND-CKA game of PEKS. We construct an algorithm $\mathcal{B}$ as follows. $\mathcal{C}$ runs $(\mathsf{pk}_{\mathsf{PEKS}}, \mathsf{sk}_{\mathsf{PEKS}}) \leftarrow \mathsf{PEKS.KG}(1^\lambda)$ and sends $\mathsf{pk}_{\mathsf{PEKS}}$ to $\mathcal{B}$. $\mathcal{B}$ prepares $(\mathsf{pk}_{\mathsf{R}}, \mathsf{pk}_{\mathsf{S}[i]}, \mathsf{pk}_{\mathsf{S}[j]})$ in accordance with the previous game, except that $\mathcal{B}$ contains $\mathsf{pk}_{\mathsf{PEKS}}$ to pk_{R}. When $\mathcal{A}$ sends an encryption query (kw, i) to $\mathcal{B}$, if (kw, i) has been queried as either an encryption query or a trapdoor query, then $\mathcal{B}$ uses $\mathsf{der}\text{-}kw$

[5] It may be an encryption query, a trapdoor query, or the challenge query. In the challenge query, $j = i^*$.

that was previously generated. Otherwise, $\mathcal{B}$ randomly chooses $\mathsf{der}\text{-}kw \xleftarrow{\$} \mathcal{KS}$, preserves $(\mathsf{der}\text{-}kw, kw, i)$, runs $\mathsf{ct_{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk_{PEKS}}, \mathsf{der}\text{-}kw)$, and returns $\mathsf{ct_{PEKS}}$ to $\mathcal{A}$. When $\mathcal{A}$ sends a trapdoor query (kw, i) to $\mathcal{B}$, if (kw, i) has been queried as either an encryption query or a trapdoor query, then $\mathcal{B}$ uses $\mathsf{der}\text{-}kw$ that was previously generated. Otherwise, $\mathcal{B}$ randomly chooses $\mathsf{der}\text{-}kw \xleftarrow{\$} \mathcal{KS}$, preserves $(\mathsf{der}\text{-}kw, kw, i)$, and sends $\mathsf{der}\text{-}kw$ to $\mathcal{C}$ as a trapdoor query. $\mathcal{C}$ runs $\mathsf{td}_{\mathsf{der}\text{-}kw} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk_{PEKS}}, \mathsf{sk_{PEKS}}, \mathsf{der}\text{-}kw)$ and sends $\mathsf{td}_{\mathsf{der}\text{-}kw}$ to $\mathcal{B}$. $\mathcal{B}$ returns $\mathsf{td}_{\mathsf{der}\text{-}kw}$ to $\mathcal{A}$. In the challenge phase, $\mathcal{A}$ sends (kw_0^*, kw_1^*, i^*) to $\mathcal{B}$. If (kw_0^*, i^*) (resp. (kw_1^*, i^*)) has appeared in an encryption query, then $\mathcal{B}$ uses $\mathsf{der}\text{-}kw_0^*$ (resp. $\mathsf{der}\text{-}kw_1^*$) that was previously generated. Otherwise, $\mathcal{B}$ randomly chooses $\mathsf{der}\text{-}kw_0^* \xleftarrow{\$} \mathcal{KS}$ (resp. $\mathsf{der}\text{-}kw_1^* \xleftarrow{\$} \mathcal{KS}$). $\mathcal{B}$ sends $(\mathsf{der}\text{-}kw_0^*, \mathsf{der}\text{-}kw_1^*)$ to $\mathcal{C}$ as the challenge query. We remark that the probability that either $\mathsf{der}\text{-}kw_0^*$ or $\mathsf{der}\text{-}kw_1^*$ has been appeared as a trapdoor query is negligible since all extended keywords are randomly chosen from $\{0,1\}^\nu$. $\mathcal{C}$ randomly selects $b \xleftarrow{\$} \{0,1\}$, runs $\mathsf{ct}^*_{\mathsf{PEKS}} \leftarrow \mathsf{PEKS.Enc}(\mathsf{pk_{PEKS}}, \mathsf{der}\text{-}kw_b^*)$, and returns $\mathsf{ct}^*_{\mathsf{PEKS}}$ to $\mathcal{B}$. $\mathcal{B}$ returns $\mathsf{ct}^*_{\mathsf{PEKS}}$ to $\mathcal{A}$. $\mathcal{B}$ simulates $\mathcal{O}_C$ and $\mathcal{O}_T$ as in the previous stage, Finally, $\mathcal{A}$ outputs b'. $\mathcal{B}$ also outputs the same b'. Then, $\mathcal{B}$ can break IND-CKA security of PEKS with the same advantage of $\mathcal{A}$. That is, $\Pr[W_6] \leq \mathsf{Adv}^{\mathsf{IND\text{-}CKA}}_{\mathsf{PEKS},\mathcal{A}}(\lambda, n)$ holds. This concludes the proof. □

Theorem 3. *Our construction is IND-IKGA secure if* PKE *is IND-CPA secure,* $\mathsf{WI\text{-}SPHF}$ *provides (approximate) correctness and adaptive smoothness, and* PRF *is pseudo-random.*

Proof Overview. The underlying PEKS scheme does not provide trapdoor privacy, i.e., from $\mathsf{td}_{\mathsf{der}\text{-}kw} \leftarrow \mathsf{PEKS.Trapdoor}(\mathsf{pk_{PEKS}}, \mathsf{sk_{PEKS}}, \mathsf{der}\text{-}kw)$, information of $\mathsf{der}\text{-}kw$ is leaked. So, the $\mathsf{PEKS.Trapdoor}$ algorithm is meaningless for hiding information of $\mathsf{der}\text{-}kw$. More concretely, from the challenge trapdoor $\mathsf{td}^*_{\mathsf{S}[i^*], kw_b^*} = \mathsf{td}_{\mathsf{der}\text{-}kw_b^*}$ where $\mathsf{der}\text{-}kw_b^*$ is the extended keyword for kw_b^*, information of $\mathsf{der}\text{-}kw_b^*$ is leaked. Thus, for providing IND-IKGA security, we need to guarantee that information of kw is not leaked from the corresponding extended keyword $\mathsf{der}\text{-}kw$. Fortunately, we have already showed that $\mathsf{der}\text{-}kw_b^*$ is indistinguishable from random. More concretely, in Game_6 of the proof of Theorem 2, the distribution of $\mathsf{der}\text{-}kw_b^*$ is identical when $b = 0$ and $b = 1$, respectively. This is sufficient to provide IND-IKGA security.

Proof. The game descriptions is the same as those of the proof of Theorem 2. In Game_6, $\mathsf{der}\text{-}kw_0^*$ and $\mathsf{der}\text{-}kw_1^*$ are randomly chosen which are independent from kw_0^* and kw_1^*. That is, the distribution of $\mathsf{der}\text{-}kw_b^*$ is identical when $b = 0$ and $b = 1$, respectively. This concludes the proof. □

We remark that adaptive smoothness does not directly guarantee unlinkability that hides information whether two hash values are computed by the same input or not. Thus, the adversary $\mathcal{A}$ above may distinguish $b = 0$ or $b = 1$ if $\mathcal{A}$ obtains a trapdoor for (kw_0^*, i^*) (or (kw_1^*, i^*)), and obtains (information of) $\mathsf{der}\text{-}kw_0^*$ (or $\mathsf{der}\text{-}kw_1^*$). Thus, we have restricted that $\mathcal{A}$ is allowed to issue a trapdoor query $(kw, i) \notin \{(kw_0^*, i^*), (kw_1^*, i^*)\}$ in the definition of IND-IKGA.

6 Conclusion and Future Work

In this paper, we proposed a generic construction of PAEKS from $\mathsf{WI\text{-}SPHF}$, PKE, PRF, and PEKS. Our construction is not only more efficient than the Liu et al. construction but also provides stronger security than that of Liu et al.

Cheng and Meng [11] claimed that their constructions employ totally different methodology from Liu et al. [29] and ours [16], which is explained that: *instead of using the shared key calculated* *by SPHF, the sender and receiver achieve keyword authentication by using their own secret key to sample a set of short vectors related to the keyword.* Proposing a generic construction based on the Cheng-Meng methodology is an interesting future work. Cheng and Meng proposed two lattice-based PAEKS schemes: one is selectively secure in the standard model and other one is adaptively secure in the random oracle model. Thus, proposing an adaptively secure lattice-based PAEKS scheme secure in the standard model or secure in the quantum random oracle model is also an interesting topic. Because Cheng and Meng did not consider consistency in the multi-sender setting, a trapdoor associated with a sender may work against ciphertexts generated by the secret key of another sender when the same keyword is associated. Thus, considering consistency in the multi-sender setting with the Cheng-Meng methodology is also an interesting future work.

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